z-TRANSFORM PAIRS

The index-domain signal is $x[n]$ for $-\infty < n < \infty$; and the z-transform is:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} \qquad \Longleftrightarrow \qquad x[n] = \frac{1}{2\pi j} \oint X(z)\, z^{n}\, \frac{dz}{z}$$

The ROC is the set of complex numbers z where the z-transform sum converges.

Signal: $x[n]$ $\quad -\infty < n < \infty$	z-Transform: $X(z)$	Region of Convergence
$\delta[n]$	1	All z
$\delta[n-n_\circ]$	$z^{-n_\circ}$	$\lvert z\rvert > 0$, if $n_\circ > 0$ $\lvert z\rvert < \infty$, if $n_\circ < 0$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n\, u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n\, u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$n\, a^n\, u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-n\, a^n\, u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$(n+1)\, a^n\, u[n]$	$\frac{1}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$[\cos\omega_\circ n]\, u[n]$	$\frac{1-[\cos\omega_\circ]z^{-1}}{1-2[\cos\omega_\circ]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$[\sin\omega_\circ n]\, u[n]$	$\frac{[\sin\omega_\circ]z^{-1}}{1-2[\cos\omega_\circ]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$[r^n\cos\omega_\circ n]\, u[n]$	$\frac{1-[r\cos\omega_\circ]z^{-1}}{1-2r[\cos\omega_\circ]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > \lvert r\rvert$
$[r^n\sin\omega_\circ n]\, u[n]$	$\frac{[r\sin\omega_\circ]z^{-1}}{1-2r[\cos\omega_\circ]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > \lvert r\rvert$
$x[n] = \begin{cases} a^n, & 0 \le n < L \\ 0, & \text{otherwise} \end{cases}$	$\frac{1-a^Lz^{-L}}{1-az^{-1}}$	$\lvert z\rvert > 0$