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TDSÑ

Apuntes de clase

Apuntes y exámenes ETSIT UPM

Si alguna vez estos apuntes te sirvieron de ayuda, piensa que tus apuntes pueden ayudar a muchas otras personas.

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TDSÑ

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Ej 2.7: Indique si estas secuencias son periódicas.

a) $x[n] = e^{j\pi n/6}$

b) $x[n] = e^{j3\pi n/4}$

c) $x[n] = \dfrac{\operatorname{sen}\left(\frac{\pi n}{5}\right)}{n\pi}$

d) $x[n] = e^{j\pi n/\sqrt{2}}$

será periódica si $x[n] = x[n+N]$

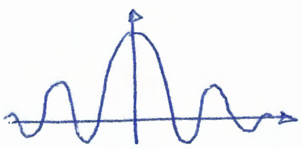
Ej: $\cos(\omega_0 n) = \cos(\omega_0(n+N)) = \cos(\omega_0 n + \omega_0 N)$

$\cos(\omega_0 n) = \cos(\omega_0 n + \omega_0 N) \Leftrightarrow \omega_0 N = 2\pi k$

luego: $N = \dfrac{2\pi k}{\omega_0}$; $k, N \in \mathbb{N}$

a) $N = \dfrac{2\pi k}{\pi/6} = 12k$; si $k=1 \Rightarrow N = 12$ (periódica con $N=12$)

b) $N = \dfrac{2\pi k}{3\pi/4} = \dfrac{8}{3}k$; si $k=3 \Rightarrow N = 8$ (periódica con $N=8$)

c)  No periódica

d) $N = \dfrac{2\pi k}{\pi/\sqrt{2}} = 2\sqrt{2}k$; $\nexists\, k \in \mathbb{N}$ para $N \in \mathbb{N} \Rightarrow$ No periódica

Ej: 2.21:

a) $x[n-2]$

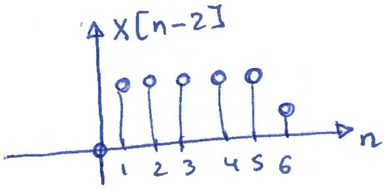
b) $x[4-n]$

c) $x[2n]$, $x[2n-1]$, $x[2n-2]$

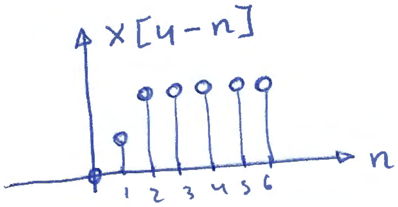
d) $x[n] \cdot u[2-n]$

e) $x[n-1] \cdot \delta[n-3]$

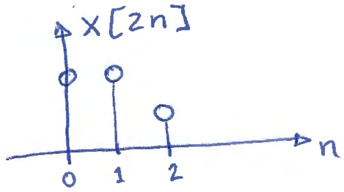
a)

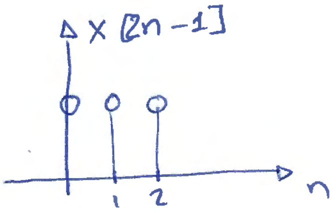


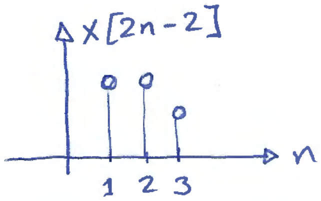
b)



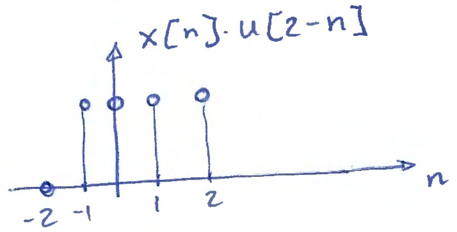
c)



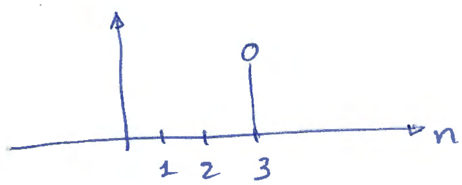




d)



e)



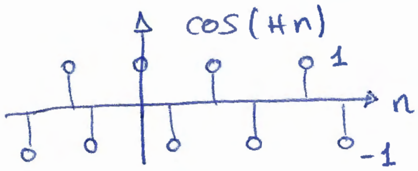
Ej 2.23: Definición de Sistemas:

a) $T\{x[n]\} = y[n] = \cos(\pi n) \cdot x[n]$

b) $y[n] = x[n^2]$

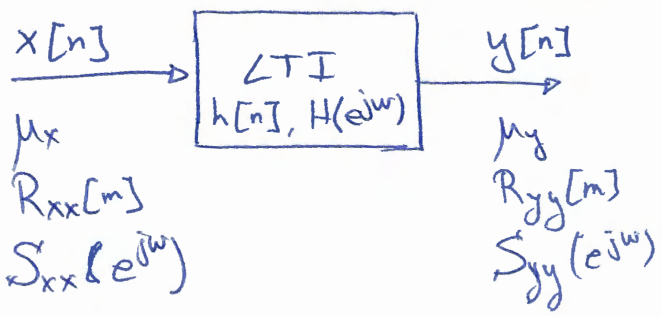
c) $y[n] = x[n] \cdot \sum_{k=0}^{\infty} \delta[n-k]$

d) $y[n] = \sum_{k=n-1}^{\infty} x[k]$



	a	b	c	d
memoria	sin	con	sin	con
causal	si	no	si	no
estable	si	si	si	no

Relación E/S de los proc. est. en sist. LTI:

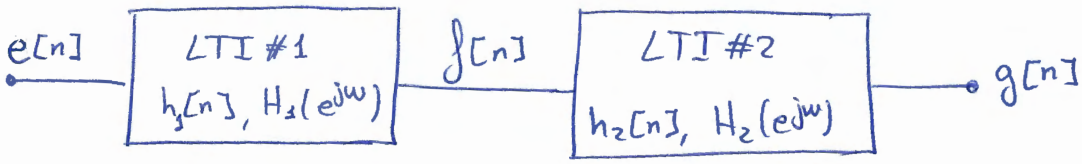


$$\begin{cases} \mu_y = \mu_x \cdot H(e^{j0}) \\ R_{yy}[m] = R_{xx}[m] * C_{hh}[m] \\ \quad C_{hh}[m] = h[m] * h[-m] \\ S_{yy}(e^{jw}) = S_{xx}(e^{jw}) \cdot |H(e^{jw})|^2 \end{cases}$$

$$R_{xy}[m] = h[m] * R_{xx}[m]$$

$$S_{xy}(e^{jw}) = H(e^{jw}) \cdot S_{xx}(e^{jw})$$

Ej 2.98:



x[n] proceso ruido blanco de media nula y potencia σ_e^2

$\#1: \; f[n] = e[n] - e[n-1]$

$\#2: \; H_2(e^{jw}) = \begin{cases} 1 & |w| \le w_c \\ 0 & w_c < |w| \le \pi \end{cases}$

$\mu_e = 0$

$R_{ee}[m] = \sigma_e^2 \cdot \delta[m]$

$S_{ee}(e^{jw}) = \sigma_e^2 \;\; \forall w$

a) Determine y dibuje $S_{ff}(e^{jw})$

b) $R_{ff}[m]$

c) Determine y dibuje $S_{gg}(e^{jw})$

d) Expresión para σ_g^2

---

a) $S_{ff}(e^{jw}) = S_{ee}(e^{jw}) \cdot |H(e^{jw})|^2$

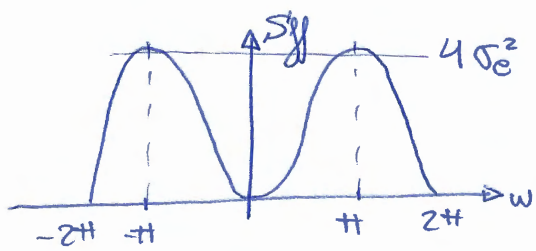
si $e[n] = \delta[n] \Rightarrow f[n] = h[n] = \delta[n] - \delta[n-1]$

$|H(e^{jw})|^2 = |1 - e^{-jw}|^2 = |1 - \cos(w) + j\,\mathrm{sen}(w)|^2 =$

$= (1-\cos(w))^2 + \mathrm{sen}^2(w) = 1^2 + \cos^2(w) - 2\cos(w) + \mathrm{sen}^2(w)$

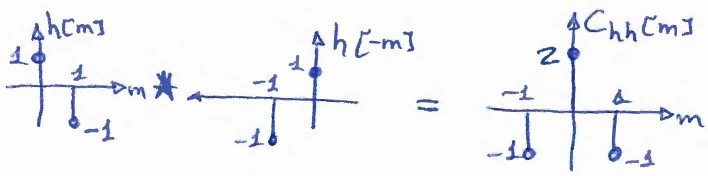
$= 2 - 2\cos(w) = 2(1 - \cos(w))$

$S_{ff}(e^{jw}) = \sigma_e^2 \cdot 2 \cdot (1 - \cos(w))$



b) $R_{ff}[m] = R_{ee}[m] * C_{hh}[m]$

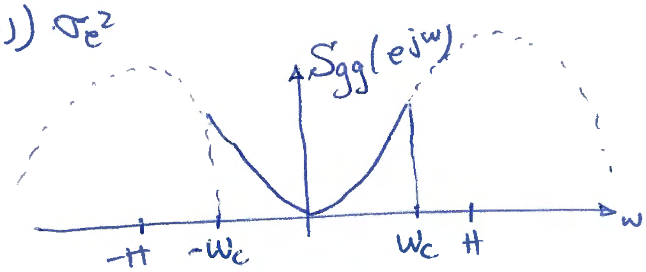
$C_{hh}[m] = h[m] * h[-m]$



$R_{ff}[m] = \sigma_e^2 \cdot \left[2\delta[m] - \delta[m-1] - \delta[m+1]\right]$

c) $S_{gg}(e^{jw}) = S_{ff}(e^{jw}) \cdot |H(e^{jw})|^2 = \begin{cases} S_{ff}(e^{jw}) & |w| \le w_c \\ 0 & w_c < |w| \le \pi \end{cases}$

siendo $S_{ff}(e^{jw}) = 2(1-\cos(w))\,\sigma_e^2$



d) $\sigma_g^2 = P = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{gg}(e^{jw})\,dw = \frac{1}{2\pi}\int_{-w_c}^{w_c} 2(1-\cos(w))\,\sigma_e^2\,dw =$

$= \frac{1}{\pi}\sigma_e^2 \int_{-w_c}^{w_c} (1-\cos(w))\,dw = \frac{2}{\pi}\sigma_e^2 \left[w - \text{sen}(w)\right]\Big|_0^{w_c} =$

$= \frac{2}{\pi}\sigma_e^2 (w_c - \text{sen}(w_c))$

Ej 2.11: LTI

$H(e^{jw}) = \dfrac{1 - e^{-2jw}}{1 + \frac{1}{2}e^{-j4w}}$, $x[n] = \text{sen}\left(\frac{\pi n}{4}\right)$, $y[n]$?

opc1:

$x[n] = \text{sen}\left(\frac{\pi n}{4}\right) = \frac{1}{2j}\left[e^{j\frac{\pi n}{4}} - e^{-j\frac{\pi n}{4}}\right]$

$X(e^{jw}) = \frac{\pi}{j}\left[\delta\left(w - \frac{\pi}{4}\right) - \delta\left(w + \frac{\pi}{4}\right)\right]$

$y[n] = TF^{-1}(X \cdot H)$

opc2:

$y[n] = H(e^{jw})\Big|_{w=\frac{\pi}{4}} \cdot \frac{1}{2j} e^{j\frac{\pi n}{4}} + H(e^{jw})\Big|_{w=-\frac{\pi}{4}} \cdot \frac{-1}{2j} e^{-j\frac{\pi n}{4}}$

$\rightarrow H(e^{jw})\Big|_{w=\frac{\pi}{4}} = \dfrac{1 - e^{-j\frac{\pi}{2}}}{1 + \frac{1}{2}e^{-j\pi}} = \dfrac{1+j}{1/2} = 2\sqrt{2}\, e^{j\pi/4}$

$\rightarrow H(e^{jw})\Big|_{w=-\frac{\pi}{4}} = \dfrac{1 - e^{j\pi/2}}{1 + \frac{1}{2}e^{+j\pi}} = \dfrac{1-j}{1/2} = 2\sqrt{2}\, e^{-j\pi/4}$

$y[n] = 2\sqrt{2}\,\frac{1}{2j}\, e^{j\left(\frac{\pi n}{4} + \pi/4\right)} - 2\sqrt{2}\,\frac{1}{2j}\, e^{-j\left(\frac{\pi n}{4} + \frac{\pi}{4}\right)} = 2\sqrt{2}\ \text{sen}\left(\frac{\pi n}{4} + \frac{\pi}{4}\right)$

Tema 2: Muestreo

PARTE 1: Muestreo ideal

1. Análisis del muestreo ideal
2. Análisis de la reconstrucción ideal
3. Procesamiento discreto de señales continuas
4. Procesamiento continuo de secuencias

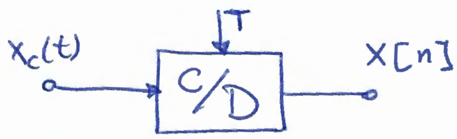
PARTE 2: Procesamiento multitasa

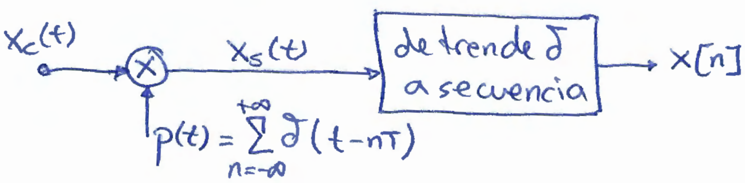
5. Reducción de la tasa por un factor entero
6. Incremento de la tasa por un factor entero
7. Cambio de la tasa por un factor racional

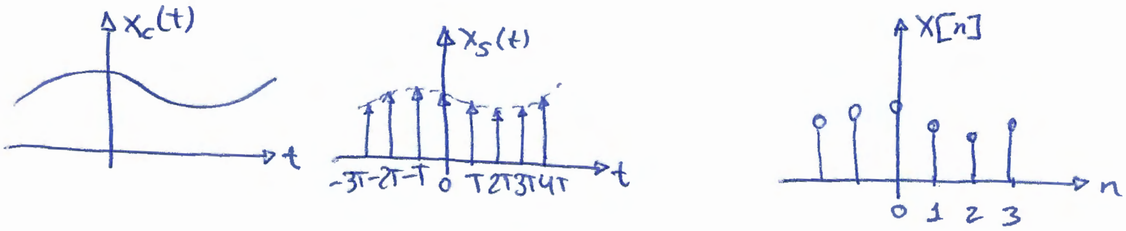
PARTE 3:

8. Aspectos prácticos de la conversión A/D, D/A
9. Sobremuestreo y conformación de ruido
   conversores Σ-Δ

1. Análisis del muestreo ideal







De señal continua a tren de δ:

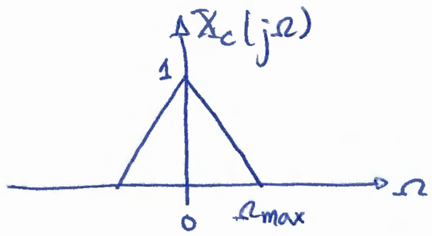
- En el tiempo:

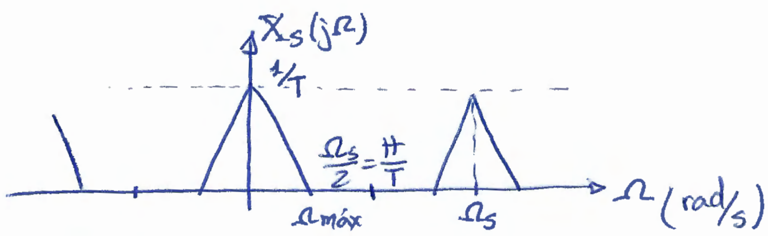
$$x_s(t) = x_c(t) \cdot \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

- En la frecuencia:

$$X_s(j\Omega) = \frac{1}{2\pi}\left[X_c(j\Omega) * \frac{2\pi}{T}\sum_{n=-\infty}^{+\infty}\delta\left(\Omega - \frac{2k\pi}{T}\right)\right] =$$

$$= \frac{1}{T}\sum_{n=-\infty}^{+\infty} X_c\left(j\left(\Omega - \frac{2k\pi}{T}\right)\right) \quad ; \quad \frac{2\pi}{T} = \Omega_s \; ; \quad \frac{1}{T} = f_s$$





Nyquist: $\Omega_s \geq 2\Omega_{máx}$

Si se cumple Nyquist:

$$X_s(j\Omega) = \frac{1}{T}X_c(j\Omega) \; ; \quad |\Omega| \leq \frac{\pi}{T} \quad ; \quad \frac{\Omega_s}{2} = \frac{\pi}{T}$$

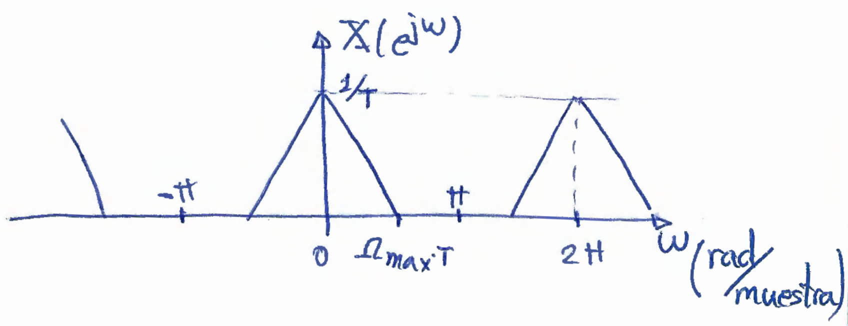
De tren de δ a secuencia:

- En el tiempo:

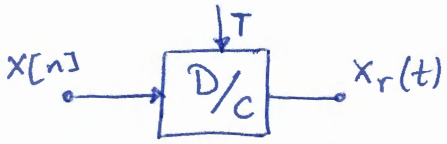
$$x[n] = x_s(nT) = x(nT) \quad ; \; t = nT \; ; \; t \in \mathbb{R}, \; n \in \mathbb{Z}$$

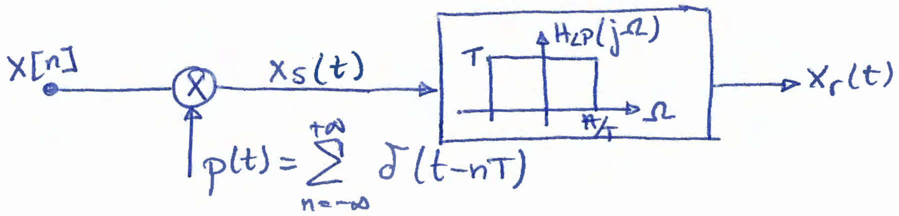
- En la frecuencia:

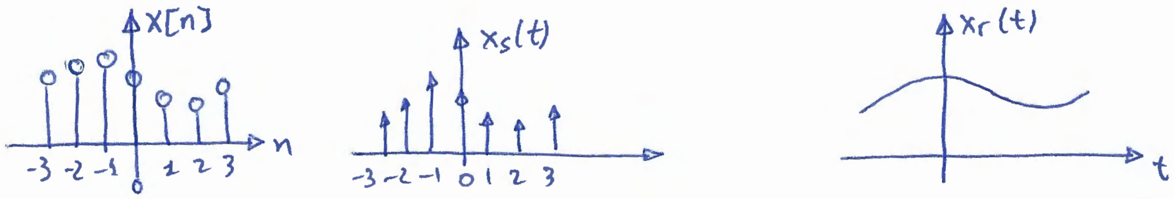
$$\omega = \Omega \cdot T$$



2. Análisis de la reconstrucción ideal







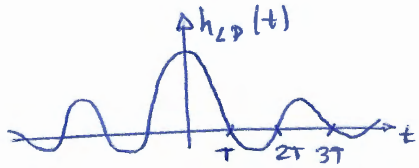
$$x_s(t) = \sum_{n=-\infty}^{+\infty} x[n]\cdot\delta(t-nT)$$

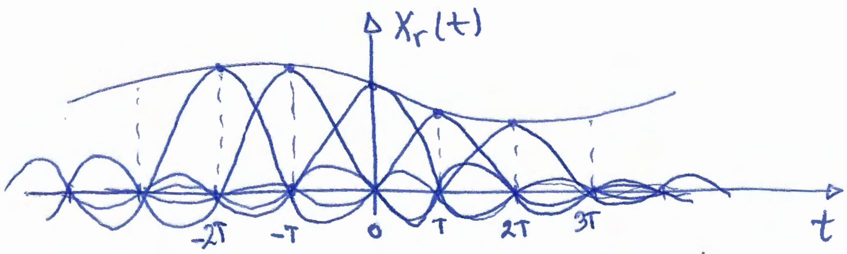
$$x_r(t) = x_s(t) * h_{LP}(t)$$

$$H_{LP}(j\Omega) = \begin{cases} T & |\Omega| \le \frac{\pi}{T} \\ 0 & \text{resto} \end{cases}$$

$$h_{LP}(t) = \frac{\text{sen}\left(\frac{\pi}{T}t\right)}{\frac{\pi}{T}t}$$

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x[n]\,\frac{\text{sen}\left(\frac{\pi}{T}(t-nT)\right)}{\frac{\pi}{T}(t-nT)}$$

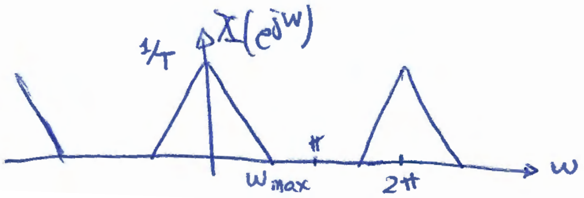


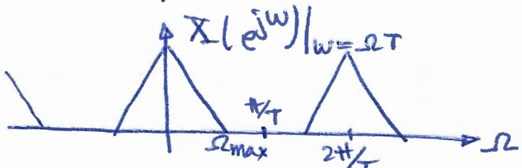


$$X(e^{jw}) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X_c\left(j\left(\frac{w}{T}-\frac{2\pi K}{T}\right)\right) \underset{\text{Nyquist}}{\overset{\text{si se cumple}}{=\!=\!=}} \frac{1}{T}X_c\left(j\frac{w}{T}\right),\ |w| \le \pi$$

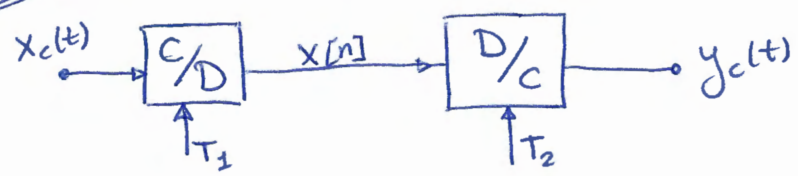
$$X_s(e^{j\Omega T}) = X_s(j\Omega) = X(e^{jw})\big|_{w=\Omega T} = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c\left(j\left(\Omega-\frac{2\pi k}{T}\right)\right)$$

$$X_r(j\Omega) = H_{LP}(j\Omega)\cdot X_s(e^{j\Omega T}) = \begin{cases} T\cdot X_s(j\Omega), & |\Omega| \le \pi/T \\ 0 & \text{resto} \end{cases}$$





