z-TRANSFORM PROPERTIES

The index-domain signal is $x[n]$ for $-\infty < n < \infty$; and the z-transform is:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} \qquad \Longleftrightarrow \qquad x[n] = \frac{1}{2\pi j} \oint X(z)\, z^{n}\, \frac{dz}{z}$$

The ROC is the set of complex numbers z where the z-transform sum converges.

Signal: $x[n] \quad -\infty < n < \infty$	z-Transform: $X(z)$	Region of Convergence
$x[n]$, $x_1[n]$ and $x_2[n]$	$X(z)$, $X_1(z)$ and $X_2(z)$	R_x, R_1 and R_2
$a x_1[n] + b x_2[n]$	$a X_1(z) + b X_2(z)$	contains $R_1 \cap R_2$
$x[n - n_\circ]$	$z^{-n_\circ} X(z)$	R_x except for the possible addition or deletion of $z = 0$ or $z = \infty$
$z_\circ^n\, x[n]$	$X(z/z_\circ)$	$\lvert z_\circ \rvert\, R_x$
$n\, x[n]$	$-z\, \dfrac{dX(z)}{dz}$	R_x except for the possible addition or deletion of $z = 0$ or $z = \infty$
$x^*[n]$	$X^*(z^*)$	R_x
$\Re e\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	contains R_x
$\Im m\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	contains R_x
$x[-n]$	$X(1/z)$	$1/R_x = \{z : z^{-1} \in R_x\}$
$x_1[n] * x_2[n]$	$X_1(z) \cdot X_2(z)$	contains $R_1 \cap R_2$
$x_1[n] \cdot x_2[n]$	$\frac{1}{2\pi j} \oint X_1(v)\, X_2(z/v)\, \frac{dv}{v}$	contains $R_1 R_2$
Parseval's Theorem:	$\sum_{n=-\infty}^{\infty} x_1[n]\, x_2^*[n] = \frac{1}{2\pi j} \oint X_1(v)\, X_2^*(1/v^*)\, \frac{dv}{v}$	
Initial Value Theorem:	$x[n] = 0$, for $n < 0 \quad \Longrightarrow \quad \lim_{z \to \infty} X(z) = x[0]$	