Gradiente, Divergencia, Rotacional, Laplaciano

Coordenadas Cartesianas $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$

$$f = f(x,y,z) \qquad ; \qquad \overrightarrow{G} = G_x \hat{\imath} + G_y \hat{\jmath} + G_z \hat{k}$$

$$\overrightarrow{\nabla} f = \frac{\partial f}{\partial x}\,\hat{\imath} + \frac{\partial f}{\partial y}\,\hat{\jmath} + \frac{\partial f}{\partial z}\,\hat{k}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{G} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z}$$

$$\overrightarrow{\nabla} \times \overrightarrow{G} = \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right)\hat{k}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Coordenadas Cilíndricas $(\boldsymbol{\rho}, \boldsymbol{\phi}, \boldsymbol{z})$

$$f = f(\rho,\phi,z) \qquad ; \qquad \overrightarrow{G} = G_\rho \hat{\rho} + G_\phi \hat{\phi} + G_z \hat{z}$$

$$\overrightarrow{\nabla} f = \frac{\partial f}{\partial \rho}\,\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\,\hat{\phi} + \frac{\partial f}{\partial z}\,\hat{z}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{G} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\, G_\rho) + \frac{1}{\rho}\frac{\partial G_\phi}{\partial \phi} + \frac{\partial G_z}{\partial z}$$

$$\overrightarrow{\nabla} \times \overrightarrow{G} = \left(\frac{1}{\rho}\frac{\partial G_z}{\partial \phi} - \frac{\partial G_\phi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial G_\rho}{\partial z} - \frac{\partial G_z}{\partial \rho}\right)\hat{\phi} + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho\, G_\phi) - \frac{\partial G_\rho}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Coordenadas Esféricas $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\phi})$

$$f = f(r, \theta, \phi) \qquad ; \qquad \overrightarrow{G} = G_r \hat{r} + G_\theta \hat{\theta} + G_\phi \hat{\phi}$$

$$\overrightarrow{\nabla} f = \frac{\partial f}{\partial r}\, \hat{r} + \frac{1}{r}\, \frac{\partial f}{\partial \theta}\, \hat{\theta} + \frac{1}{r \operatorname{sen} \theta}\, \frac{\partial f}{\partial \phi}\, \hat{\phi}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{G} = \frac{1}{r^2}\, \frac{\partial}{\partial r} \left(r^2\, G_r\right) + \frac{1}{r \operatorname{sen} \theta}\, \frac{\partial}{\partial \theta} \left(\operatorname{sen} \theta\, G_\theta\right) + \frac{1}{r \operatorname{sen} \theta}\, \frac{\partial G_\phi}{\partial \phi}$$

$$\begin{aligned} \overrightarrow{\nabla} \times \overrightarrow{G} &= \frac{1}{r \operatorname{sen} \theta} \left[\frac{\partial}{\partial \theta} \left(\operatorname{sen} \theta\, G_\phi\right) - \frac{\partial G_\theta}{\partial \phi}\right] \hat{r} \\ &+ \left[\frac{1}{r \operatorname{sen} \theta}\, \frac{\partial G_r}{\partial \phi} - \frac{1}{r}\, \frac{\partial}{\partial r} \left(r\, G_\phi\right)\right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} \left(r\, G_\theta\right) - \frac{\partial G_r}{\partial \theta}\right] \hat{\phi} \end{aligned}$$

$$\nabla^2 f = \frac{1}{r^2}\, \frac{\partial}{\partial r} \left(r^2\, \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \operatorname{sen} \theta}\, \frac{\partial}{\partial \theta} \left(\operatorname{sen} \theta\, \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \operatorname{sen}^2 \theta}\, \frac{\partial^2 f}{\partial \phi^2}$$