Ej 4.29:



$X_c(j\Omega) = 0, \quad |\Omega| \geq \pi/T_1$ Exprese $y_c(t) = f(x_c(t))$

como $X(j\Omega) = 0$ para $|\Omega| \geqslant \pi/T_1 \Rightarrow$ cumple Nyquist

$$X(e^{j\omega}) = \frac{1}{T_1} X_c\left(j\frac{\omega}{T_1}\right) \; ; \; |\omega| \leq \pi$$

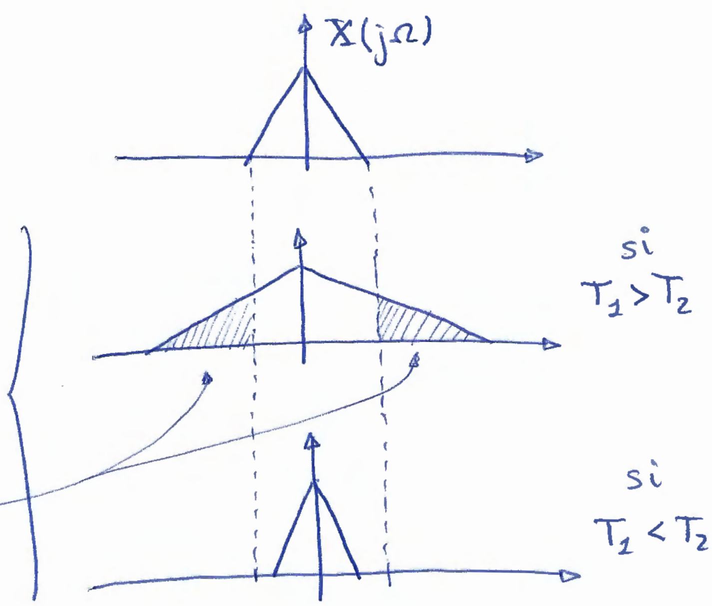
$$Y_c(j\Omega) = H_{LP}(j\Omega) \cdot X(e^{j\omega})\Big|_{\omega = \Omega T_2} = \begin{cases} T_2 \frac{1}{T_1} X_c\left(j\frac{\Omega T_2}{T_1}\right) & |\Omega| \leq \pi/T_2 \\ 0 & \text{resto} \end{cases}$$

$$= \frac{1}{T_1/T_2} \cdot X_c\left(j\frac{\Omega}{T_1/T_2}\right) \; , \; |\Omega| \leq \pi/T_2$$

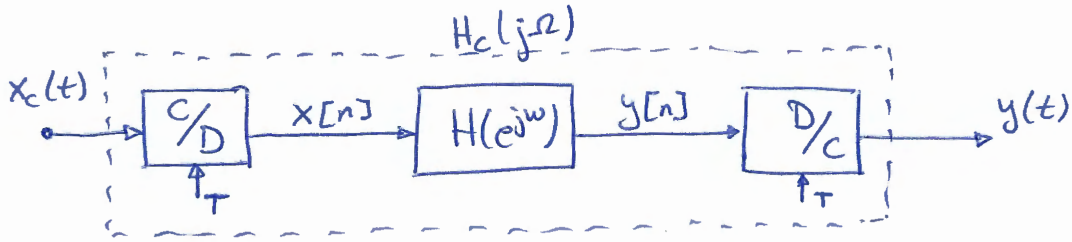
sabiendo que: $x(at) \xleftrightarrow{F} \frac{1}{|a|} X_c\left(j\frac{\Omega}{a}\right)$

$$\underline{y_c(t) = x_c\left(\frac{T_1}{T_2} t\right)}$$

Transformaciones NO lineales, pues se crean frecuencias nuevas



3. Procesamiento discreto de señales continuas

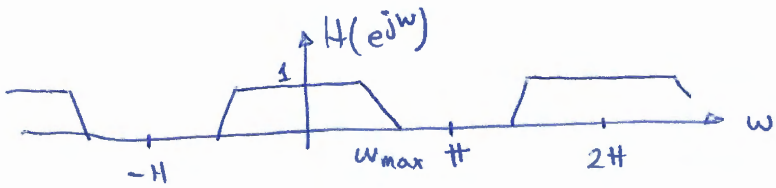


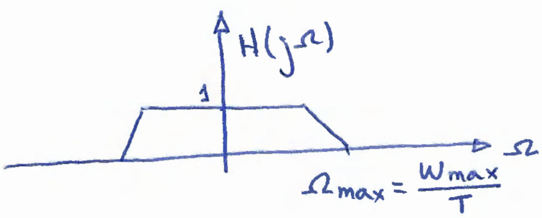
para ser un sistema LTI necesitamos:

→ frecuencia de muestreo y reconstrucción iguales

→ no exista aliasing

$$H_c(j\Omega) = \begin{cases} H(e^{j\Omega T}) & |\Omega| \leq \pi/T \\ 0 & \text{resto} \end{cases}$$





$$H(e^{jw}) = H_c\left(j\frac{w}{T}\right) \quad |w| \leq \frac{\pi}{T} \iff H_c(j\Omega) = 0 \quad |\Omega| \geq \frac{\pi}{T}$$

Demostración:

$$X(e^{jw}) = \frac{1}{T} X_c\left(j\frac{w}{T}\right), \quad |w| \leq \pi$$

$$Y(e^{jw}) = \frac{H(e^{jw})}{T} \cdot X_c\left(j\frac{w}{T}\right), \quad |w| \leq \pi$$

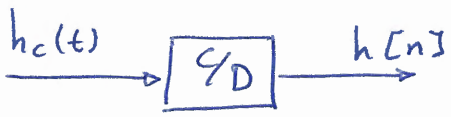
$$Y(j\Omega) = H_{LP}(j\Omega) \cdot Y(e^{j\Omega T}) = \begin{cases} T \cdot X_c(j\Omega), & |\Omega| \leq \pi/T \\ 0 & \text{resto} \end{cases}$$

$$H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)} = \begin{cases} H(e^{j\Omega T}), & |\Omega| \leq \pi/T \\ 0 & \text{resto} \end{cases}$$

Invarianza al impulso:

$$H(e^{jw}) = H_c\left(j\frac{w}{T}\right) \quad |w| \leq \pi$$

$$h[n] = T \cdot h_c(nT)$$

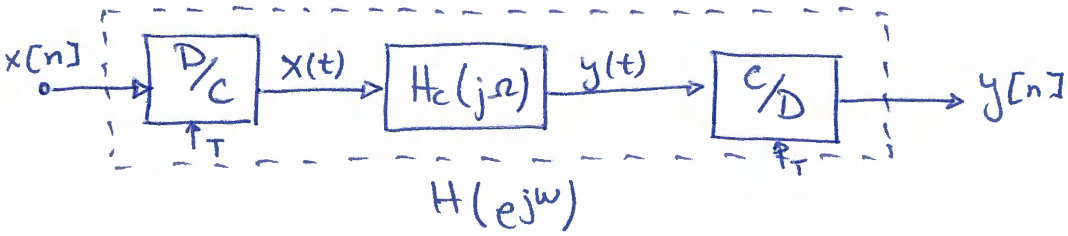


$$\begin{cases} h[n] = h_c(nT) \\ H(e^{jw}) = \frac{1}{T} H_c\left(j\frac{w}{T}\right) \quad |w| \leq \pi \end{cases}$$

proceso:

- → partimos de un filtro $H_c(s)$
- → transformada de Laplace: $\mathcal{L}^{-1}\{H(s)\} = h(t)$
- → muestreo de la respuesta al impulso: $h[n] = T \cdot h_c(nT)$
- → transformada $\mathcal{Z}$ ó $\mathcal{F}$: $H(e^{jw})$ ó $H(z)$

4. Procesamiento continuo de secuencias



para ser un sistema LTI necesitamos:

- → frecuencias de muestreo y reconstrucción iguales
- → en este procesamiento no existe aliasing posible

$$H_{eff}(j\Omega) = \begin{cases} H_c(j\Omega), & |\Omega| \leq \pi/T \\ 0, & \text{resto} \end{cases}$$

$$H(e^{jw}) = H_{eff}\left(j\frac{w}{T}\right), \ |w| \leq \pi$$

$$H(j\Omega) = \begin{cases} H(e^{j\Omega T}), & |\Omega| \leq \pi/T \\ 0, & \text{resto} \end{cases}$$

nos cubrimos las espaldas evitando un posible filtro con $Bw = \infty$ ya que la señal no excederá Ω_{max}

Ej 3.46: sistema LTI causal

si $x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}\, 2^n\, u[n-1]$

la Transf. Z de salida es: $Y(z) = \dfrac{1+z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})(1-2z^{-1})}$

a) $X(z)$?

b) ROC de $Y(z)$?

c) $h[n]$?

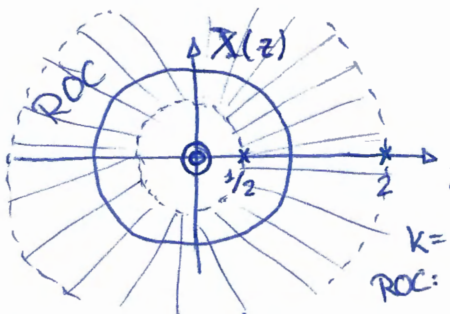
d) Es estable?

a) $X(z) = \underbrace{\dfrac{-1/3}{1-\frac{1}{2}z^{-1}}}_{|z|>1/2} + \underbrace{\dfrac{+4/3}{1-2z^{-1}}}_{|z|<2} = \dfrac{1}{1-\frac{5}{2}z^{-1}+z^{-2}} = \dfrac{1}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$

$\frac{1}{2} < |z| < 2$: ROC

polo $z=2$

polo $z=\frac{1}{2}$



$K=1$

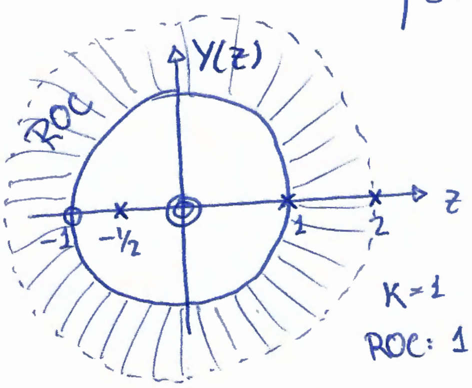
ROC: $\frac{1}{2}<|z|<2$

$X(z) = X(z)\cdot\dfrac{z^2}{z^2} = \dfrac{z^2}{(z-\frac{1}{2})\cdot(z-2)}$

2x ceros en $z=0$

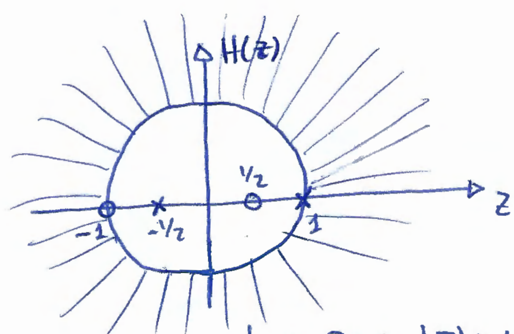
b) $Y(z) \longrightarrow$ polo: $z=1$, $z=-\frac{1}{2}$, $z=2$

$Y(z) = Y(z)\cdot\dfrac{z^2}{z^2} \Rightarrow$ cero: $z=0$ x2, $z=-1$



$K=1$

ROC: $1<|z|<2$



causal $\Rightarrow$ ROC: $|z|>1$

c) $H(z) = \dfrac{(1+z^{-1})(1-\frac{1}{2}z^{-1})}{(1-z^{-1})(1+\frac{1}{2}z^{-1})} = \dfrac{1+\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}}{1-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}} = 1 + \dfrac{z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})}$

división polinomios

$= 1 + \dfrac{A}{1-z^{-1}} + \dfrac{B}{1+\frac{1}{2}z^{-1}}$

$$A = \left.\frac{z^{-1}}{1+\frac{1}{2}z^{-1}}\right|_{z=1} = \frac{1}{3/2} = \frac{2}{3}$$

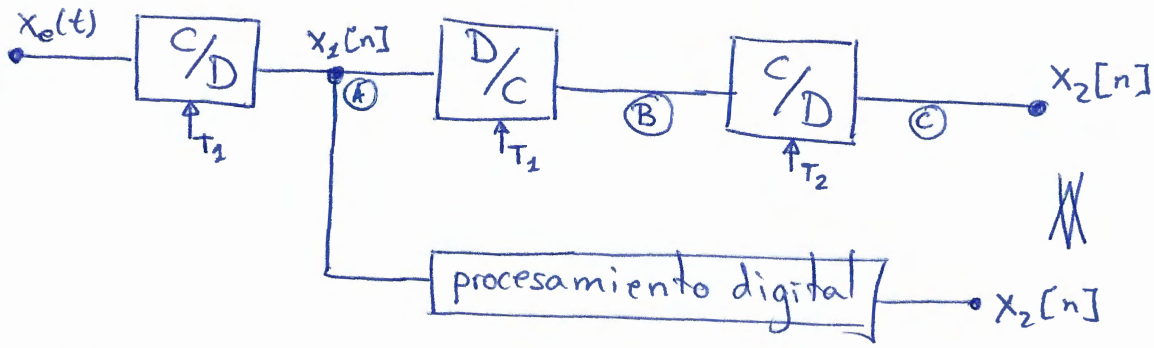
$$B = \left.\frac{z^{-1}}{1-z^{-1}}\right|_{z=-1/2} = \frac{-2}{1+2} = \frac{-2}{3}$$

$$H(z) = 1 + \frac{2/3}{1-z^{-1}} + \frac{-2/3}{1+\frac{1}{2}z^{-1}}$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \delta[n] + \frac{2}{3}u[n] + \frac{-2}{3}\left(-\frac{1}{2}\right)^n u[n]$$

d) No estable, pues existe un cero en el origen

5. Reducción de la tasa por un factor entero

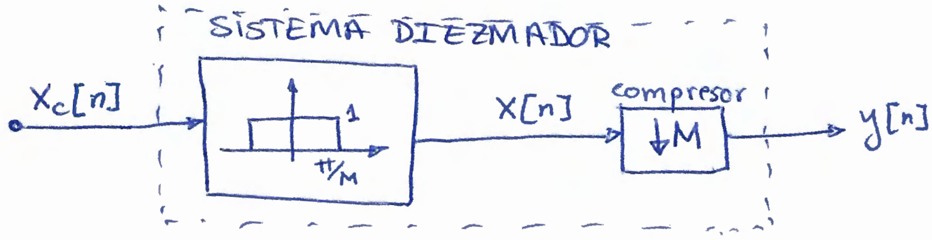


En A tenemos una señal $x_c(t)$ muestreada a $f_{s1} = \frac{1}{T_1}$: $x_1[n]$

En B tenemos la misma señal en tiempo continuo

En C tenemos la señal $x_c(t)$ muestreada a $f_{s2} = \frac{1}{T_2}$: $x_2[n]$

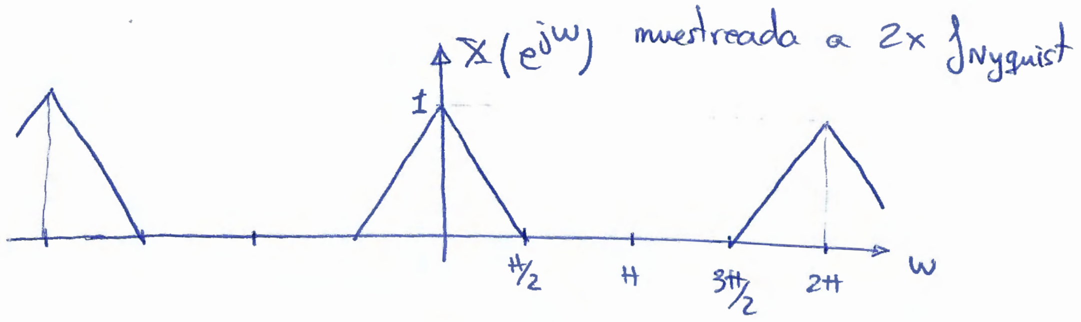
Hemos cambiado la frecuencia de muestreo

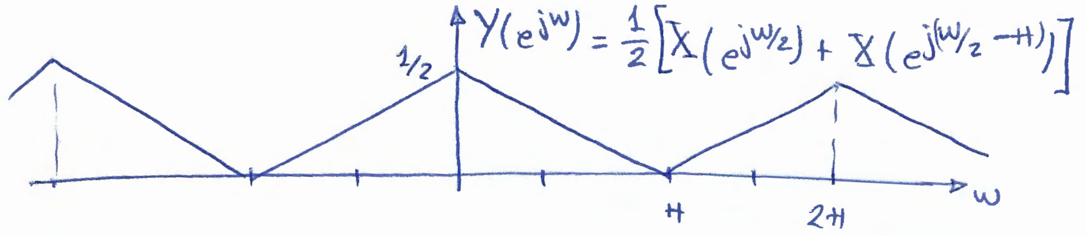


compresor:

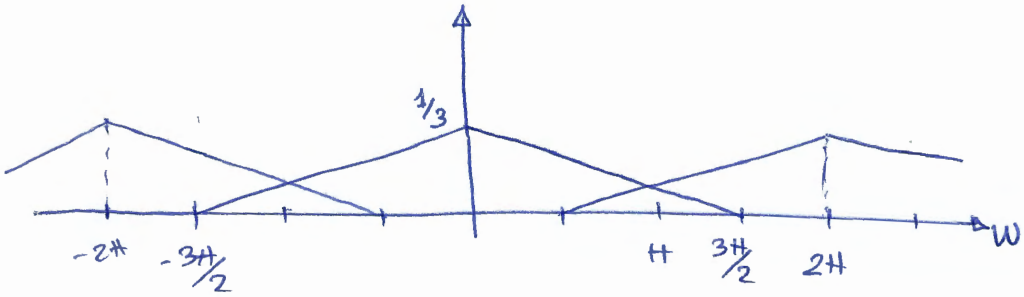
$$y[n] = x[Mn]$$

$$Y(e^{jw}) = \frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{j\left(\frac{w}{M} - \frac{2k\pi}{M}\right)}\right)$$



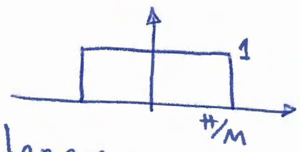


compresión con $M=2$



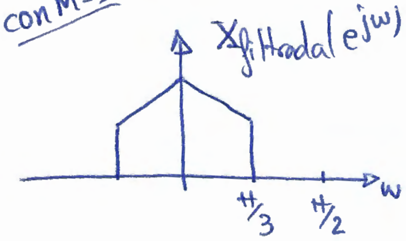
compresión con $M=3$

si pasamos $X(e^{jw})$ por el filtro

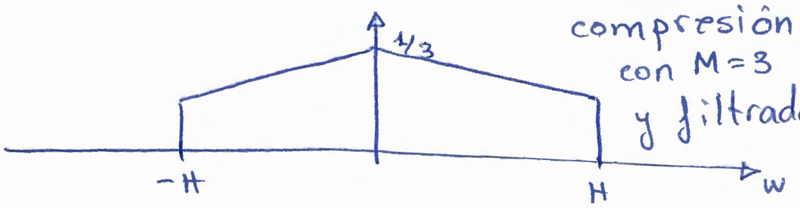


(llamado filtro antialiasing) obtenemos:

con $M=3$:



$\Rightarrow$

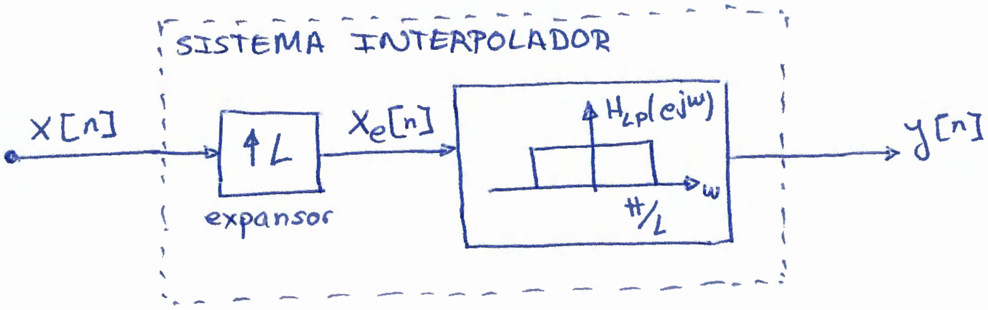


compresión con $M=3$ y filtrado

aunque hemos perdido frecuencias tras filtrar, nos hemos ahorrado el aliasing, no solapándose los espectros

Si no existiese aliasing, el filtro será totalmente transparente, luego lo pondremos siempre antes de un compresor.

