$Y' = AY$ (I)

sea $e^{AX} := \sum_{n=0}^{\infty} \frac{A^n x^n}{n!}$ veamos que:

PROP) $\forall c_1, \dots, c_n \in \mathbb{R}$ $\forall v_1, \dots, v_n \in \mathbb{R}^n$ linealmente independientes

$\cancel{Y(x)}$ $Y(x) := c_1 e^{Ax} v_1 + \dots + c_n e^{Ax} v_n$ es solucion de (I)

dem) $Y'(x) = c_1 A e^{Ax} v_1 + \dots + c_n A e^{Ax} v_n = A(c_1 e^{Ax} v_1 + \dots + c_n e^{Ax} v_n) =$

$A Y(x) \Rightarrow Y(x)$ es solucion de (I) ✓

$\boxed{\text{PROBLEMA: } e^{AX} = ???}$

~~sea~~

SOLUCION: sea $A = P^{-1} J A$ $\cancel{J = P^{-1} A P}$, su forma de Jordan $\Rightarrow J = D + M$

donde $D = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$ y M nilpotente.

$DM = MD \Rightarrow e^{Jx} = e^{(D+M)x} = e^{Dx} e^{Mx}$.

• $e^{Dx} = \sum_{k=0}^{\infty} \frac{D^k x^k}{k!} = \sum_{k=0}^{\infty} \frac{\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}^k x^k}{k!} = \sum_{k=0}^{\infty} \frac{\begin{pmatrix} \lambda_1^k & & \\ & \ddots & \\ & & \lambda_n^k \end{pmatrix} x^k}{k!} = \sum_{k=0}^{\infty} \begin{pmatrix} \lambda_1^k x^k/k! & & \\ & \ddots & \\ & & \lambda_n^k x^k/k! \end{pmatrix} = \begin{pmatrix} e^{\lambda_1 x} & & \\ & \ddots & \\ & & e^{\lambda_n x} \end{pmatrix}$

$\Rightarrow e^{Dx}$ calculable!!!

• M nilpotente $\Rightarrow \exists r \in \mathbb{N}$ tq $M^r = 0 \Rightarrow \sum_{k=0}^{\infty} \frac{M^k x^k}{k!} = \sum_{k=0}^{r} \frac{M^k x^k}{k!} \Rightarrow$

e^{Mx} calculable!!!

CONCLUSION: $e^{Jx} = e^{Dx} e^{Mx}$ es calculable!!!

DE ESTA FORMA: $e^{Ax} = \sum_{k=0}^{\infty} \frac{A^k x^k}{k!} = \sum_{k=0}^{\infty} \frac{(P^{-1} J P)^k x^k}{k!} = \sum_{k=0}^{\infty} \frac{(P^{-1} J^k P) x^k}{k!} =$

$P^{-1} \left(\sum_{k=0}^{\infty} \frac{J^k x^k}{k!} \right) P = P^{-1} e^{Jx} P \Rightarrow \boxed{e^{Ax} = P^{-1} e^{Jx} P}$

~~$A = P^{-1} J P \Rightarrow J = A(\{v_1 \dots v_n\})$~~

sea $v_1 \dots v_n$ la base de la forma de Jordan $\Rightarrow J = A(\{v_1 \dots v_n\}) \Rightarrow$

si $J = \begin{pmatrix} j_{11} & j_{1n} \\ j_{n1} & j_{nn} \end{pmatrix}$ entonces $Av_\lambda = \sum_{k=1}^{n} j_{k\lambda} v_k \quad \forall \lambda = 1 \dots n$

$e^{Ax} = P^{-1} e^{Jx} P \Rightarrow e^{Jx} = e^{Ax}(\{v_1 \dots v_n\}) \Rightarrow$

si $e^{Jx} = \begin{pmatrix} a_{11}(x) & \dots & a_{n1}(x) \\ & & \vdots \\ a_{n1}(x) & \dots & a_{nn}(x) \end{pmatrix}$ entonces $e^{Ax} v_\lambda = \sum_{k=1}^{n} a_{k\lambda}(x) v_k \quad \forall \lambda = 1 \dots n$

$$\Rightarrow Y(x) = C_1 \underbrace{e^{Ax} v_1}_{= \sum_{k=1}^{n} a_{k1}(x) v_k} + \dots + C_n \underbrace{e^{Ax} v_n}_{= \sum_{k=1}^{n} a_{kn} v_k}$$