Heat transfer in Creeping motion. or Stokes flow

Temperature and heat transfer in Stokes flow can be computed from the linearized energy equation

$$\rho C_p . U \frac{dT}{dx} \simeq K \nabla^2 T \quad (\text{eq. } 1)$$

wher U is the constant freestream velocity. This relation is entirely uncoupled from the velocity distribution (Stokes- Oseen). Typical boundary conditions would be known temperatures at the wall T_w, and in the stream T_∞

Non dimensionalization of eq. 1. would yield a single parameter, the Peclet number

$$\boxed{P_e = P_r . R_e = \rho . U . C_p . \frac{L}{K}}$$ Peclet number

$$P_r = \frac{\nu}{\alpha} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}} = \frac{C_p . \mu}{K} \quad \left(\begin{array}{c}\text{Prandtl}\\ \text{number}\end{array}\right)$$

$$P_r = \frac{\text{Kinematic viscocity}}{\text{Thermal diffusivity}} = \frac{\mu}{\rho} . \frac{1}{\frac{K}{\rho C_p}} = \frac{C_p . \mu}{K} .$$

$$P_r . \begin{cases} \text{low . . conductive transfer strong} \\ \text{High - convective transfer strong} \end{cases}$$

In heat transfer problems, the Prandtl numbers controls the relative thickness of the momentum and thermal boundary layers, so Pr is small, the heat diffuses very quickly compared to the velocity (momentum). For liquid metals the thickness of the thermal boundary layer is bigger than the velocity boundary layer.

$$Pe = \frac{\text{heat transport by convection}}{\text{heat transport by conduction}} = \frac{\rho^*_* C_{p*} U_* (T_1 - T_0)/l}{k_* \frac{(T_1 - T_0)}{l^2}} \quad (2)$$

ρ: density

C_p: heat capacity

U: Free stream velocity ~~kinematic viscosity~~.

k: thermal coefficient

l: characteristic length.

$$\boxed{Pe = \rho . U . C_p . \frac{L}{k}}$$

From equation (2) we obtain the Pe. number expression.

We define. Nusselt number. as.

$$\overline{Nu}_m = \frac{\bar{q}_w . L}{k(T_w - T_\infty)} \quad (3)$$

and Num. would vary only with the Peclet number and geometry.

* The solution for flow past a sphere $(L = 2R)$ was given by Tomotika.

$$Nu_{m\,sphere} = 2.0 + 0.5\, Pr.\, Re + O(Pr^2.\, Re^2) + \dots \quad (5)$$

The first term is Stokes term (2.0) and the second term is the Orseen corrections.

* The solution for flow past a circular cylinder. $(L = 2R)$. The mean heat transfer is given by

$$Nu_{m\,cylinder} = B - \frac{Pr^2.\, Re^2 (16 + B^2)}{12} \quad (6)$$

where $B = \dfrac{2}{\ln\left(\dfrac{8}{Pr\, Re}\right) - \Gamma}$

where $\Gamma = 0.577$.

A better formula was suggested by Kramers (1946).

$$Nu_m = 0.42\, Pr^{0.2} + 0.57\, Pr^{1/3}.\, Re_D^{1/2} \quad (7)$$

(7) is valid for $0.1 < Re_D < 10^4$

Note: Eq. (7) is used to design the hot-wire anemometer, a fundamental instrument for the study of turbulence.

Ex. Lubricating oil at 20°C with $\rho = 890 \frac{Kg}{m^3}$ $\mu = 0.8$ Pa.S., $K = 0.15 \frac{W}{m\text{-}K}$ and $C_p = 1800 \frac{J}{Kg\text{-}K}$ is to be cooled by flowing at an average velocity of 2m/s through a 3cm diameter whose walls are at 10 °C. Estimate the heat loss $\left(\frac{W}{m^2}\right)$ at $x = 10$ Cm.

Solution

Calculate $P_r = \frac{C_p . \mu}{K}$ and $Re_D = \frac{\rho . U . D}{\mu}$

$$P_r = \frac{1800 \frac{J}{Kg\text{-}K} \times 0.8 \frac{Kg.m}{m^2 s^2} . s}{0.15 \frac{W}{m\text{-}K}} = 9600$$

$$Re_D = \frac{890 \frac{Kg}{m^3} \times 2 \times \frac{M}{S} \times 0.03 m}{0.8 \frac{Kg}{m\text{-}s}} = 66.75$$

We calculate $\bar{N}u_m$ for Cylinder.

$$\bar{N}u_m = 0.42 \times P_r^{0.2} + 0.57 \times P_r^{1/3} \times Re_D^{1/2}$$

$$\bar{N}u_m = 101.5637$$

We know that

$$\overline{Nu}_m = \frac{\bar{q}_w L}{K(T_w - T_\infty)} \Rightarrow \bar{q}_w = \frac{\overline{Nu}_m \cdot K(T_w - T_\infty)}{L}$$

where $T_w = 283\,K$ and $T_\infty = 293\,K$; $L = 0.1\,m$.

$$\hat{q}_w = \frac{101.56 \times 0.15\,(283 - 293)}{0.1}\left(\frac{W}{m\text{-}K} \times \frac{K}{m}\right) = \frac{W}{m^2}$$

$\hat{q}_w = 1523.4.\ \frac{W}{m^2}$