6. Incremento de la tasa por un factor entero

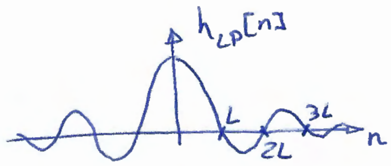


<u>expansor:</u> mete $L-1$ "0" entre muestra y muestra

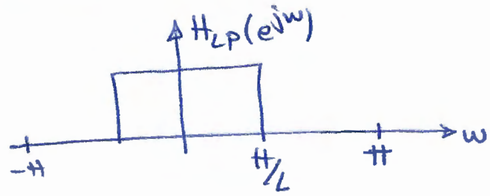
$$x_e[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-kL] = \begin{cases} x[n/L] & n = 0, \pm L, \pm 2L \dots \\ 0 & \text{resto} \end{cases}$$

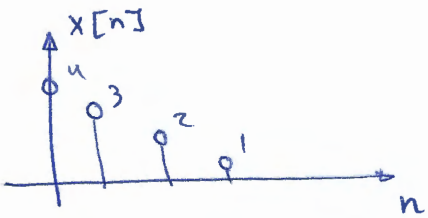
<u>filtro:</u>

$$h[n] = \frac{\text{sen}(\pi/L \cdot n)}{\pi/L \cdot n} \longrightarrow$$

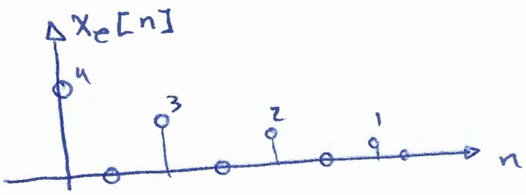


$$H(e^{jw}) = \begin{cases} 1 & |w| \leq \pi/L \\ 0 & \frac{\pi}{L} \leq |w| < \pi \end{cases} \longrightarrow$$

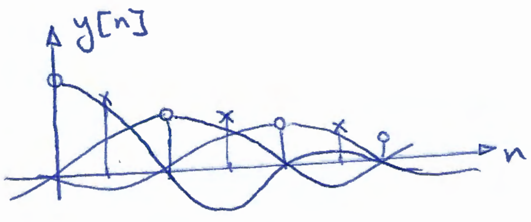




<u>con $L=2$</u>



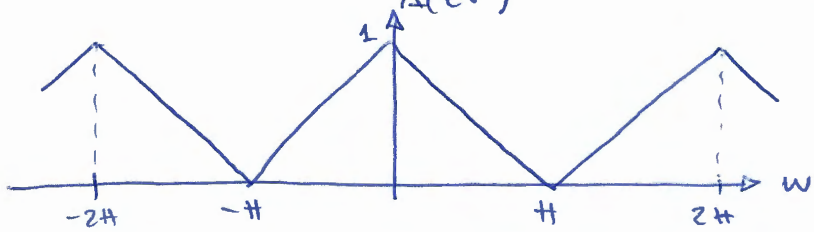
← tras el expansor:
se "meten" ceros entre muestras



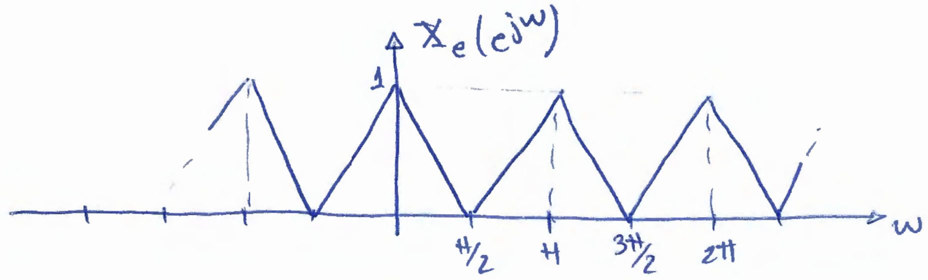
← al sumar las sincs, "nacen" valores entre las muestras, produciéndose la interpolación.

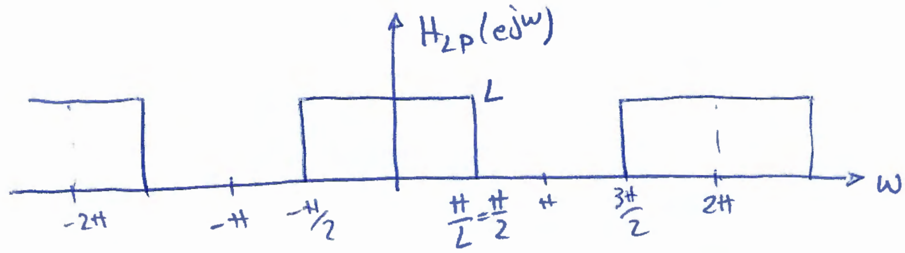
$$Y(e^{jw}) = \begin{cases} L \cdot X(e^{jw}) & |w| \leq \pi/L \\ 0 & \pi/L < |w| < \pi \end{cases}$$

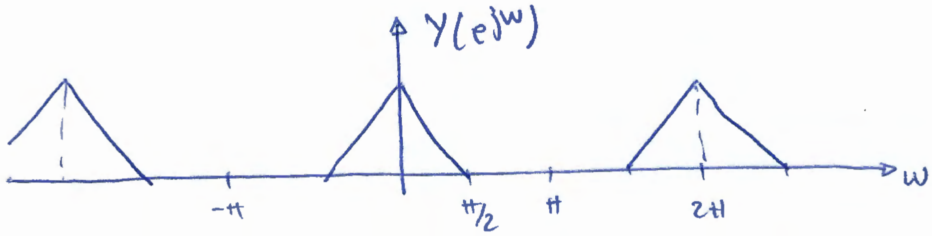
$X(e^{jw})$ muestreada a $f_{Nyquist}$



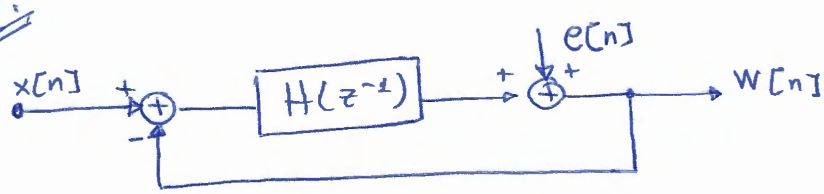
$L=2$







Ej 3.42:



a) $W(z)$?

b) Si $H(z) = \dfrac{z^{-1}}{1-z^{-1}}$

a) $$W(z) = H_x(z)\Big|_{e[n]=0} \cdot X(z) + H_e(z)\Big|_{x[n]=0} \cdot E(z)$$

$H_x(z)$: $$W(z) = \left(X(z) - W(z)\right) \cdot H(z)$$

$$W(z)\left(1+H(z)\right) = X(z)\cdot H(z) \Rightarrow H_x(z) = \frac{W(z)}{X(z)} = \frac{H(z)}{1+H(z)}$$

$$H_e(z): \quad W(z) = E(z) - W(z)\cdot H(z)$$

$$W(z)\,(1+H(z)) = E(z) \Rightarrow H_e(z) = \frac{W(z)}{E(z)} = \frac{1}{1+H(z)}$$

b)

$$H_x(z) = \frac{z^{-1}}{(1-z^{-1})\left(1+\frac{z^{-1}}{1-z^{-1}}\right)} = \frac{z^{-1}}{1-z^{-1}+z^{-1}} = z^{-1}$$

$$H_e(z) = \frac{1}{1+\frac{z^{-1}}{1-z^{-1}}} = \frac{1-z^{-1}}{1-z^{-1}+z^{-1}} = 1-z^{-1}$$

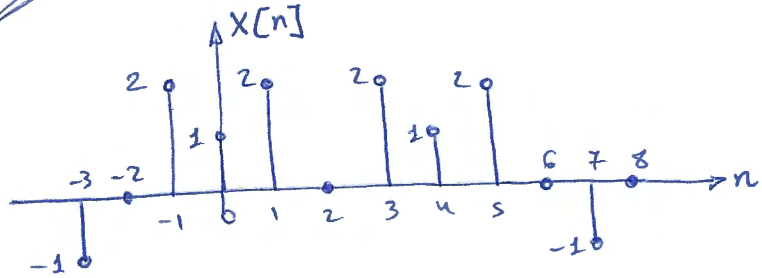
c) Estabilidad de $H(z)$, $He(z)$ y $Hx(z)$

$H(z)$ tiene un polo en $z=1$ $\Rightarrow$ inestable

$H_x(z)$ es un retardo (sistema FIR) $\Rightarrow$ estable

$He(z)$ es un filtro paso alto $\Rightarrow$ estable

Ej 2.55:



a) $X(e^{jw})\big|_{w=0}$

b) $X(e^{jw})\big|_{w=\pi}$

c) $\measuredangle X(e^{jw})$

d) $\int_{-\pi}^{\pi} X(e^{jw})\,dw$

e) Determinar y Dibujar $x'[n]$; $X'(e^{jw}) = X(e^{-jw})$

f) " " $x''[n]$; $X''(e^{jw}) = \mathrm{Re}\{X(e^{jw})\}$

---

a) $$X(e^{jw})\big|_{w=0} = \sum_{n=-\infty}^{+\infty} x[n]\,\cancel{e^{-jwn}}^{\,1} = \sum_{n=-\infty}^{+\infty} x[n] = 8$$

b) $$X(e^{jw})\big|_{w=\pi} = \sum_{n=-\infty}^{+\infty} (-1)^n \cdot x[n] = -4$$

c) $x[n] = x_1[n] + \delta[n-2] \xrightarrow{\mathcal{F}} X_1\left(e^{-j2w}\right) \Rightarrow \measuredangle X(e^{jw}) = 0 + (-2w)$

$\Downarrow$

$Im\{X(e^{jw})\} = 0$

$\measuredangle X_1(e^{jw}) = 0$

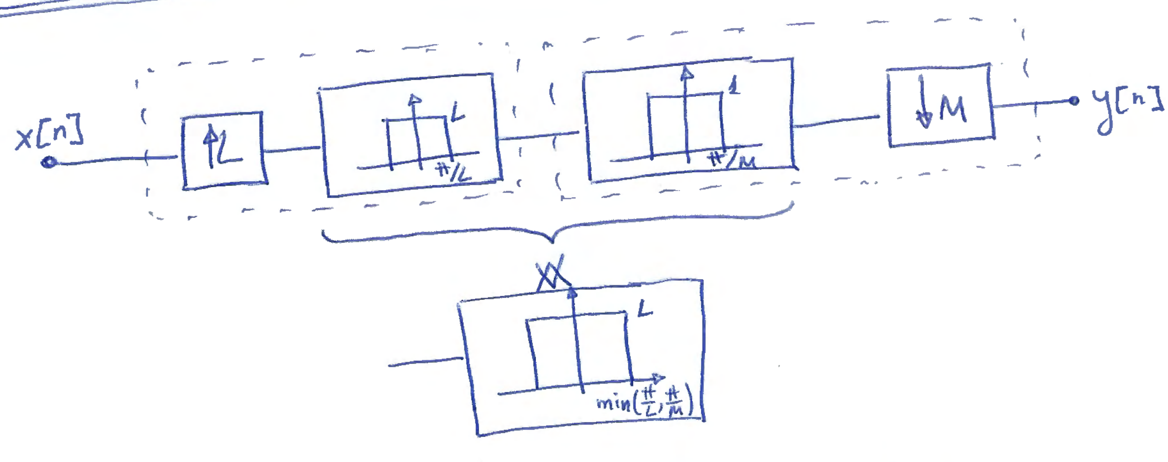
d) $$x[n]\Big|_{n=0} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})\cdot e^{+jwn}\, dw \Big|_{n=0} \Rightarrow 2\pi \cdot x[0] = 2\pi$$

sabemos que $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})\, e^{jwn}\, dw$

necesitamos anular (hacer 1) e^{jwn} para lograr lo que nos piden: $\int_{-\pi}^{\pi} X(e^{jw})\, dw$ $(n=0)$

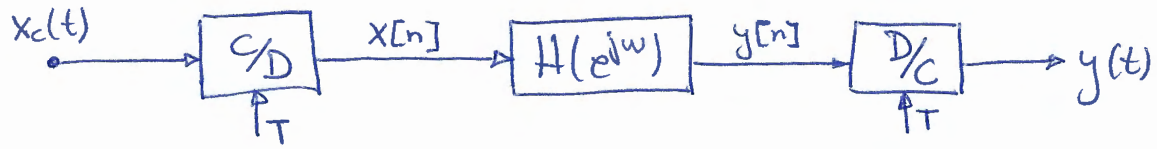
luego: $\int_{-\pi}^{\pi} X(e^{jw})\, dw = 2\pi \cdot x[0] = 2\pi$

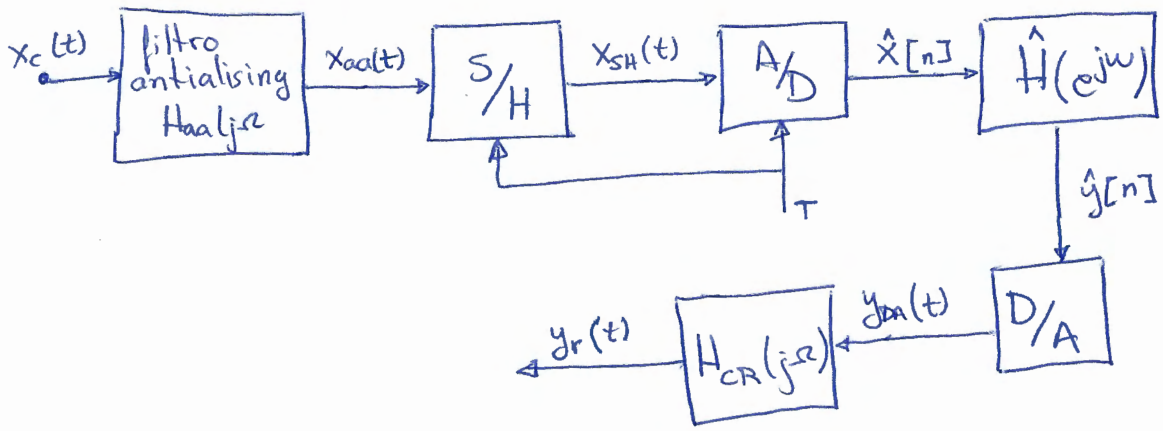
7. Cambio de la tasa por un número racional



Hay que respetar el orden: primero L, luego M pues el filtro antialiasing de M podría hacernos perder información

8. Aspectos prácticos de conversiones C/D, D/C



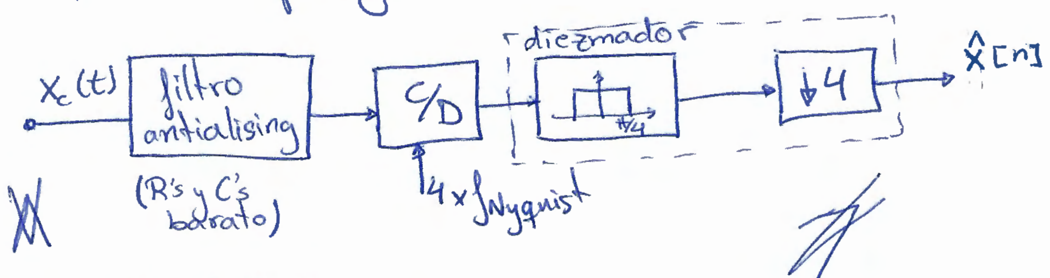


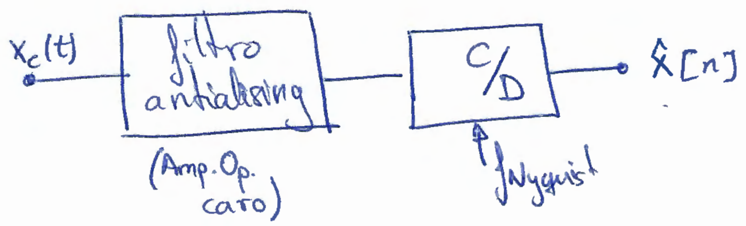
⊛ $H_{aa}(j\Omega)$: filtro antialising

$$H_{aa}(j\Omega) = \begin{cases} 1 & |\Omega| \leq \pi/T \\ 0 & \text{resto} \end{cases}$$

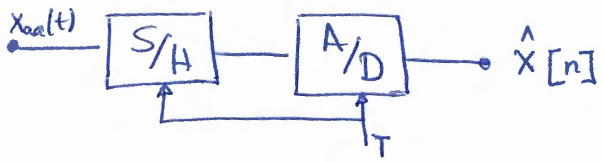
Es caro hacer un buen filtro de este tipo. Usaremos: 4xoversampling que es más barato

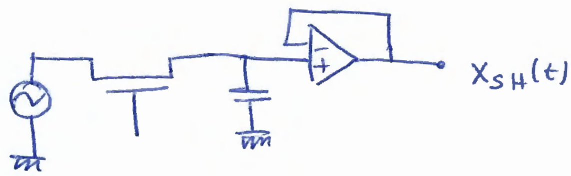
4x oversampling:

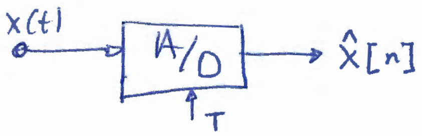


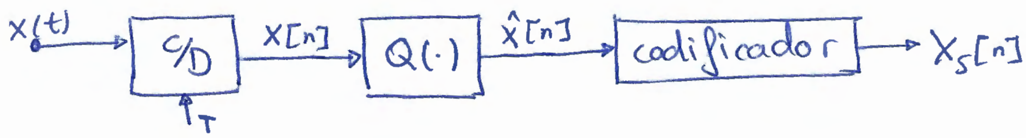


⊛ S/H : Sample & Hold

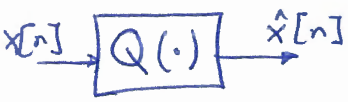


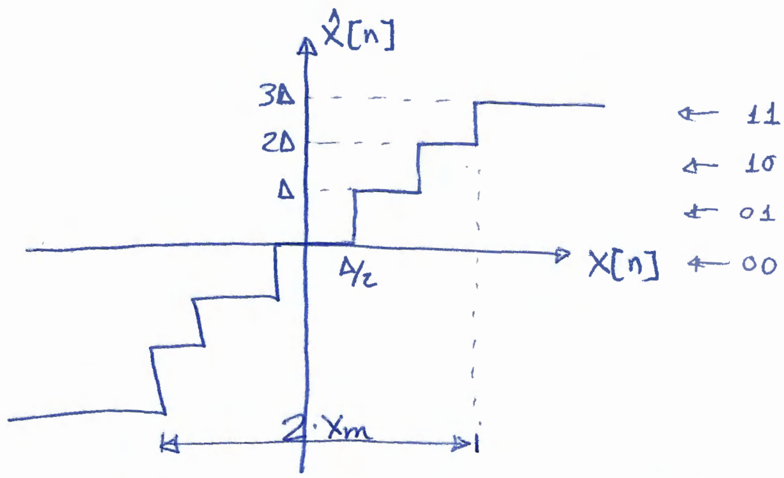






⊛ Cuantificador:





Sólo se permiten un número finito de valores de amplitud

Escalón de cuantificación : Δ
Fondo de escala : $2 \cdot X_m$
Número de bits : $B+1$

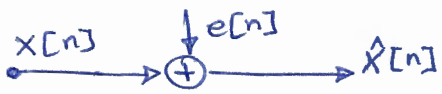
$$\Delta = \frac{2X_m}{2^{B+1}} = \frac{X_m}{2^B}$$

$$\hat{x}[n] = x[n] + e[n]$$

$$-\frac{\Delta}{2} \leq e[n] \leq \Delta/2$$

Se trata de un sistema NO lineal

Modelo lineal del cuantificador:

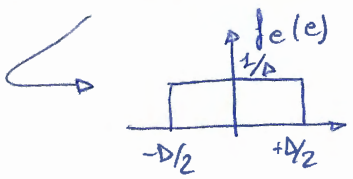


$e[n]$: proceso estocástico estacionario

$e[n]$: incorrelado con $x[n]$ (suma de potencias)

$e[n]$: proceso ruido blanco con media nula

$e[n]$: tiene distribución uniforme $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$



$$\Rightarrow \begin{cases} \mu_e = \int_{-\infty}^{\infty} e \cdot f_e(e)\, de = 0 \\ \sigma_e^2 = \int_{-\Delta/2}^{\Delta/2} e^2 f_e(e)\, de = \frac{\Delta^2}{12} = \frac{X_M^2}{12 \cdot 2^{2B}} \\ R_{ee}[m] = \sigma_e^2\, \delta[m] \\ S_{ee}(e^{jw}) = \sigma_e^2 \quad \forall w \end{cases}$$

Asumiendo señal con media nula:

$$SNR\,(dB) = 10 \cdot \log_{10}\left(\frac{P_{señal}}{P_{ruido}}\right) = 10 \cdot \log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right) = 10 \cdot \log_{10}\left(\frac{\sigma_x^2}{X_M^2}\, 12 \cdot 2^{2B}\right) =$$

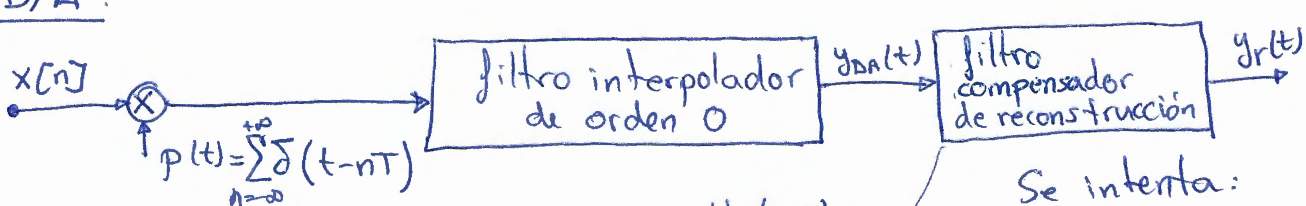
$$= -20 \log_{10}\left(\frac{X_M}{\sigma_x}\right) + 10{,}8 + 6{,}02 \cdot B$$

- Si el diseño del cuantificador cuida que no se sature con la señal
- Si la señal posee una distribución gaussiana y $X_M = 4\sigma_x$

Entonces: $SNR(dB) = 6 \cdot n^o bits - 7{,}25 \approx 6 \cdot n^o bits$

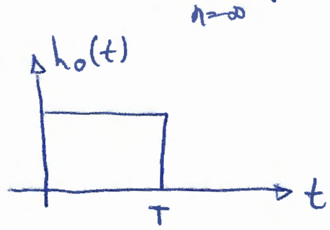
si nºbits ↑↑

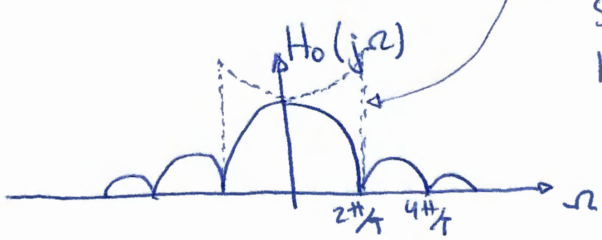
D/A:



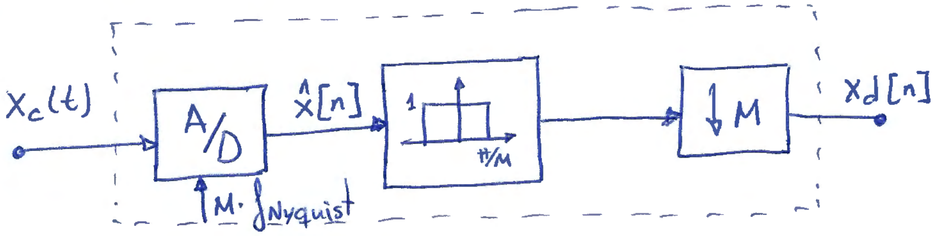
Se intenta:

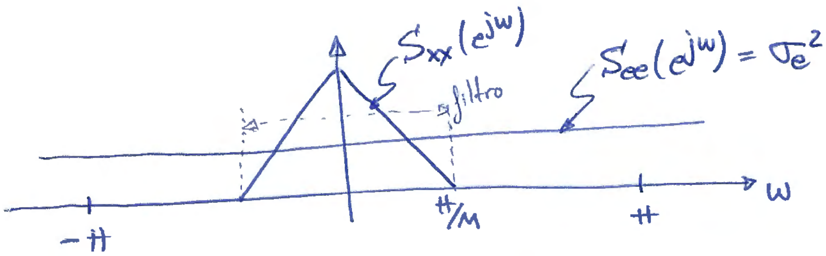
$H_0(j\Omega) \cdot H_{CR}(j\Omega) \approx$ ideal



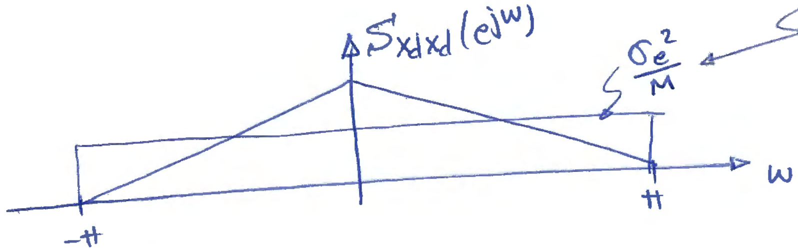


9. Sobremuestreo y conformación de ruido





$$P_{ruido} = \frac{1}{2\pi}\int_{-\pi/M}^{\pi/M} \sigma_e^2\, dw = \frac{\sigma_e^2}{M}$$

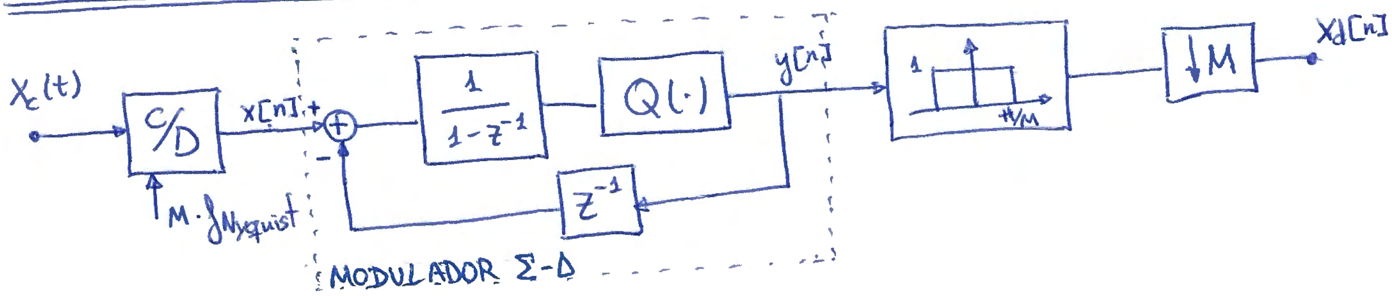


$M = 2^r$

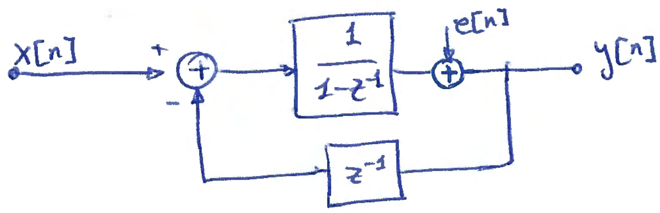
$$SNR\,(dB) = 10\cdot \log_{10}\left(\frac{P_{señal}}{P_{ruido}}\right) = 10\cdot\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\cdot M\right) = SNR\,(dB)\Big|_{\substack{cuantif\\directa}} + 3{,}01\cdot r$$

Cada vez que duplico la frecuencia de muestreo, gano 3 dB en SNR o lo que es lo mismo: el cuantificador equivalente se incrementa en $\frac{1}{2}$ bit

Conversores Σ - Δ:



Modulador Σ-Δ:



$$\begin{cases} Y(z) = H_x(z)\cdot X(z) + H_e(z)\cdot E(z) \\ H_x(z) = z^{-1} \\ H_e(z) = 1 - z^{-1} \end{cases}$$

$$S_{yy}(e^{jw}) = S_{xx}(e^{jw})\cdot |H_x(e^{jw})|^2 + S_{ee}(e^{jw})\cdot |H_e(e^{jw})|^2$$

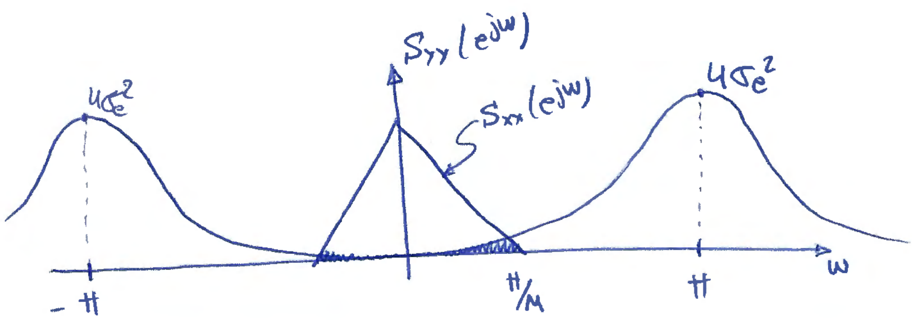
$$|H_x(e^{jw})|^2 = |e^{-jw}|^2 = 1$$

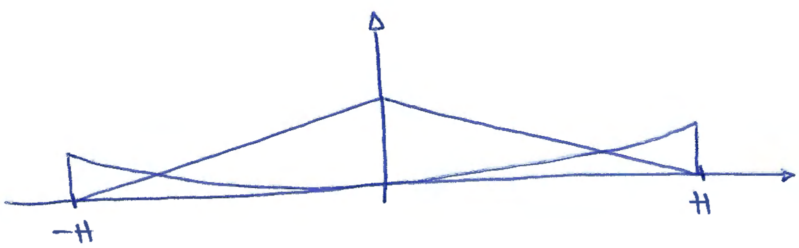
$$|H_e(e^{jw})|^2 = |1-e^{-jw}|^2 = |1-\cos(w) + j\,\mathrm{sen}(w)|^2 - (1-\cos(w))^2 + \mathrm{sen}^2(w) =$$

$$= 1+\cos^2(w) - 2\cos(w) + \mathrm{sen}^2(w) = 2(1-\cos(w)) = 4\,\mathrm{sen}^2\left(\tfrac{w}{2}\right)$$

$$\mathrm{sen}^2(\theta) = \tfrac{1}{2}(1-\cos(2\theta))$$

$$S_{yy}(e^{jw}) = S_{xx}(e^{jw}) + 4\sigma_e^2\,\mathrm{sen}^2\left(\tfrac{w}{2}\right)$$





$$P_{ruido} = \frac{1}{2\pi}\int_{-\pi/M}^{\pi/M} 4\sigma_e^2\cdot \mathrm{sen}^2\left(\tfrac{w}{2}\right) dw$$

$$= \{\theta \downarrow\downarrow \Rightarrow \mathrm{sen}(\theta)\approx\theta\} =$$

$$= \frac{\sigma_e^2\,\pi^2}{3\cdot M^3}$$

$$SNR(dB) = 10\cdot\log_{10}\left(\frac{S}{N}\right) = 10\cdot\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\,\frac{3}{\pi^2}\,M^3\right) = SNR(dB)\Big|_{\substack{\text{cuantif}\\ \text{directa}}} - 5{,}17 + 9{,}03\cdot r$$

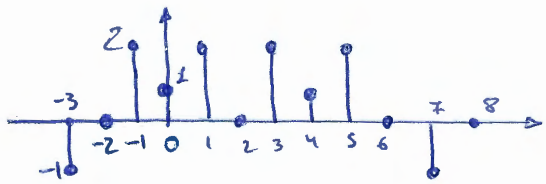
$M = 2^r$

Si duplico $f \Rightarrow$ SNR aumenta 9 dB

Ej 2.55:

e) $x'[n]$; $\mathcal{F}\{x'[n]\} = X(e^{-jw})$

f) $x''[n]$; $\mathcal{F}\{x''[n]\} = \mathrm{Re}\{X(e^{jw})\}$



e) $$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-jwn}$$

$$X(e^{-jw}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{+jwn} = \{\ell = -n\} = \sum_{\ell=-\infty}^{+\infty} x[-\ell] \cdot e^{-jw\ell} = \mathcal{F}\{x[-\ell]\}$$

luego: $x'[n] = x[-n]$

f) $$x[n] = \underset{\downarrow}{x_{par}[n]} + \underset{\downarrow}{x_{impar}[n]} \Rightarrow x''[n] = x_{par}[n]$$

$$\mathrm{Re}\{X(e^{jw})\} + j\,\mathrm{Im}\{X(e^{jw})\} \qquad x_{par}[n] = \frac{1}{2}\left[x[n] + x[-n]\right]$$

Ej 2.84:

$$x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{jw})$$

$$y[n] \overset{\mathcal{F}}{\longleftrightarrow} Y(e^{jw})$$

a) $g[n]? \overset{\mathcal{F}}{\longleftrightarrow} X(e^{jw}) \cdot Y^*(e^{jw})$

b) $$\sum_{n=-\infty}^{+\infty} x[n] \cdot y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw}) \cdot Y^*(e^{jw})\, dw$$

c) $$\sum_{n=-\infty}^{+\infty} \frac{\mathrm{sen}\left(\frac{\pi}{4} n\right)}{2\pi n} \cdot \frac{\mathrm{sen}\left(\frac{\pi}{6} n\right)}{5 \cdot \pi \cdot n}$$

a) $$Y(e^{jw}) = \sum_{n=-\infty}^{+\infty} y[n]\, e^{-jwn}$$

$$Y^*(e^{jw}) = \sum_{n=-\infty}^{+\infty} y^*[n]\, e^{+jwn} = \{\ell = -n\} = \sum_{\ell=-\infty}^{+\infty} y^*[-\ell] \cdot e^{-jw\ell} = \mathcal{F}\{y^*[-\ell]\}$$

si $G(e^{jw}) = X(e^{jw}) \cdot Y^*(e^{jw}) \Rightarrow g[n] = x[n] * y^*[-n]$

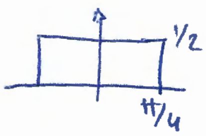
b) $g[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})\cdot Y^*(e^{jw})\; e^{+jwn}\, dw$

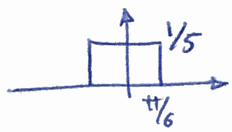
será igual al enunciado si $e^{+jwn} = 1 \Leftrightarrow n=0$

$g[n]\Big|_{n=0} = \underbrace{x[n] * y^*[-n]}_{a)} = \sum_{k=-\infty}^{+\infty} x[k]\cdot y^*[n+k]\Big|_{n=0} \Rightarrow \sum x[k]\cdot y^*[k] = g[0]$

c) $\sum_{-\infty}^{+\infty} \text{sinc}(x) = \infty$, pues decrece $\frac{1}{n}$ (no converge)

pero $\sum_{-\infty}^{+\infty} \text{sinc}^2(x) \neq \infty$, pues decrece $\frac{1}{n^2}$ (converge)

$\frac{1}{2\pi}\int_{-\pi}^{\pi}$ 

$\cdot$ 

$dw = \frac{1}{2\pi}\int_{-\pi/6}^{\pi/6} \frac{1}{2}\cdot\frac{1}{5}\, dw = \frac{1}{60}$

Ej 4.3:

$x_c(t) = \cos(4000\pi t)$ muestreada a T

$x[n] = \cos\left(\frac{\pi n}{3}\right)$

a) T?

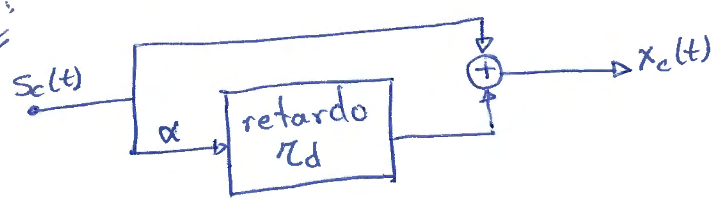
$x[n] = x_c(nT) = x_c(t)\Big|_{t=nT} = \cos(4000\pi nT) = \cos\left(\frac{\pi n}{3}\right) \Leftrightarrow T = 1/12000$

$f_s = 12\,kHz$

b) T es única?

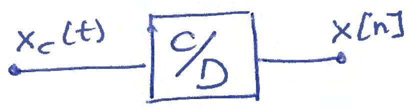
NO. Todas las frecuencias superiores al doble de la frec. orig.

Ej 4.7:



$S_c(j\Omega) = 0, \quad |\Omega| \geq \frac{\pi}{T}$

a)



$\mathcal{F}\{x_c(t)\}$? $\mathcal{F}\{x[n]\}$? en función de $S_c(j\Omega)$

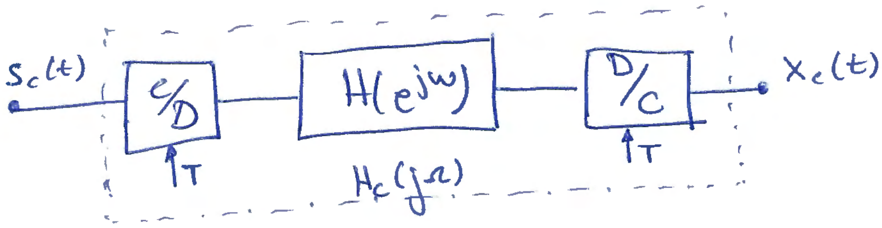
$$X_c(j\Omega) = S_c(j\Omega) + \alpha S_c(j\Omega) e^{-j\Omega\tau_d} = S_c(j\Omega)(1 + \alpha e^{-j\Omega\tau_d})$$

$$H_c(j\Omega) = \frac{X_c(j\Omega)}{S_d(j\Omega)} = 1 + \alpha e^{-j\Omega\tau_d}$$

$$X(e^{jw}) = \frac{1}{T} X_c\left(j\frac{w}{T}\right); \quad |w| \leq \pi$$

$$= \frac{1}{T} S_c\left(j\frac{w}{T}\right) \cdot \left(1 + \alpha e^{-j\left(\frac{w}{T}\tau_d\right)}\right) \quad |w| \leq \pi$$

b)



$H(e^{jw})$?

$$H_c(j\Omega) = 1 + \alpha e^{-j\Omega\tau_d}$$

$$H_{eff}(j\Omega) = \begin{cases} H_c(j\Omega) & |\Omega| \leq \pi/T \\ 0 & \text{resto} \end{cases}$$

$$H(e^{jw}) = H_{eff}\left(j\frac{w}{T}\right), \quad |w| \leq \pi/T \Rightarrow H(e^{jw}) = 1 + \alpha e^{-jw\frac{\tau_d}{T}}$$

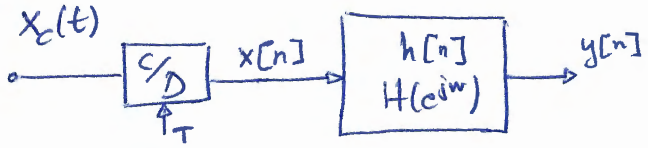
c) $h[n]$ si $\tau_d = T$ y $\tau_d = T/2$?

$$h[n] = \delta[n] + \alpha \frac{\text{sen}\left(\pi\left(n - \frac{\tau_d}{T}\right)\right)}{\pi\left(n - \frac{\tau_d}{T}\right)}$$

si $\tau_d = T \Rightarrow h[n] = \delta[n] + \alpha \cdot \delta[n-1]$

si $\tau_d = \frac{T}{2} \Rightarrow h[n] = \delta[n] + \alpha \dfrac{\text{sen}(\pi(n - 1/2))}{\pi(n - 1/2)}$

Ej 4.8



$X_c(j\Omega) = 0 \quad |\Omega| \geq 2\pi \cdot 10^4$

$x[n] = x_c(nT)$

$y[n] = T \sum_{k=-\infty}^{+\infty} x[k]$

a) Tmax para evitar aliasing?

$T_{max} = \frac{1}{f_{min}} = \frac{1}{2\cdot 10^4}$ (Nyquist)

b) h[n]?

$h[n] = T \sum_{k=-\infty}^{+\infty} \delta[k] = T \cdot u[n]$

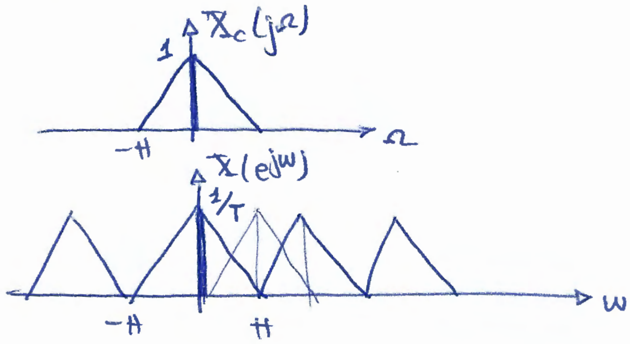
c) $y[n]\big|_{n\to\infty}$ en función de $X(e^{jw})$?

$y[n]\big|_{n\to\infty} = T \sum_{k=-\infty}^{+\infty} x[k] = T\, X(e^{jw})$

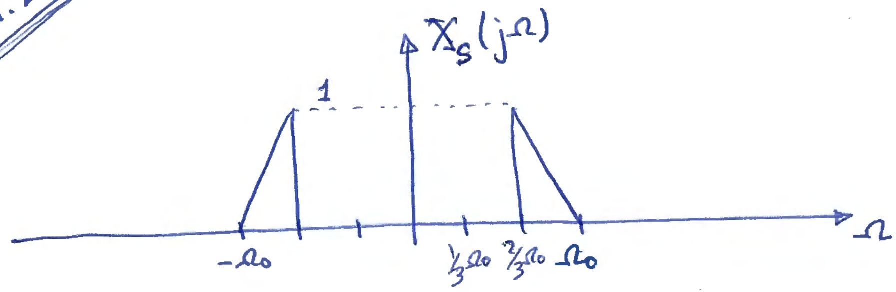
d) $\exists\, T$ que cumpla $y[n]\big|_{n\to\infty} = \int_{-\infty}^{+\infty} x_c(t)\,dt$?

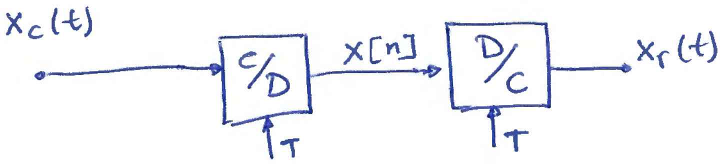
$T\, X(e^{j0}) = X_c(j0)$

$f_{min} > 10^4 \Rightarrow T_{min} < \frac{1}{10^4}$



Ej 4.19:





¿ rango de valores de T para que $x_c(t) = x_r(t)$?

$$X_s(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(j\left(\Omega - \frac{2\pi k}{T}\right)\right)$$

Evitemos solapamiento:

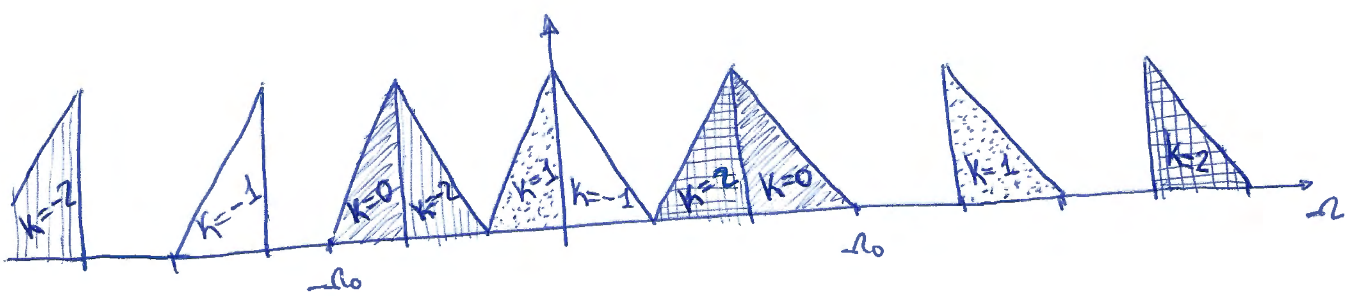
$$\left.\begin{array}{l} \frac{2}{3}\Omega_0 + i\Omega_s \leq \frac{2}{3}\Omega_0 \\ -\Omega_0 + (i+1)\Omega_s \geq \Omega_0 \end{array}\right\} \begin{array}{l} \rightarrow i\Omega_s \leq \frac{4}{3}\Omega_0 \\ \rightarrow (i+1)\Omega_s \geq 2\Omega_0 \end{array} \Bigg\} \quad \frac{2\Omega_0}{i+1} \leq \Omega_s \leq \frac{4\Omega_0}{3i}$$

si $i=0$: $2\Omega_0 \leq \Omega_s \leq \infty$ ← si no metemos más señales $\Omega_s \geq 2\Omega_0$ (Nyquist)

si $i=1$: $\Omega_0 \leq \Omega_s \leq \frac{4}{3}\Omega_0$ ← si metemos 1 réplica

si $i=2$: $\frac{2}{3}\Omega_0 \leq \Omega_s \leq \frac{2}{3}\Omega_0$

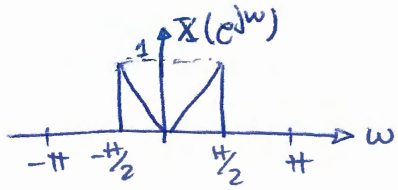
si $i=3$: $\frac{\Omega_0}{2} \leq \Omega_s \leq \frac{4}{9}\Omega_0$ ⇒ imposible recuperar con 3 réplicas



Ej 4.32

$$x[n] \to \boxed{\uparrow L} \xrightarrow{x_e[n]} \boxed{H(e^{jw})} \xrightarrow{y_e[n]} \boxed{\downarrow M} \to y[n]$$

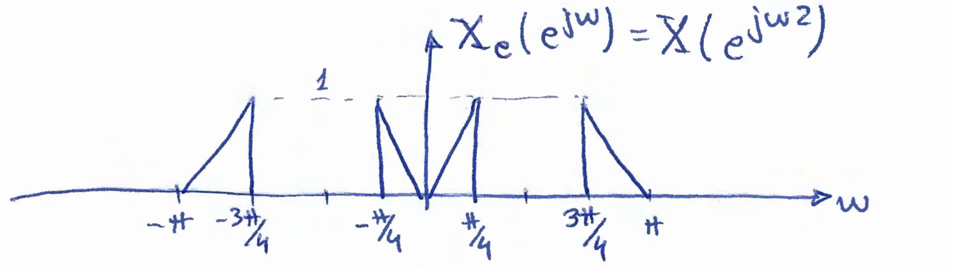
$$H(e^{jw}) = \begin{cases} M & |w| \leq \pi/4 \\ 0 & \pi/4 < |w| < \pi \end{cases}$$

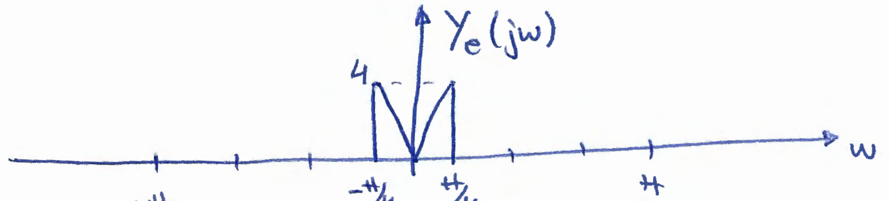


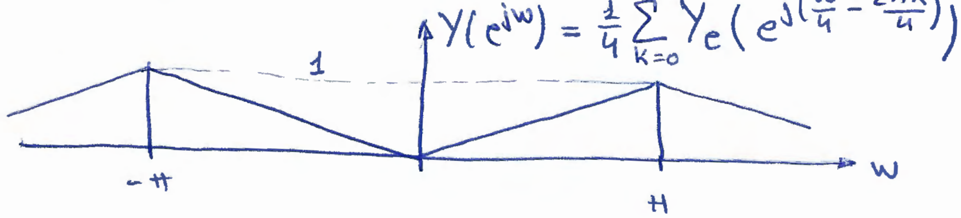
a) $L = 2$, $M = 4$, dibujar T.F.

b) $L = 2$, $M = 8$, dibujar T.F.

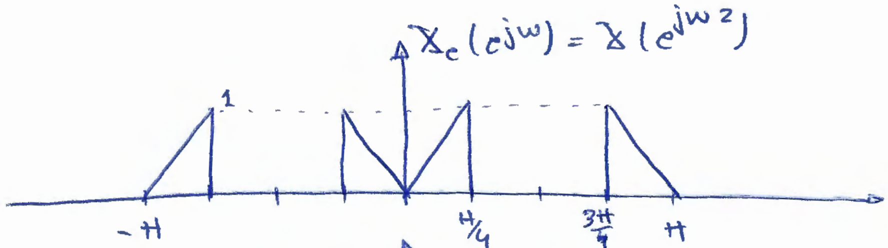
a)

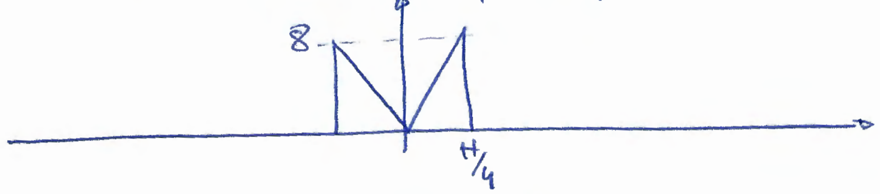


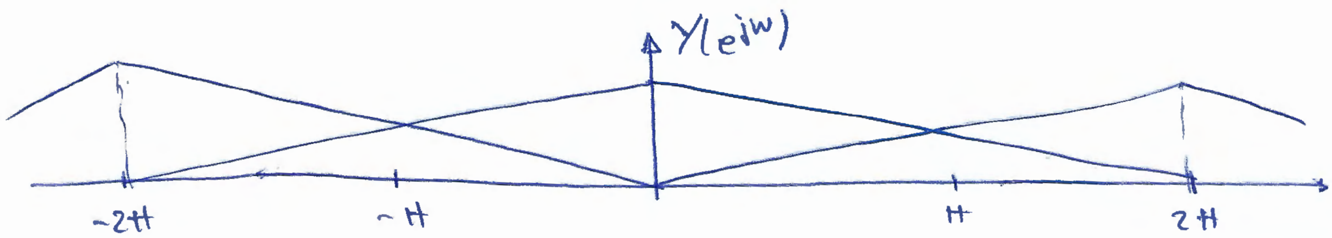




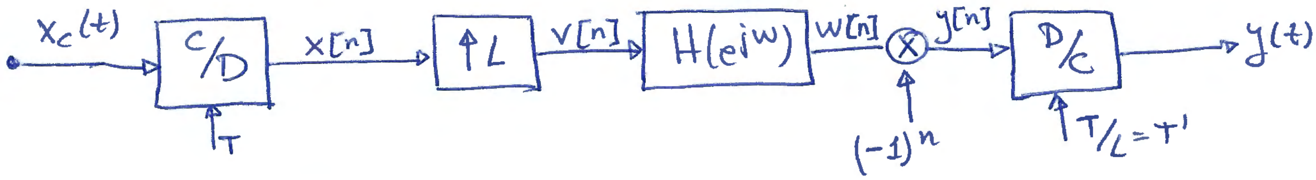
b)





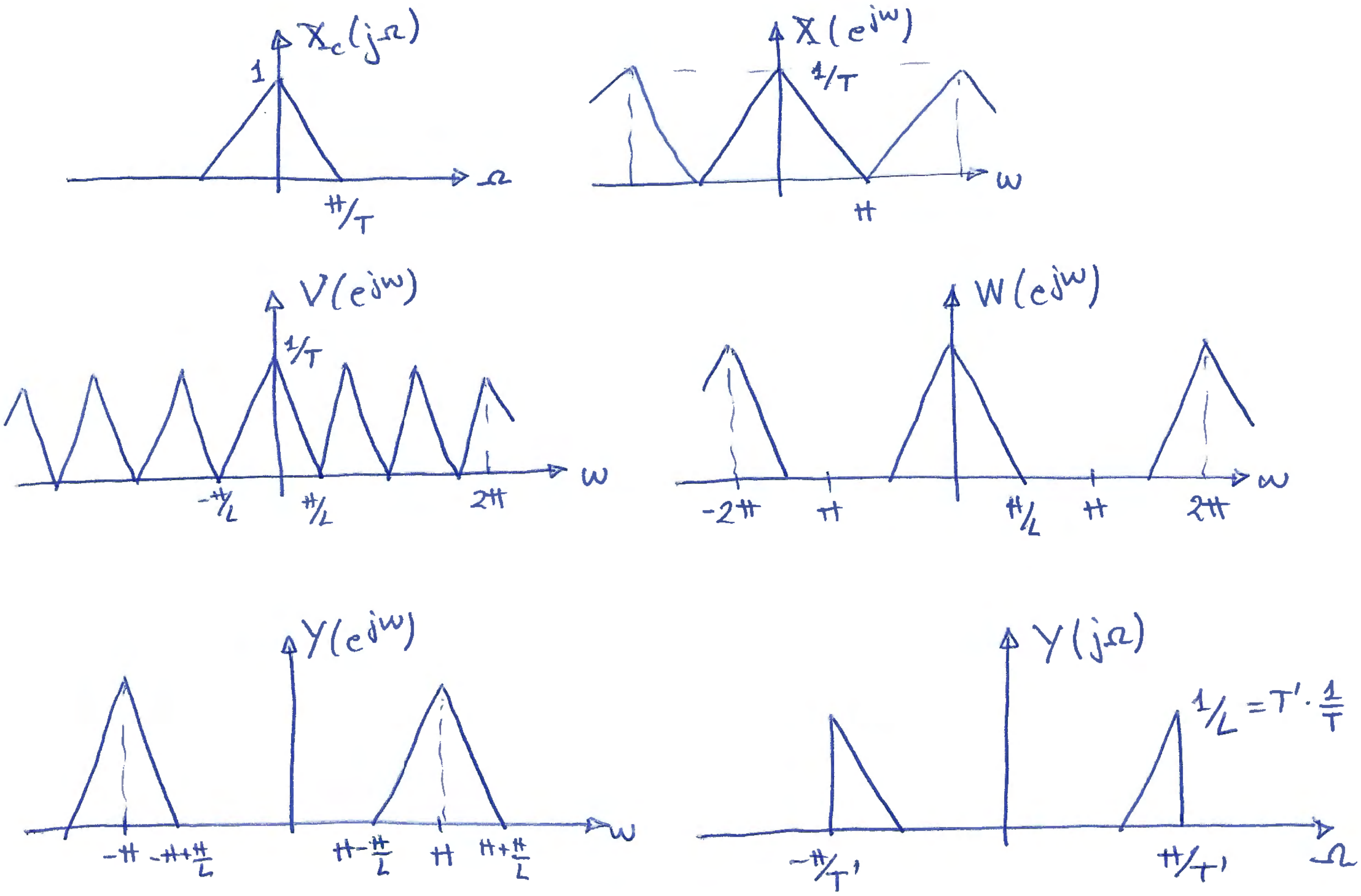


Ej 4.38:

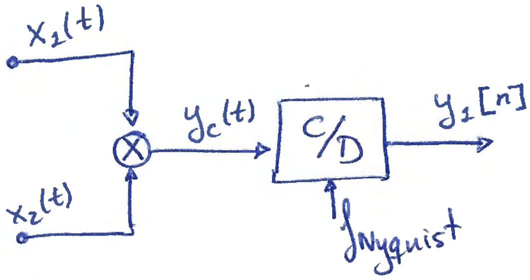


$$H(e^{jw}) = \begin{cases} 1 & |w| \le \pi/L \\ 0 & \pi/L < |w| \le \pi \end{cases}$$

dibujar las T.F.

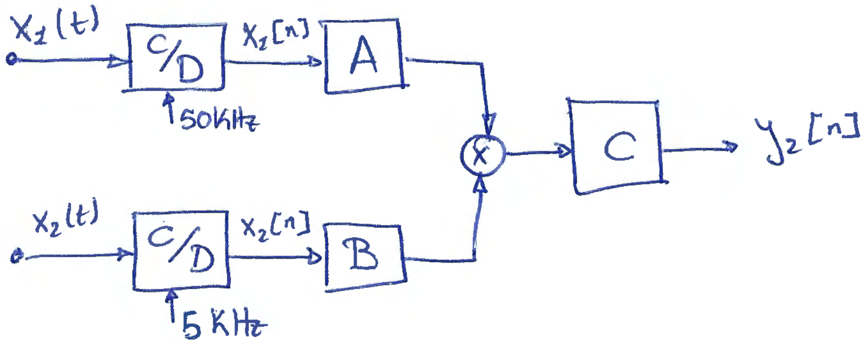


Ej 4.64:



$x_1 : Bw = 25\,kHz$

$x_2 : Bw = 2{,}5\,kHz$



A, B, C pueden ser: 

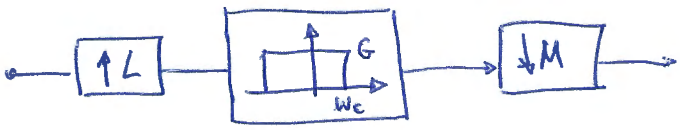
qué es A, B, C para que $y_1[n] = y_2[n]$

$$\left.\begin{aligned} Bw_1 &= 25\,kHz \\ Bw_2 &= 2{,}5\,kHz \end{aligned}\right\} \Rightarrow Bw(y_c(t)) = 27{,}5\,kHz \quad \text{(convolución de 25 con 2's)}$$

$$\hookrightarrow f_{Nyq} = 55\,kHz \Rightarrow Bw(y_2[n]) = 55\,kHz$$

necesitamos que $x_1[n]$ y $x_2[n]$ tengan el mismo Bw para poder ser multiplicadas $\Rightarrow$ A y B serán M ó L

$x_1 : \dfrac{f_{out}}{f_{in}} = \dfrac{55}{50} = \dfrac{L}{M} = \dfrac{11}{10}$



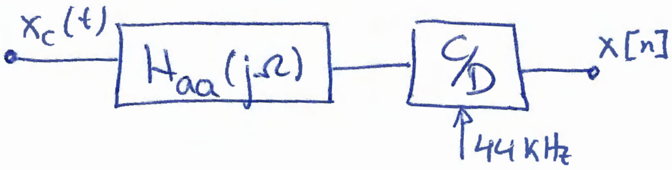
A: $L = 11$ $\quad G = 11$, $w_c = \pi/11$ $\quad M = 10$

$x_2 : \dfrac{f_{out}}{f_{in}} = \dfrac{55}{5} = \dfrac{L}{M} = \dfrac{11}{1}$

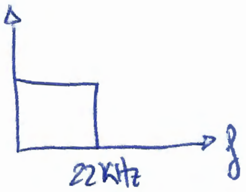
B: $L = 11$ $\quad G = 11$, $w_c = \pi/11$ $\quad$ ~~$M = 1$~~

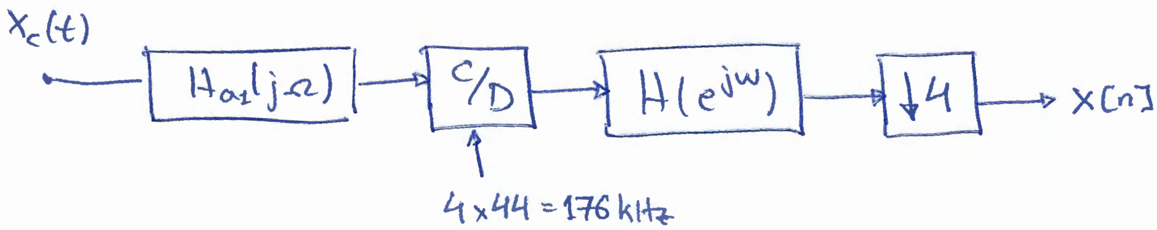
C: NADA

Ej 4.66:



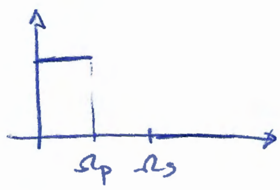
$$H_a(j\Omega) = \begin{cases} 1 & \Omega \leq 2\pi \cdot 22 \text{ kHz} \\ 0 & \text{resto} \end{cases}$$

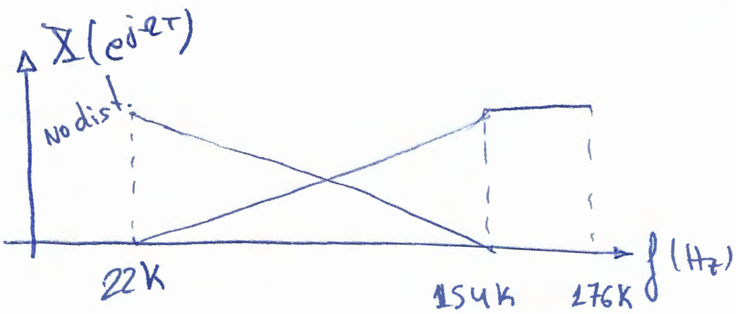


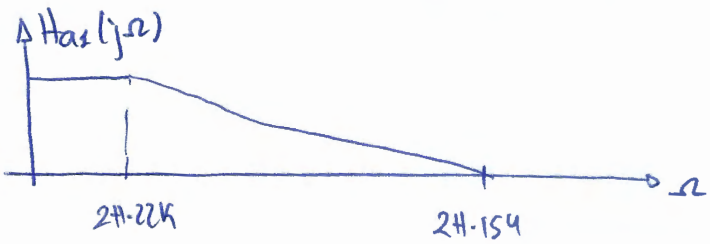


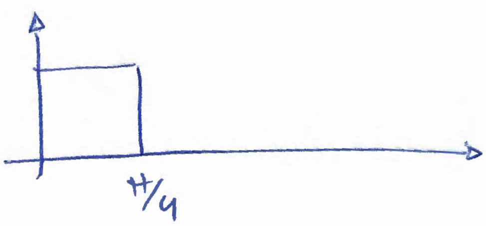
$$H(e^{jw}) = \begin{cases} 1 & |w| \leq \pi/4 \\ 0 & \pi/4 < |w| \leq \pi \end{cases}$$

$$H_{a1}(j\Omega) = \begin{cases} 1 & |\Omega| \leq \Omega_p \\ 0 & |\Omega| > \Omega_s \end{cases}$$

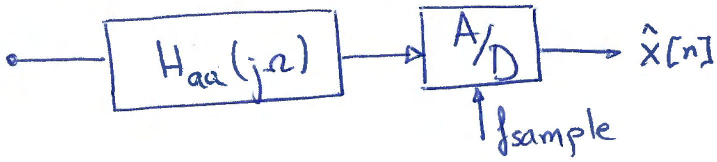




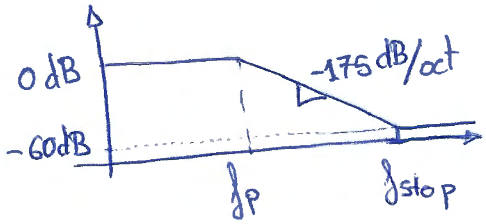




Ej:



$x_c(t) \cdot B_W = 10\,KHz$



→ muestreo a $f_{Nyquist}$
→ ruido de cuantif. = ruido solap. espectr.
→ cuantif. uniforme

$X_M = 4\sigma_x$, $SNR = 6 \cdot n^o bits - 7{,}25$

a) f_{SAMPLE}? b) f_p? c) nº de bits?

a) $f_{sample} = 20\,KHz$

b) $\dfrac{f_2}{f_1} = 2^{n^o \text{ de octavas}} \Leftrightarrow n^o \text{ octavas} = \log_2\left(\dfrac{f_2}{f_1}\right)$

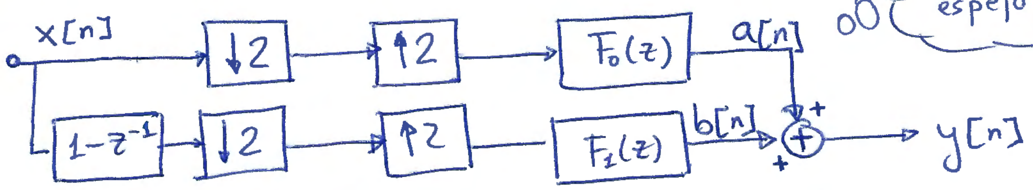
$\text{pendiente} = \dfrac{A}{n^o \text{ octavas}} \Rightarrow -175 = \dfrac{-60}{\log_2\left(\frac{10KHz}{f_p}\right)}$; $f_{stop} = 10\,KHz = \dfrac{f_{SAMPLE}}{2}$

$\hookrightarrow f_p = \dfrac{10\,KHz}{2^{60/175}} = 7884{,}8\,KHz$

c) ruido = + 60 dB

$SNR = 6 \cdot n^o bits - 7{,}25 \Rightarrow n^o \text{ bits} = \dfrac{67{,}25}{6} \geq 11{,}2 \Rightarrow n^o bits = 12$

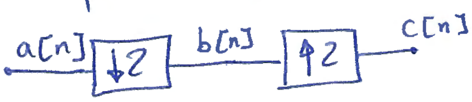
feb 08:



QMF: Filtros espejo en cuadratura

a) Condición para LTI

Sea:



$$\begin{cases} B(e^{jw}) = \frac{1}{2}\left[A(e^{jw/2}) + A(e^{j(w/2 - \pi)})\right] \\ C(e^{jw}) = B(e^{2jw}) = \frac{1}{2}\left[A(e^{jw}) + A(e^{jw-\pi})\right] \end{cases}$$

Basándonos en el a y b del dibujo inicial (enunciado):

$$A(e^{jw}) = \frac{F_0(e^{jw})}{2}\left[X(e^{jw}) + X(e^{j(w-\pi)})\right]$$

$$B(e^{jw}) = \frac{F_1(e^{jw})}{2}\left[X(e^{jw})(1-e^{-jw}) + X(e^{j(w-\pi)})\underbrace{\left(1-e^{-j(w-\pi)}\right)}_{1+e^{-jw}}\right]$$

$$Y(e^{jw}) = X(e^{jw})\cdot\left[\frac{F_0(e^{jw})}{2} + \frac{F_1(e^{jw})}{2}(1-e^{-jw})\right] +$$

$$+ X(e^{j(w-\pi)})\left[\frac{F_0(e^{jw})}{2} + \frac{F_1(e^{jw})}{2}(1-e^{-jw})\right]$$

para que sea LTI necesitamos $Y(e^{jw}) = X(e^{jw})\cdot$ algo

por lo que necesitamos que $\left[\frac{F_0(e^{jw})}{2} + \frac{F_1(e^{jw})}{2}(1+e^{-jw})\right] = 0$

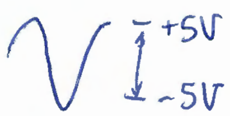
b) Si $y[n] = x[n-1]$, $F_0(z)$? $F_1(z)$?

$$\left.\begin{array}{l} F_0(z) + F_1(z)(1+z^{-1}) = 0 \\ F_0(z) + F_1(z)(1-z^{-1}) = 2z^{-1} \end{array}\right\} \begin{array}{l} F_0(z) = -F_1(z)(1+z^{-1}) \\ F_1(z)\cdot(1-z^{-1}-1-z^{-1}) = 2z^{-1} \end{array}$$

$$F_1(z) = \frac{2z^{-1}}{-2z^{-1}} = -1$$

$$F_0(z) = 1 + z^{-1}$$

sepOU: $x_a(t) \Rightarrow B_W = 4\,\text{KHz}$ requerimos $SNR = 40\,dB$



muestreo a $8\,kHz \Rightarrow SNR = 50\,dB$

$$x_a(t) \longrightarrow \boxed{\text{atenuador}} \xrightarrow{x'_a(t)} \boxed{A/D} \longrightarrow x[n]$$

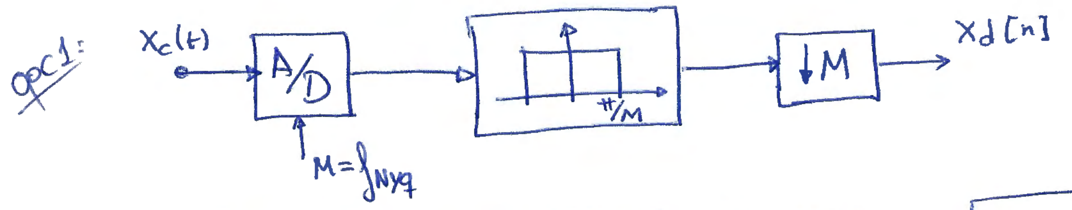
a) Att para que $SNR = 40\,dB$

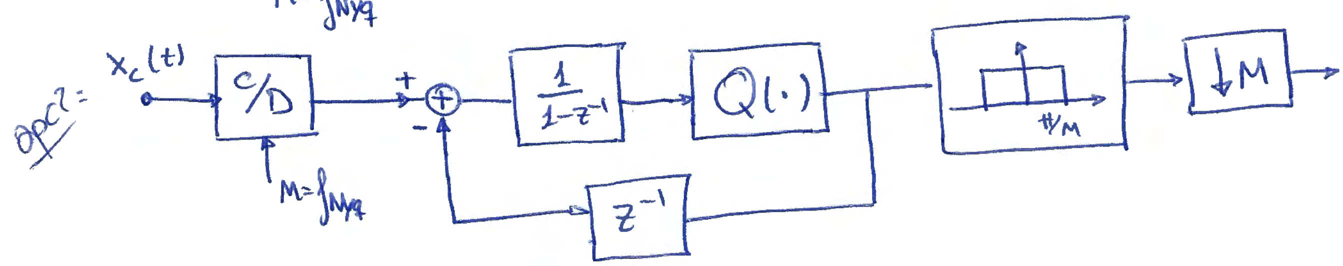
b) si $Att = 20\,dB$, estudiar posibles esquemas para cumplir los requisitos

a) $SNR = 10 \cdot \log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right) = P_{señal}(dB) - P_{ruido}(dB)$

$Att = 10\ dB$

b) si $Att = 20\ dB$ necesito $\begin{cases} SNR = SNR|_{\substack{quant\\directa}} + 3{,}01 \cdot r \\ SNR = SNR|_{\substack{quant\\directa}} + 9{,}03 \cdot r \end{cases}$ $\quad M = 2^r$

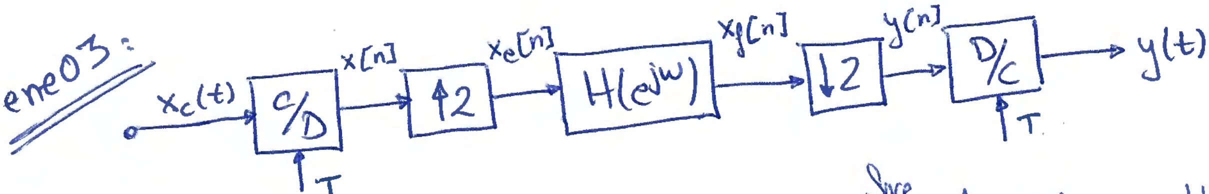




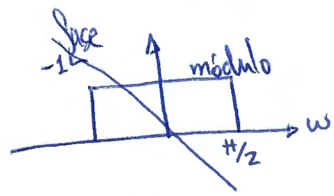
opc1: $3{,}01 \cdot r = 10 \Rightarrow r = 3{,}3 \Rightarrow M = \lceil 2^{3,3} \rceil$

opc2: $9{,}03 \cdot r = 10 \Rightarrow r = 1{,}1 \Rightarrow M = \lceil 2^{1,1} \rceil$

ene03:

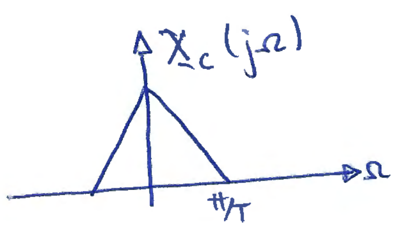


$H(e^{jw}) = \begin{cases} e^{-jw} & |w| \leq \pi/2 \\ 0 & \pi/2 < |w| \leq \pi \end{cases} \iff$

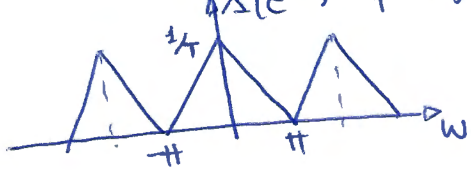


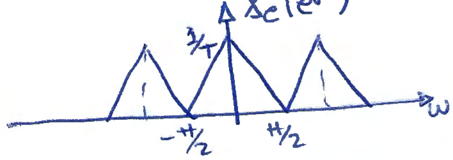
$|H(e^{jw})| = 1$

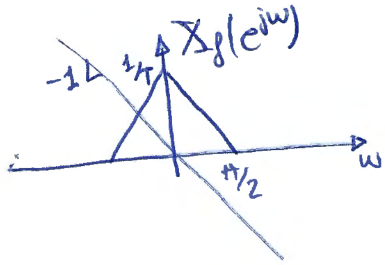
$\measuredangle H(e^{jw}) = -w$

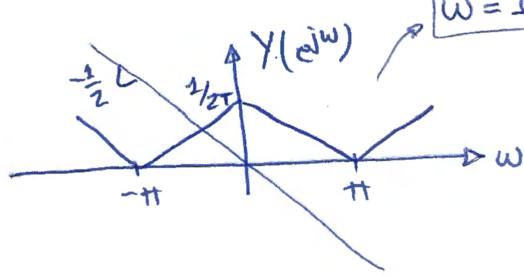


$X(e^{jw}) = \frac{1}{T} X_c(j\frac{w}{T}) \quad |w| \leq \pi$

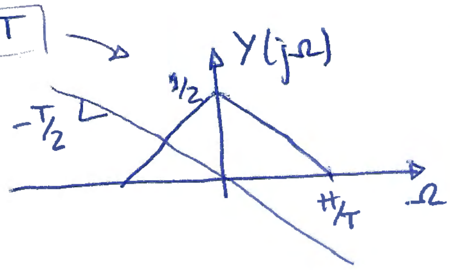




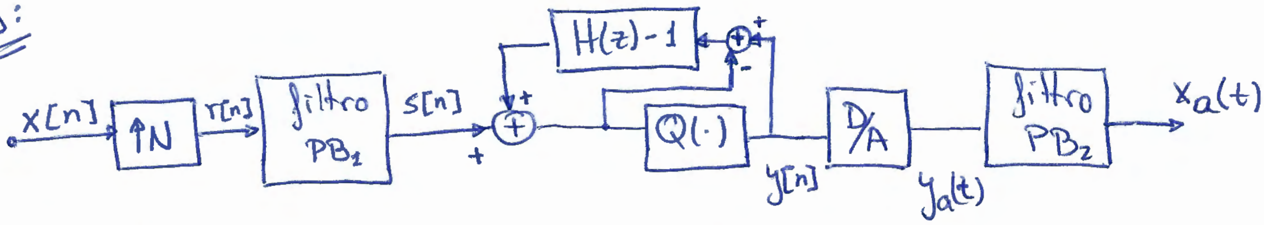


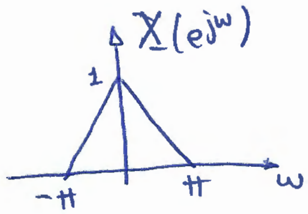


$\boxed{w = \Omega T}$



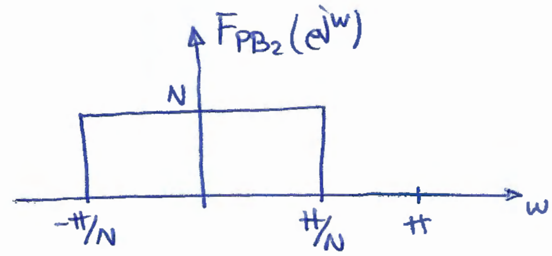
ene09:



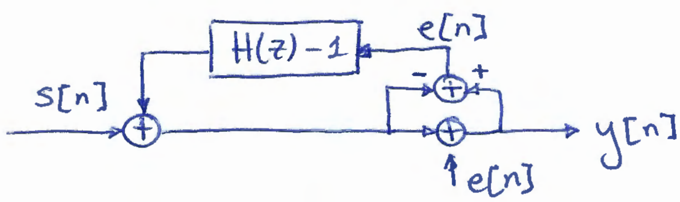


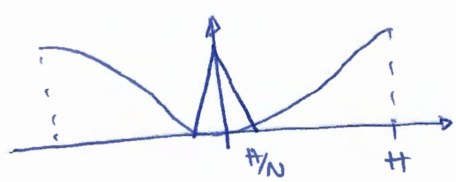
$$H(z) = (1 - z^{-1})^3$$

a) Dibujar $R(e^{jw})$ y especificar PB_1



b) Dibujar $S_{yy}(e^{jw})$





$$H_s(z) = 1$$

$$Y(z) = E(z) \cdot (H(z) - 1) + E(z)$$

$$H_e(z) = \frac{Y(z)}{E(z)} = H(z) = (1 - z^{-1})^3$$

Tema 3: Análisis transformado de Sistemas LTI

1. Sistemas de Ecuaciones en Diferencias de coef. ctes.
2. Respuesta en frecuencia con TF racional
3. Sistemas Racionales Básicos: Factorización
4. Estructuras Básicas para sistemas IIR
5. Estructuras Básicas para sistemas FIR
6. Realizaciones de filtros de orden alto

1. Sistemas de Ecuaciones en Diferencias con coef. ctes

Sea una ecuación en diferencias, con condiciones iniciales nulas:

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k] \Leftrightarrow \left(\sum_{k=0}^{N} a_k z^{-k}\right) Y(z) = \left(\sum_{k=0}^{M} b_k z^{-k}\right) X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} = k \cdot \frac{\prod_{k=1}^{M} (1 - c_k z^{-1})}{\prod_{k=1}^{N} (1 - p_k z^{-1})} \; ; \; k = \frac{b_0}{a_0}$$

Si es causal:

$H(z)$ No tiene potencias en z positivas $\Rightarrow$ no tiene polos en $z=\infty$

→ ROC incluye al infinito

→ Grado del denominador $\geq$ Grado del numerador

→ Número de polos $\geq$ Número de ceros

Será estable si: (y sólo si):

→ ROC incluye a la circunferencia unidad

→ $\exists$ transformada de Fourier

si es causal será estable $\Leftrightarrow$ todos sus polos están dentro del círculo unidad

Filtros FIR y IIR:

	FIR	IIR
Ec. en dif.:	$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$	$\sum_{k=0}^{N} a_k \, y[n-k] = \sum_{k=0}^{M} b_k \, x[n-k]$
$h[n]$	finita: $h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$	Infinita: exp y cos amortiguados
$H(z)$	$H(z) = \sum_{k=0}^{M} b_k z^{-k} = B(z)$	$H(z) = \dfrac{\sum b_k z^{-k}}{\sum a_k z^{-k}} = \dfrac{B(z)}{A(z)}$
diagr. polo-cero	sólo ceros (polos en el origen)	existen polos fuera del origen
estabilidad	siempre estables	pueden ser inestables
posible fase lineal	sí	no

2. Respuesta en frecuencia con TF racional

$$Y(e^{j\omega}) = H(e^{j\omega}) \cdot X(e^{j\omega}) \Leftrightarrow \begin{cases} |Y(e^{j\omega})| = |H(e^{j\omega})| \cdot |X(e^{j\omega})| \\ \measuredangle Y(e^{j\omega}) = \measuredangle H(e^{j\omega}) + \measuredangle X(e^{j\omega}) \end{cases}$$

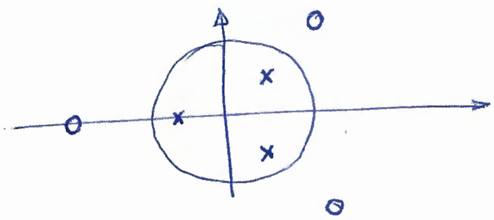
- Ante un polo, el módulo tiene un máximo, mayor cuanto más cerca de la circunferencia unidad se encuentre.
- Si el polo estuviese sobre la circunferencia unidad, el módulo valdría infinito.
- Ante un cero, el módulo tiene un mínimo, menor cuanto más cerca de la circunferencia unidad se encuentre.
- Si el cero estuviese sobre la circunferencia unidad, el módulo valdría cero.
- Las fases de un sistema racional son continuas, excepto si existe un cero sobre la circunferencia unidad, que se produce un salto de fase de $\pm\pi$.

3. Sistemas Racionales Básicos: Factorización

Si un sistema tiene un cero en una posición determinada, o en su posición recíproca conjugada, posee el mismo módulo (atenuado o amplificado por un factor de ganancia), sin embargo, la fase es diferente.

sistema paso todo:

$$\text{cero} = \left(\frac{1}{\text{polo}}\right)^*$$



sistema de fase mínima:

aquel que tiene todos los ceros y los polos dentro de la circunferencia unidad.

sistema de fase máxima:

aquel que tiene todos los ceros fuera del círculo unidad

sistema de fase mixta:

aquel con ceros dentro y fuera del círculo unidad

- El sistema inverso de un sistema de fase mínima causal y estable es también causal y estable

Sistemas FIR causales y reales de fase lineal

Tipo I: centro de simetría en una muestra (nº de muestras impar), M es par

Tipo II: centro de simetría entre muestras (nº de muestras par), M es impar

Tipo III: centro de "simetría" en una muestra (nº de muestras impar), M es par

Tipo IV: centro de "simetría" entre muestras (nº de muestras par), M es impar

"simetría" ⇒ 1ª muestra = − última muestra
2ª muestra = − penúltima muestra
⋮
⇒ En tipo III $h[0] = 0$

Sistemas FIR Tipos I y II:

- si existe un cero en determinada posición, también existirá otro cero en su recíproca conjugada y en las conjugadas de las anteriores (ceros en cuádruplas)
- si existe un cero en la circunferencia unidad, existirá en su posición conjugada (ceros en duplas)
- si existe un cero en el eje Real existirá en su posición conjugada (ceros en duplas)
- si existe un cero en $z_0 = \pm 1$ éste puede aparecer aislado

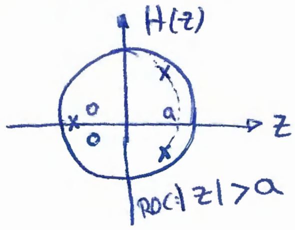
En Tipo II: existe siempre un cero aislado en $z_0 = -1$

En Tipo III: existe siempre un cero aislado en $z_0 = -1$ y en $z_0 = +1$

En Tipo IV: existe siempre un cero aislado en $z_0 = +1$

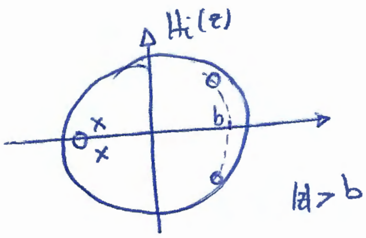
Ej 5.10

Sistema LTI causal



puede ser el sistema inverso causal y estable?

$$H_i(z) = \frac{1}{H(z)} \Rightarrow \begin{cases} H_i(z) \cdot H(z) = 1 \\ ROC_{H(z)} \cap ROC_{H_i(z)} \neq \{\emptyset\} \end{cases}$$



núm ceros > núm polos $\Rightarrow$ NO causal

sí estable

Ej 5.28

Sistema causal

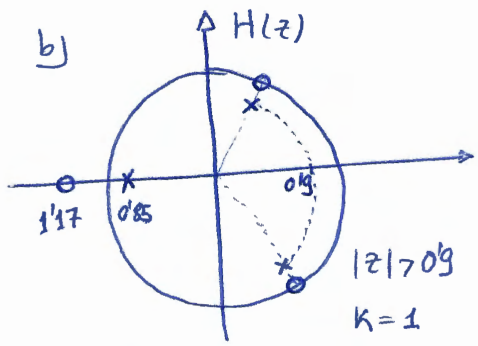
$$H(z) = \frac{(1-e^{j\frac{\pi}{3}} z^{-1})(1-e^{-j\pi/3} z^{-1})(1+1{,}1765\, z^{-1})}{(1-0{,}9\cdot e^{j\pi/3} z^{-1})(1-0{,}9\cdot e^{-j\pi/3} z^{-1})(1+0{,}85\cdot z^{-1})}$$

a) Ec. en diferencias

b) Diagrama de polos y ceros

c) Dibujar $|H(e^{jw})|$

b)



a) $H(z) = \frac{Y(z)}{X(z)}$

siendo: $(1-re^{jw_0}z^{-1})(1-re^{-jw_0}z^{-1}) = 1 - 2r\cos(\theta)z^{-1} + r^2 z^{-2}$

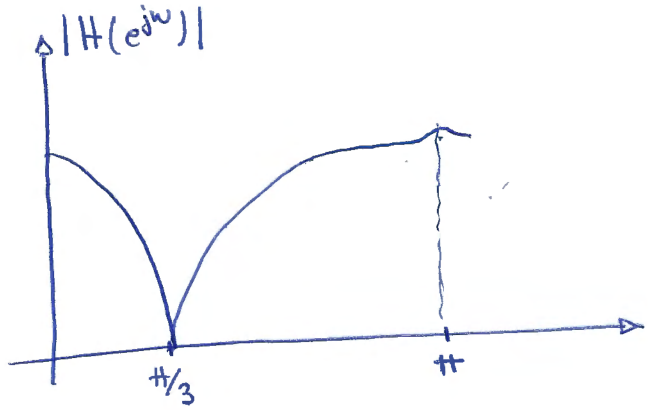
$$H(z) = \frac{(1-2\cos(\pi/3)z^{-1}+z^{-2})(1+1{,}1765\, z^{-1})}{(1-1{,}8\cos(\pi/3)z^{-1}+0'81 z^{-2})(1+0'85\, z^{-1})} =$$

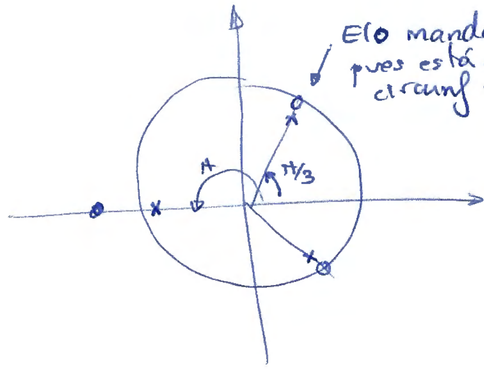
$$= \frac{1 - 2\cos(\pi/3)z^{-1} + z^{-2} + 1{,}17 z^{-1} + 2\cdot 1{,}17\cdot\cos(\pi/3)\cdot z^{-2} + 1{,}17\cdot z^{-3}}{1 - 1{,}8\cdot\cos(\pi/3)z^{-1} + 0'81\cdot z^{-2} + 0'85 z^{-1} + 1'8\cdot 0'85\cos(\pi/3)z^{-2} + 0'81\cdot 0'85 z^{-3}}$$

$$y[n] - (1{,}8\cdot\cos(\pi/3) + 0'85)\, y[n-1] + (0'81 + 1{,}8\cdot 0'85\cos(\pi/3))\, y[n-2] + (0{,}81\cdot 0'85)\, y[n-3] =$$

$$= x[n] + (-2\cos(\pi/3) + 1{,}17)\cdot x[n-1] + (1 + 2\cdot 1{,}17\cos(\pi/3))\, x[n-2] + 1{,}17\cdot x[n-3]$$

c) $$H(z) = \frac{(1 - e^{j\pi/3} z^{-1})(1 - e^{-j\pi/3} z^{-1})(1 + 1{,}1765\, z^{-1})}{(1 - 0{,}9\, e^{j\pi/3} z^{-1})(1 - 0{,}9 \cdot e^{-j\pi/3} z^{-1})(1 + 0'85 \cdot z^{-1})}$$





El o manda pues está sobre circunf r=1

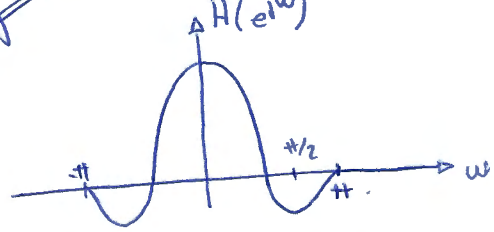
d) V ó F:

- Estable : V
- la respuesta al impulso si $n\uparrow\uparrow$ se aproxima a una cte $\neq 0$

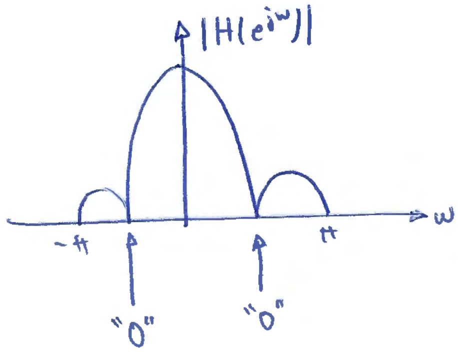
  $\lim_{n \to \infty} h[n] = \text{cte} \neq 0$ F (si estable $\Rightarrow$ tiene que ser sumable)
- máximo local en $w = \pi/3$ F
- sistema de fase mínima F
- $\exists$ sistema inverso causal y estable F

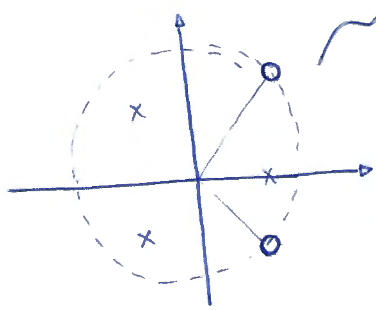
Ej 5.50: La T.F. de un sistema es real

tiene sistema inverso estable?



$\Downarrow$

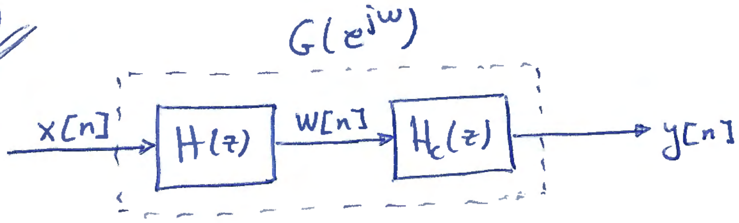




No puede haber ningún polo en el círculo unidad, luego NO existe sistema inverso estable

los polos tienen varias interpretaciones, pero deberán estar dentro de la circunf. unidad

Ej 5.70



$H(z)$ NO es de fase mínima

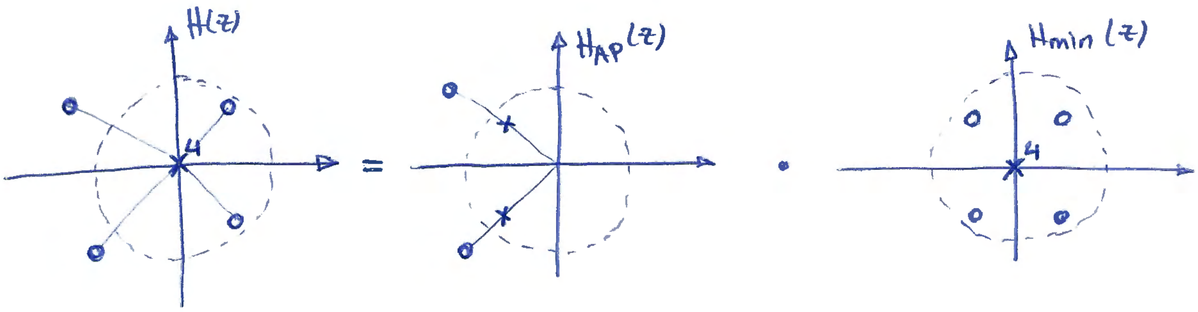
a) $H_c(z)$? que sea: $\begin{cases} \text{causal y estable} \\ |H(e^{jw})| \cdot |H_c(e^{jw})| = 1 \\ H(z) = H_{AP}(z) \cdot H_{min}(z) \end{cases}$

$$H(z) = H_{AP}(z) \cdot H_{min}(z) \Rightarrow H_c(z) = \frac{1}{H_{min}(z)}$$

b) $G(z)$?

$$G(z) = H_{AP}(z) \cdot \cancel{H_{min}(z)} \cdot \frac{1}{\cancel{H_{min}(z)}} = H_{AP}(z)$$

c) $H(z) = (1 - 0'8\, e^{j0,3\pi} z^{-1})(1 - 0'8\, e^{-j0'3\pi} z^{-1})(1 - 1,2\, e^{j0'7} z^{-1})(1 - 1,2\, e^{j0'7} z^{-1})$



$$H_{AP}(z) = \frac{\left(\frac{1}{1'2}\right)^2 - \frac{2}{1'2} \cdot \cos(0,7\pi)\, z^{-1} + z^{-2}}{1 - \frac{2}{1,2} \cdot \cos(0'7\pi) z^{-1} + \underbrace{\left(\frac{1}{1,2}\right)^2}_{k} z^{-2}} \quad \begin{cases} \text{ceros en } 1,2 \cdot e^{\pm j0'7\pi} \\ \text{polos en } \frac{1}{1'2}\, e^{\pm j0'7\pi} \end{cases}$$

$k = 1$ $=$ $k = \left(\frac{1}{1'2}\right)^2$ $\cdot$ $k = (1'2)^2$

ROC: $\forall z - \{0\}$ $\quad$ ROC: $|z| > \frac{1}{1'2}$ $\quad$ ROC: $\forall z - \{0\}$

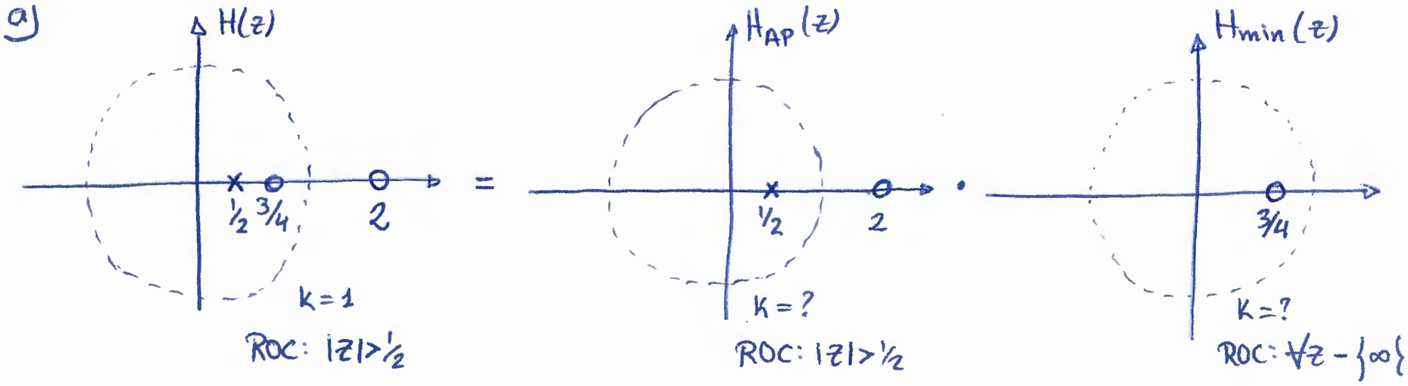
Ej 5.33

$$H(z) = \frac{(1-2z^{-1})(1-0'75\cdot z^{-1})}{z^{-1}\cdot(1-0'5z^{-1})} = \frac{(z-2)(z-0'75)}{(z-0,5)}$$

a) $H(z) = H_{AP}(z)\cdot H_{min}(z)$; $|H_{AP}(e^{jw})| = 1 \ \forall w$

b) $H(z) = H_{minz}(z)\cdot H_{lin}(z)$

a)

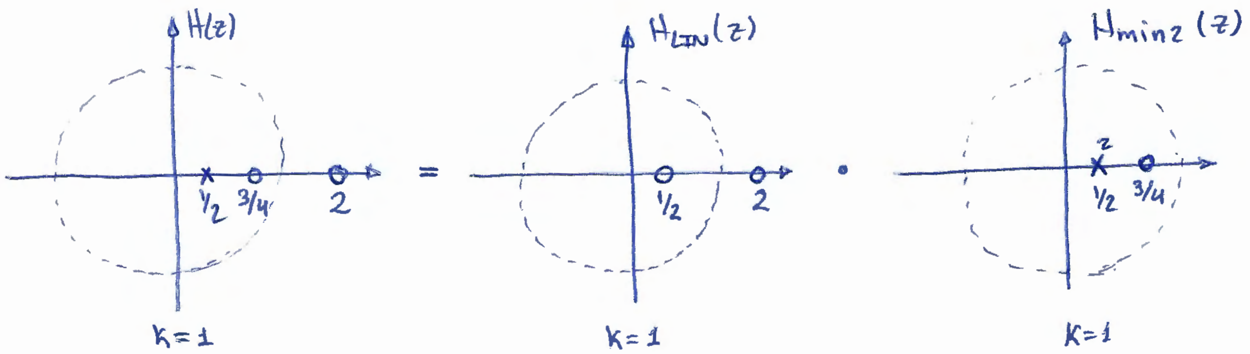


$$H_{AP}(z) = \frac{K(1-2z^{-1})}{(1-\frac{1}{2}z^{-1})} \qquad H_{min}(z) = (z-0'75)$$

"0" en 3/4

$$H_{AP}(z)\Big|_{z=1} = 1 = \frac{K(1-2)}{1-1/2} \Rightarrow K = \left(-\tfrac{1}{2}\right)^{-1} \Rightarrow K = -2$$

b)



$$H_{LIN}(z) = (z-1/2)(z-2) \qquad H_{minz}(z) = \frac{(z-0'75)}{(z-1/2)^2}$$

Ej 5.65 Efecto antiblurring : (fotografía)

$$\underset{\text{deseada}}{x[n]} \longrightarrow \boxed{h[n]} \longrightarrow \underset{\text{observada}}{y[n]}$$

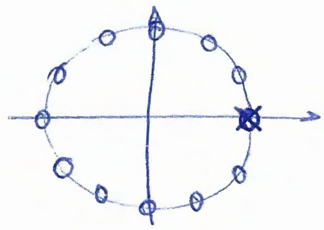
$$h[n] = \begin{cases} 1 \ , & 0 \le n \le M-1 \\ 0 \ , & \text{resto} \end{cases}$$

se quiere obtener la imagen no movida:

$$y[n] \longrightarrow \boxed{H_{inv}} \longrightarrow \hat{x}[n]$$

a) $H_{inv}(z) = \dfrac{1}{H(z)}$

$$H(z) = \sum_{n=0}^{M-1} 1 \cdot z^{-n} = \frac{1 - z^{-1} \cdot z^{-(M-1)}}{1 - z^{-1}} = \frac{1 - z^{-M}}{1 - z^{-1}}$$



No es un sistema de fase mínima $\Rightarrow$ No tiene inverso

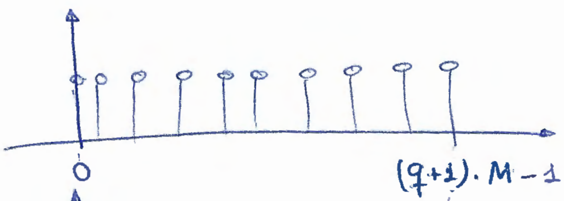
b) $H_{inv}(z) = H_1(z) \cdot H_2(z)$

$$y[n] \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{h_2[n]} \longrightarrow w[n] \simeq \hat{x}[n]$$

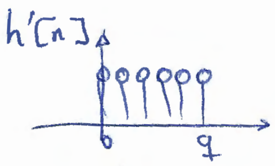
$$h_1[n] = \sum_{k=0}^{q} \delta[n - k \cdot M]$$

$$h_2[n] = \delta[n] - \delta[n-1]$$

$h[n] * h_1[n]$



$h[n] * h_2[n]$



$$H'(z) = \frac{1 - z^{-(q+1)}}{1 - z^{-1}} \Rightarrow H_1(z) = \frac{1 - z^{-M(q+1)}}{1 - z^{-M}}$$

$$H_{TOT}(z) = H(z) \cdot H_1(z) \cdot H_2(z) = 1 - z^{-M(q+1)}$$

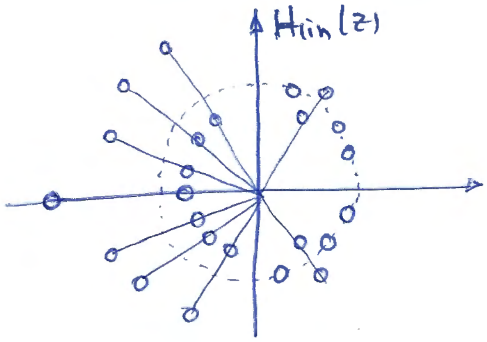
$$h_1[n] = [h'[n]]_{\uparrow M}$$

$$h'[n] \longrightarrow \boxed{\uparrow M} \longrightarrow h_1[n]$$

Ej 5.76. Filtro FIR causal, real y de fase lineal

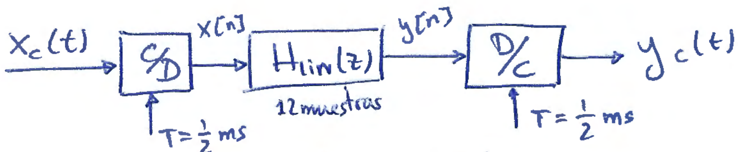
Diagrama de ceros:

tiene tantos polos como ceros (24)



a) $0'5_{\angle 153}$

b)

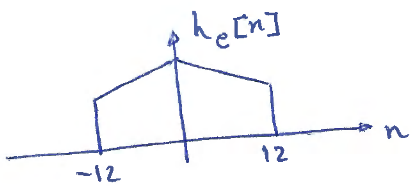


¿retardo de $x_c(t)$ a $y_c(t)$?

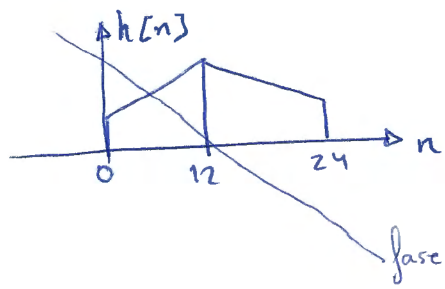
c) Dibujar $20 \cdot \log_{10} |H_{lin}(e^{j\omega})|$

a) $0'5_{\angle 153}$, $0'5_{\angle -153}$, $2_{\angle 153}$, $2_{\angle -153}$ (aparecen de 4 en 4)

b)



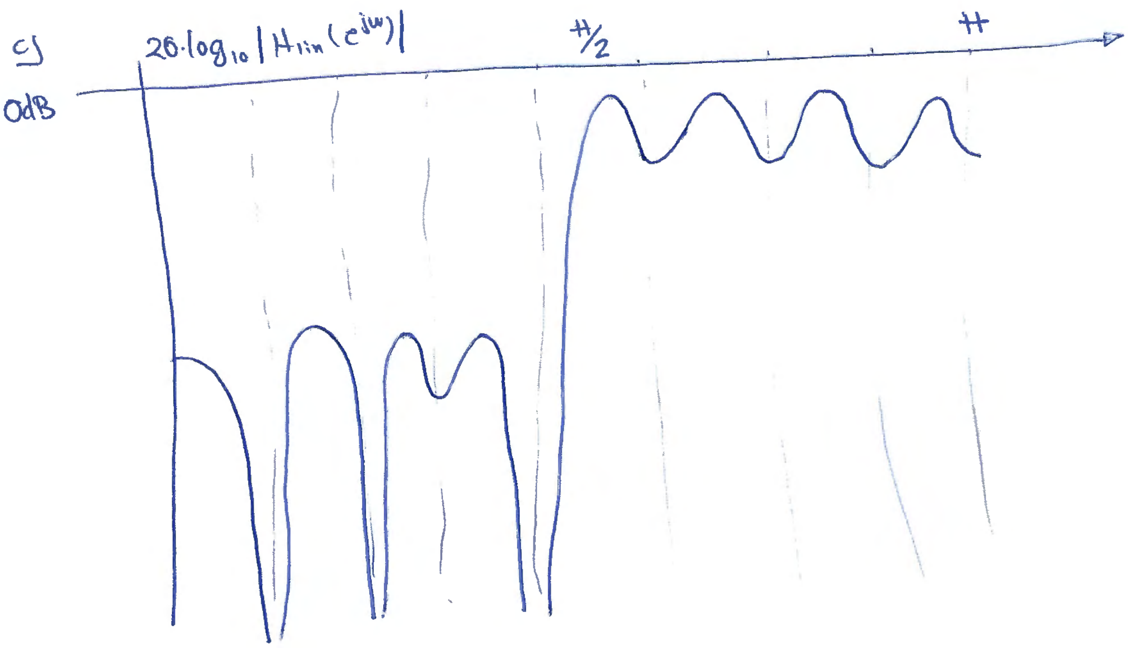
$\rightarrow$



$\measuredangle = 0 \ \forall n$

12 muestras $\Rightarrow$ 6 ms

c)

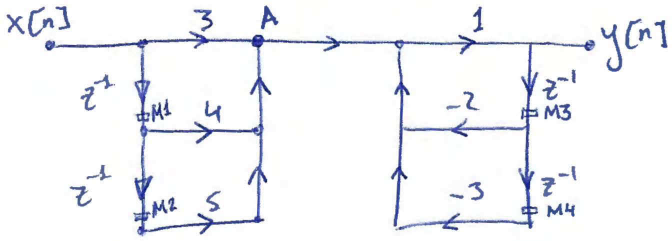


Ej:

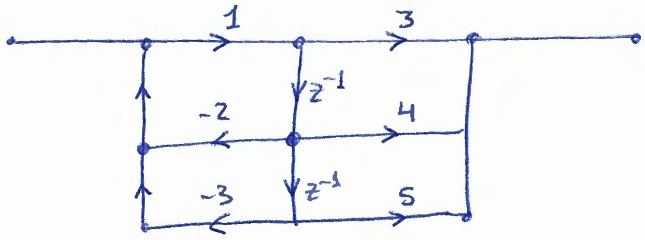
$$H(z) = \frac{3 + 4z^{-1} + 5z^{-2}}{1 + 2z^{-1} + 3z^{-2}}$$

FIR ⇒ numerador
IIR ⇒ denominador

Directa tipo I

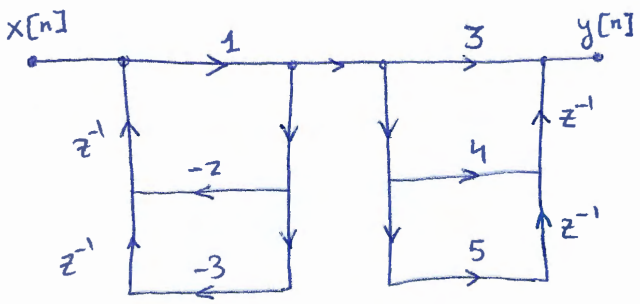


Directa tipo II o canónica

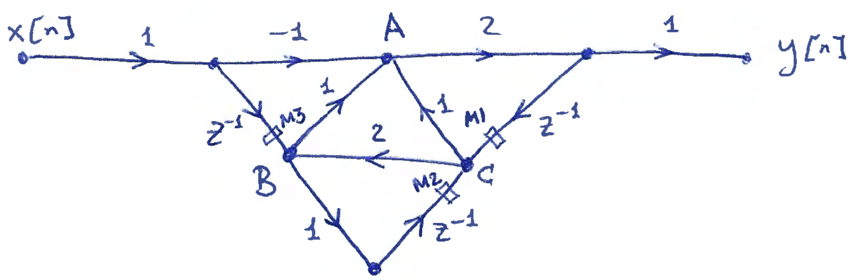



```
M1 = M2 = M3 = M4 = 0
for  i = 1 : dataLength
    A = 3 x[i] + 4 M1 + 5 M2;
    y[i] = A + (-2) M3 + (-3) M4;
    M2 = M1;
    M1 = x[i];
    M4 = M3;
    M3 = y[i];
end;
```

Traspuesta tipo I



Ej 6.12



a) Ec. en diferencias:

$$A = (-1)\cdot X(z) + B + C$$
$$B = X(z)\cdot z^{-1} + 2C$$
$$C = Y(z)\cdot z^{-1} + B z^{-1}$$
$$Y(z) = 2\cdot A$$

$$B = X(z)\cdot z^{-1} + 2\left(Y(z)\cdot z^{-1} + B z^{-1}\right)$$
$$\hookrightarrow B(1-2z^{-1}) = X(z)\cdot z^{-1} + 2\cdot z^{-1} Y(z)$$

$$C = Y(z) z^{-1} + z^{-1}\left(\frac{X(z) z^{-1} + 2z^{-1}\cdot Y(z)}{1-2z^{-1}}\right)$$
$$\hookrightarrow C = Y(z)\cdot\left(z^{-1} + \frac{2z^{-2}}{1-2z^{-1}}\right) + \frac{X(z)\, z^{-2}}{(1-2z^{-1})}$$

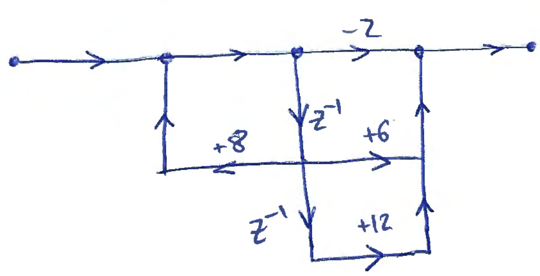
$$Y(z) = 2\left[(-1)X(z) + \frac{X(z) z^{-1} + 2z^{-1} Y(z)}{(1-2z^{-1})} + \right.$$
$$\left. + Y(z)\left(z^{-1} + \frac{2z^{-2}}{1-2z^{-1}}\right) + \frac{X(z)\cdot z^{-2}}{1-2z^{-1}}\right]$$

$$Y(z)(1-2z^{-1}) = 2(-1)(1-2z^{-1})X(z) + 2X(z)\cdot z^{-1} + 4z^{-1}Y(z) + 2z^{-1}(1-2z^{-1})Y(z) +$$
$$+ 4z^{-2}Y(z) + 2z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-2 + 6z^{-1} + 2z^{-2}}{1-8z^{-1}}$$

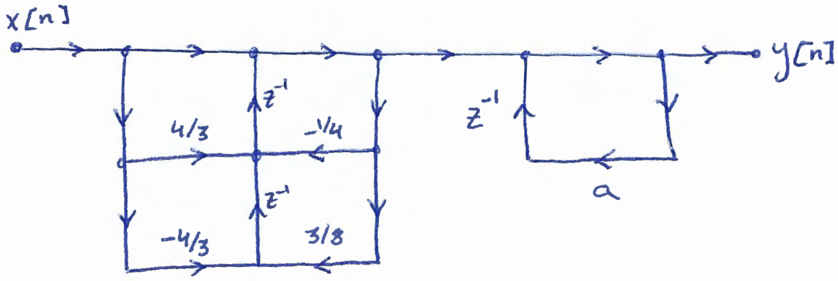
$$\hookrightarrow y[n] = -2x[n] + 6x[n-1] + 2x[n-2] + 8y[n-1]$$

b) Pinta y colorea




```
M1 = M2 = M3 = 0
for (i = 0; i < dataLength ; i++) {
    A = (-1)·x[i] + M3 + 2 (M1 + M2) + (M1 + M2);
    y[i] = 2·A;
    M2 = M3 + 2(M1 + M2)
    M1 = y[i]
    M3 = x[i]
}
```

Ej 6.18:



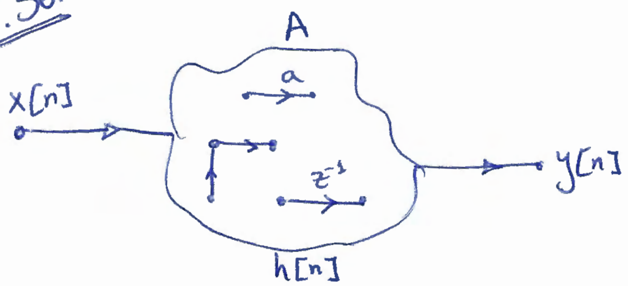
Para ciertos valores de 'a' el diagrama representa a un sistema de segundo orden. ¿Qué valores?

$$H(z) = \frac{1 + \frac{4}{3} z^{-1} - \frac{4}{3} z^{-2}}{1 + \frac{1}{4} z^{-1} - \frac{3}{8} z^{-2}} \cdot \frac{1}{1 - a z^{-1}}$$

$$r_{1,2} = \frac{-4/3 \pm \sqrt{\frac{16}{9} + \frac{16}{3} \cdot \frac{48}{9}}}{2} = \frac{-4/3 \pm 8/3}{2} = \begin{cases} 2/3 \\ -2 \end{cases}$$

$$\boxed{\begin{aligned} a_1 &= 2/3 \\ a_2 &= -2 \end{aligned}}$$

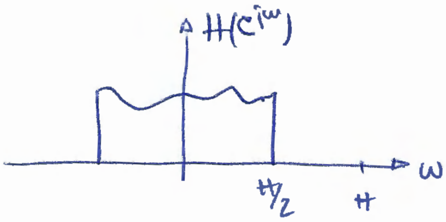
Ej 6.36:



Se quiere modificar A para crear otro de forma que:

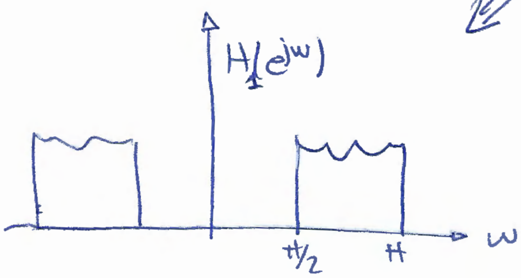
$$h_1[n] = (-1)^n \cdot h[n]$$

a) Si $H(e^{jw})$ es: Dibujar $H_1(e^{jw})$



$$h_1[n] = (-1)^n \cdot h[n] = e^{j\pi n} \cdot h[n]$$

$$e^{jw_0 n} \cdot x[n] \leftrightarrow X(e^{j(w-w_0)})$$



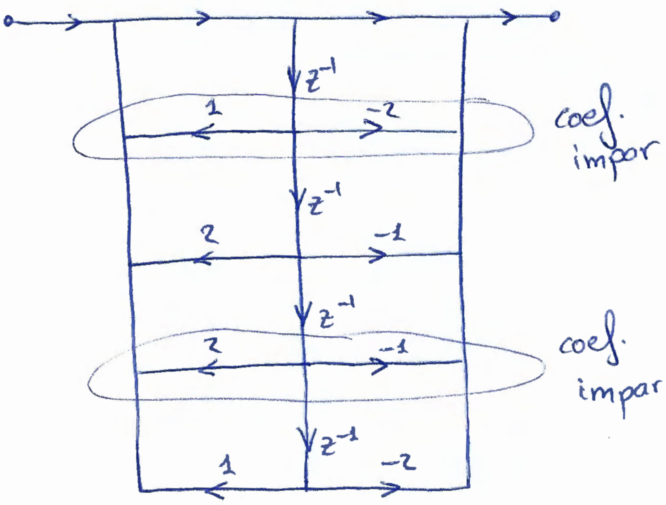
b) Cómo modificar A para obtener A_1

$$e^{j\pi n} h[n] \leftrightarrow H(-z)$$

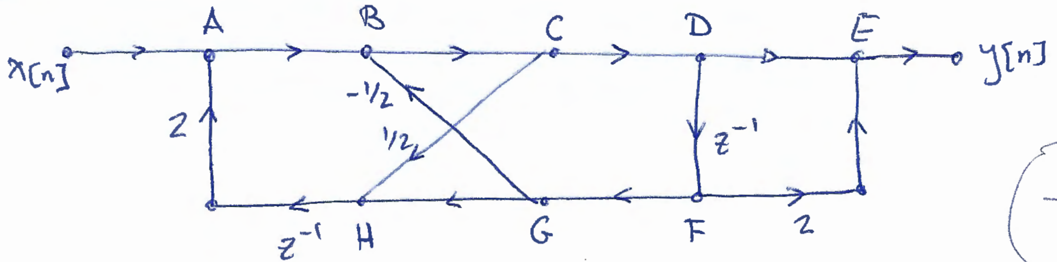
$$H_1(z) = H(-z) = \frac{\sum_{k=0}^{M} (-1)^k b[k] z^{-k}}{\sum_{k=0}^{N} (-1)^k a[k] \cdot z^{-k}}$$

Habría que multiplicar los coef. impares por -1

c) Sea A:

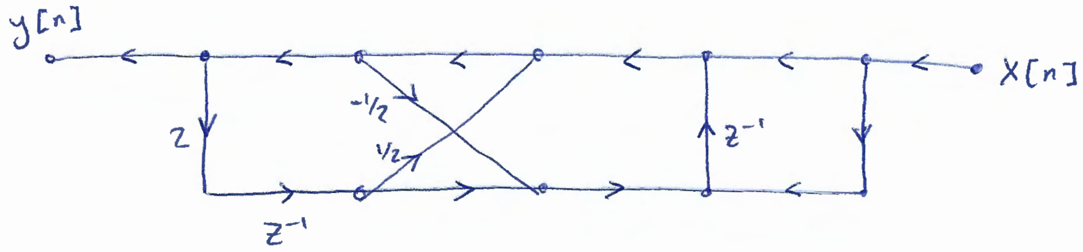


Ej 6.32:



Nota: es causal sólo tiene sumas y retardos (z^{-1})

a) Flujograma traspuesto:



b) Ec. en diferencias:

$A = X(z) + 2z^{-1} \cdot H$

$B = A + \left(\frac{-1}{2}\right) G$

$C = B$

$D = C$

$E = D + 2F$

$F = D \cdot z^{-1}$

$G = F$

$H = G + \frac{1}{2} C$

$Y(z) = E$

$B = X(z) + 2 \cdot z^{-1} \cdot H + \frac{-1}{2} F$

$F = D \cdot z^{-1}$

$H = \frac{1}{2} B + F$

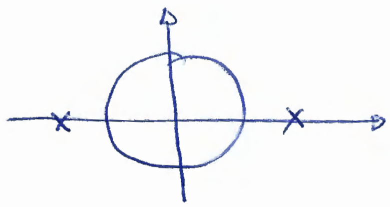
$Y(z) = B + 2F$

$$Y(z)\left(1 - \frac{1}{2}z^{-1} - 2z^{-2}\right) = X(z) \cdot (1 + 2z^{-1})$$

$$y[n] = x[n] + 2x[n-1] + \frac{1}{2}y[n-1] + 2y[n-2]$$

c) Estabilidad:

$$H(z) = \frac{1 + 2z^{-1}}{1 - \frac{1}{2}z^{-1} - 2z^{-2}} \Rightarrow \frac{+\frac{1}{2} \pm \sqrt{\frac{33}{4}}}{2} = \begin{cases} 1,68 \\ -1,18 \end{cases}$$



$\Rightarrow$ No estable

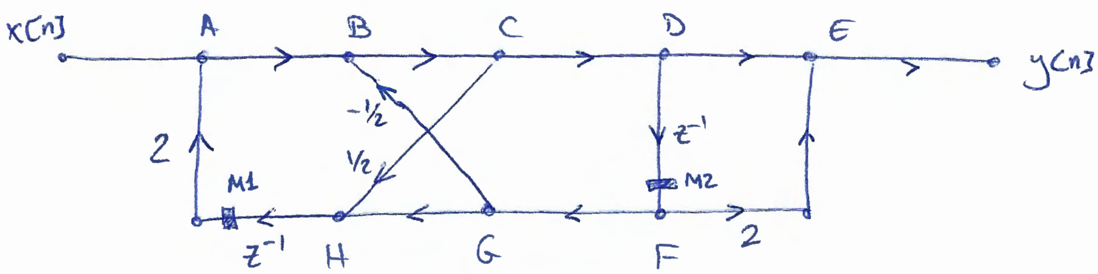
d) $x[0] = 1$, $x[1] = \frac{1}{2}$, $x[2] = \frac{1}{4}$ ... ; $y[2]$?

$$y[0] = x[0] + 2\underbrace{\cancel{x[-1]}}_{0} + \frac{1}{2}\underbrace{\cancel{y[-1]}}_{0} + 2\underbrace{\cancel{y[-2]}}_{0} = 1$$

$$y[1] = x[1] + 2x[0] + \frac{1}{2}y[0] + 2y[-1] = 3$$

$$y[2] = x[2] + 2x[1] + \frac{1}{2}y[1] + 2y[0] = \underline{4,75}$$

e) Programación:

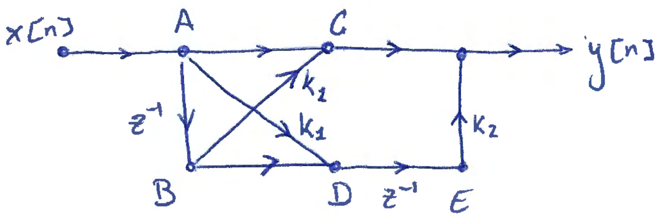



```
M1 = M2 = 0;
for i=1:datalength
  A = x[i] + 2·M1;
  F = M2;
  G = F;
  B = A+(-1/2) G;
  C = B;
  D = C;
  E = D+2F;
  H = 1/2 C + G;
  y[i] = E;
  M1 = H;
  M2 = D;
end;
```

feb 06:

$E \leftarrow D$
$B \leftarrow A$ } Actualización memorias
$A \leftarrow \text{leer}(x[n])$
$D \leftarrow B + k_1 \cdot A$
$C \leftarrow k_1 \cdot B + A$
$Y \leftarrow k_2 \cdot E + C$ } Resolución flujograma

obtener flujograma

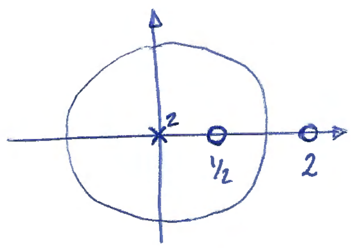


sep 11:

$H(z)$ es LTI causal y estable y:

- → FIR de fase lineal (real y causal)
- → tiene 3 coeficientes distintos de cero $(n=0,1,2)$
- → tiene 1 cero en $z = 1/2$
- → $H(z)\big|_{z=1} = 1$

a) $H(z)$ y $h[n]$



ROC: $\forall z - \{0\}$

$K = ?$

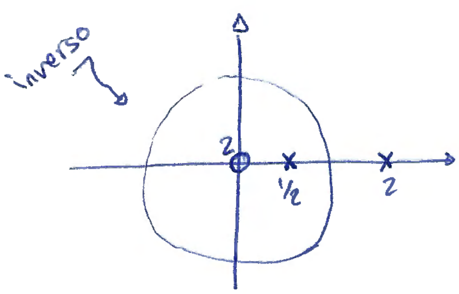
$$H(z) = k\,(1-\tfrac{1}{2}z^{-1})(1-2z^{-1})$$

$$H(z)\big|_{z=1} = 1 = k\,(1-\tfrac{1}{2})(1-2) \Rightarrow k = -2$$

$$H(z) = -2\left[1 - 2z^{-1} - \tfrac{1}{2}z^{-1} + z^{-2}\right]$$

$$h[n] = -2\,\delta[n] + 5\,\delta[n-1] - 2\,\delta[n-2]$$

b) Tiene inverso estable?



será estable si $1/2 < |z| < 2$

$$H_{inv}(z) = \frac{-1}{2} \cdot \frac{1}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

$$H_{inv}(z) = \frac{A}{(1-\frac{1}{2}z^{-1})} + \frac{B}{(1-2z^{-1})} \; ; \; A = 1/6 \, , \; B = \frac{-2}{3}$$

$$h_{inv}[n] = \frac{1}{6}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{3}\, 2^n\, u[-n-1]$$

Tema 4: DFT (Discrete Fourier Transform)

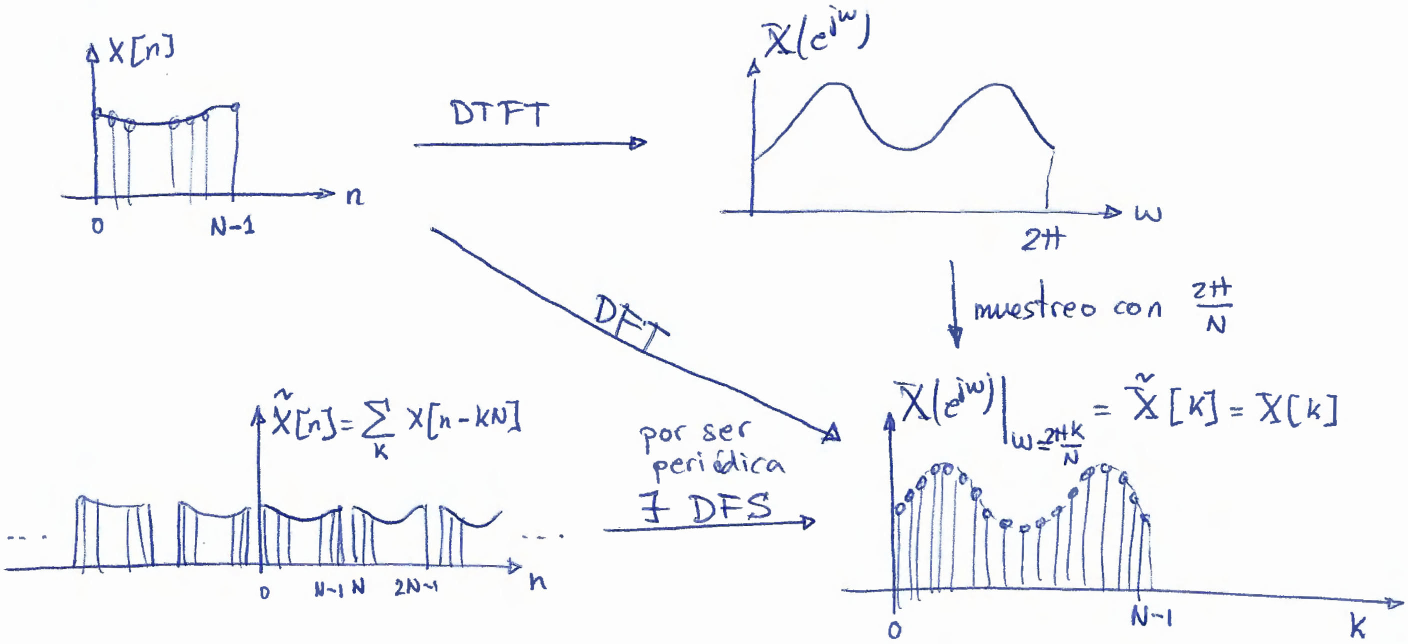
1. Secuencias periódicas
   - a) DFS (Discrete Fourier Series)
   - b) Propiedades de la DFS
   - c) Relación entre la DFS y la TF de secuencias (DTFT)
2. Muestreo de la Transformada de Fourier (DTFT)
   - a) Conclusiones respecto al muestreo de la TF
   - b) Relación entre la secuencia original y la recuperada
3. La DFT
   - a) Interpretaciones de la DFT
   - b) Propiedades de la DFT
4. Aplicación de la DFT: filtrados
   - a) Filtrado de secuencias
   - b) Filtrado de secuencias muy largas
5. La DCT (Discrete Cosine Transform)

La DFT se aplica a señales discretas causales y de dimensión finita.

Son muestras equiespaciadas de la transformada de Fourier (DTFT)

Importantísima importancia en la implementación de algoritmos (alta eficiencia)

DTFT = Discrete Time Fourier Transform



$$\tilde{X}(e^{jw}) = \frac{2\pi}{N} X(e^{jw})\Big|_{w=2\pi k/N}$$

Nomenclatura empleada:

secuencias aperiódicas

$x[n]$

$x[n]\Big|_0^{N-1}$

secuencias periódicas

$\tilde{x}[n]$

$x[(n \text{ módulo } N)] = x[((n))_N]$

$\llcorner$ secuencia periódica de periodo N donde $x[n]$ es su periodo

se define la señal W_N como:

$$W_N = e^{-j\frac{2\pi}{N}}$$

extracción de un periodo de una sec. per.

$\tilde{x}[n] \quad 0 \le n \le N-1$

$x[((n))_N] \quad 0 \le n \le N-1$

1. Secuencias Periódicas

$$\tilde{x}[n] = \tilde{x}[n + r\cdot N] \quad ; \quad -\infty < r < +\infty$$

construcción de secuencias periódicas a partir de sec. aperiódicas:

$$\tilde{x}[n] = x[n] + \sum_{k=-\infty}^{+\infty} \delta[n-kN] = \sum_{k=-\infty}^{+\infty} x[n-kN]$$

Existen infinitas señales que producen la misma $\tilde{x}[n]$, pero sólo existe una que ocupe menos que su periodo N $(x[n])$

Fórmulas de Poisson:

$$x[n] \xrightarrow{TF} X(e^{jw})$$

$$\tilde{x}[n] = \sum_{k} x[n-kN] \xrightarrow{TF} \frac{2\pi}{N} \sum_{k} X(e^{jw}) \cdot \delta\left(w - \frac{2\pi k}{N}\right)$$

$$x(t) \xrightarrow{TF} X(j\Omega)$$

$$\tilde{x}(t) = \sum_{k} x(t-kT) \xrightarrow{TF} \frac{2\pi}{T} \sum_{k} X(j\Omega)\, \delta\left(\Omega - \frac{2\pi k}{T}\right)$$

a) DFS

$$e^{j\frac{2\pi k}{N} n} = e^{j\frac{2\pi k}{N}(n+N)} = e^{j\frac{2\pi k}{N} n} \cdot \underbrace{e^{j\frac{2\pi k}{\cancel{N}}\cancel{N}}}_{=0 \text{ si } k\in\mathbb{Z}}$$

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]\, e^{-j\frac{2\pi k}{N} n} = \sum_{n=0}^{N-1} \tilde{x}[n] \cdot W_N^{k\cdot n} \quad ; \quad -\infty < k < \infty$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k]\, e^{+j\frac{2\pi k}{N} n} = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] \cdot W_N^{-kn} \quad ; \quad -\infty < n < \infty$$

Ej: $\tilde{x}[n] = \sum_{k=-\infty}^{+\infty} \delta[n-kN] \Rightarrow \tilde{X}[k] = \sum_{n=0}^{N-1} \delta[n]\, e^{-j\frac{2\pi k}{N} n} = 1 \quad \forall k$

$$\tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} e^{j\frac{2\pi k}{N} n}$$

b) Propiedades de la DFS

$\tilde{x}_1[n]$ y $\tilde{x}_2[n]$ sean de periodo N

- linealidad:

$$\left.\begin{array}{l} \tilde{x}_1[n] \xrightarrow{DFS} \tilde{X}_1[k] \\ \tilde{x}_2[n] \xrightarrow{DFS} \tilde{X}_2[k] \end{array}\right\} a\cdot\tilde{x}_1[n] + b\,\tilde{x}_2[n] \longrightarrow a\tilde{X}_1[k] + b\,\tilde{X}_2[k]$$

- desplazamiento:

$$\tilde{x}[n] \xrightarrow{DFS} \tilde{X}[k]$$

$$\tilde{x}[n-m] \longrightarrow \tilde{X}[k]\cdot e^{-j\frac{2\pi k}{N}m} = \tilde{X}[k]\cdot W_N^{km}$$

- simetría:

  ídem que para DTFT

- convolución circular:

$$\tilde{x}_3[n] = \tilde{x}_1[n] \circledast \tilde{x}_2[n] = \underbrace{\sum_{m=0}^{N-1} \tilde{x}_1[m]\cdot\tilde{x}_2[n-m]}_{\text{muy feo!}} \longrightarrow \tilde{X}_1[k]\cdot\tilde{X}_2[k]$$

mejor hacemos:

① extracción de periodos:

$$x_1[n] = \tilde{x}_1[n] \quad ; \quad 0 \le n \le N-1$$
$$x_2[n] = \tilde{x}_2[n] \quad ; \quad 0 \le n \le N-1$$

② convolución lineal:

$$x_3[n] = x_1[n] * x_2[n]$$

③ extensión periódica:

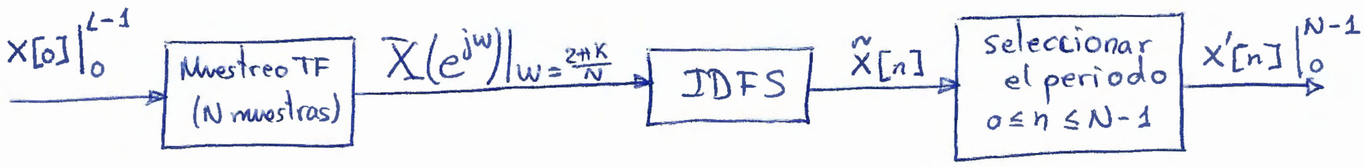
$$\tilde{x}_3[n] = \sum_{k=-\infty}^{+\infty} x_3[n-kN]$$

c) Relación entre DFS y DTFT

$$\tilde{x}[n] = \sum_{k=-\infty}^{+\infty} x[n-kN] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}[k]\, e^{+j\frac{2\pi k}{N}\cdot n}$$

$$\tilde{X}[k] = X(e^{jw})\Big|_{w=\frac{2\pi k}{N}} \qquad 0 \leq k \leq N-1$$

2. Muestreo de TF



1) muestreo de TF:

$$\tilde{X}[k] = X(e^{jw})\Big|_{w=\frac{2\pi k}{N}} \qquad 0 \leq k \leq N-1$$

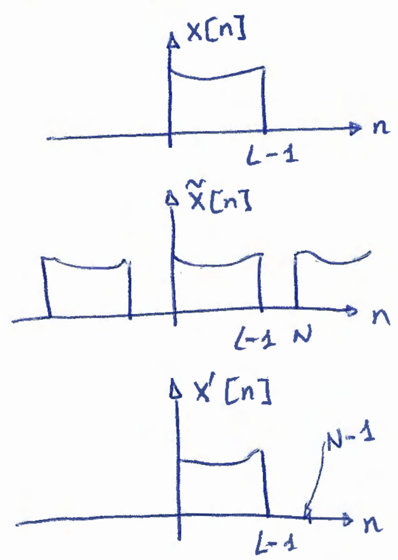
2) obtención de la secuencia periódica:

$$\tilde{x}[n] = \sum_{k=0}^{\infty} x[n-kN]$$

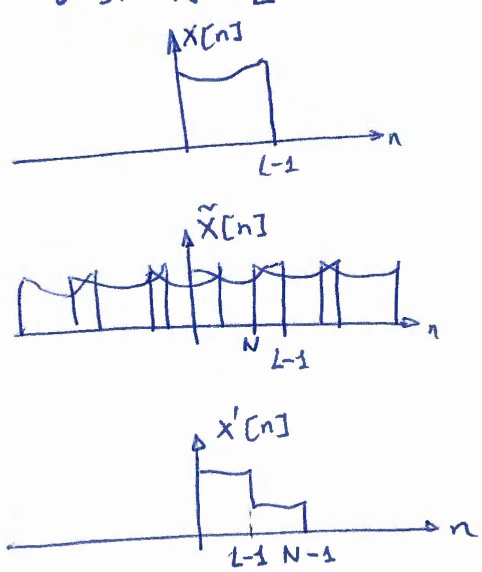
3) selección del periodo:

$$x'[n] = \tilde{x}[n] \qquad 0 \leq n \leq N-1$$

• si $N \geq L$:



• si $N < L$



b) Relación entre la secuencia original $x[n]\big|_0^{L-1}$ y la recuperada $x'[n]\big|_0^{N-1}$

- $N \geq L$:

$$x'[n] = \begin{cases} x[n] & 0 \leq n \leq L-1 \\ 0 & L-1 \leq n \leq N-1 \end{cases} \Leftrightarrow x[n] \text{ rellenada con ceros}$$

- $N < L$:

$$x'[n] = \sum_{K=-\infty}^{+\infty} x[n-KL] \quad 0 \leq n \leq N-1$$

3. La DFT

$$DFT^{(N)}\{x[n]\} = X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi K}{N} n} = \sum_{n=0}^{N-1} x[n] \cdot W_N^{kn} \quad 0 \leq K \leq 1$$

$$IDFT^{(N)}\{X[k]\} = x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi kn}{N}} = \frac{1}{N} \sum_{K=0}^{N-1} X[k] W_N^{-kn} \quad 0 \leq n \leq N-1$$

- si $N \geq L$ se rellena con ceros (zero-padding)
- si $N < L$ $\nexists$ la DFT

jun08 2 son de fase lineal, 3 tienen el mismo módulo:

$$H_1(z) = 1 - 1,4 z^{-1} + 0,2 z^{-2} - 0,04 z^{-3} + 0,7 z^{-4} - 0,88 z^{-5} + 0,3 z^{-6}$$

$$H_2(z) = 0,1 - 0,5 z^{-1} + 2,5 z^{-2} + 2,5 z^{-4} - 0,5 z^{-5} + 0,1 z^{-6}$$

$$H_3(z) = 0,3 - 0,88 z^{-1} + 0,7 z^{-2} - 0,04 z^{-3} + 0,2 z^{-4} - 1,4 z^{-5} + z^{-6}$$

$$H_4(z) = 0,09 - 0,4 z^{-1} + z^{-2} - 1,4 z^{-3} + z^{-4} - 0'4 z^{-5} + 0'09 z^{-6}$$

$$H_5(z) = 0'8 - z^{-1} + 0'084 z^{-2} - 0'18 z^{-3} + 1,1 z^{-4} - 1,1 z^{-5} + 0,4 z^{-6}$$

a) Es de fase lineal si 1^er^ y último coeficiente son iguales: H_2 y H_4

b) Si tienen mismo módulo tendrán misma energía:

$E_1 = \sum (\ldots)^2 = 4.7$; $E_2 = 13,4$; $E_3 = 4,7$; $E_4 = 4,7$; $E_5 = 4,7 \Rightarrow H_1, H_3, H_5$ (No H_4)

c) H_2 y H_4 son de fase mixta

H_1 y H_3 tienen sus coeficientes cruzados

H_1 es de fase mínima (más energía en los 1os coef.)

H_3 es de fase máxima (más energía en los últimos coef.)

H_5 es de fase mixta

d) Todos son estables

e) $H_1(z)$ tiene inverso estable

f) Filtro paso todo será: $H_{AP}(z) = \dfrac{H_3(z)}{H_1(z)}$

b) Propiedades de la DFT

- Linealidad:

$$x_1[n] \xrightarrow{DFT^{(N)}} X_1[k] \qquad 0 \le k \le N-1$$

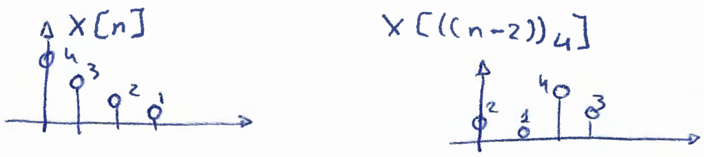
$$x_2[n] \xrightarrow{DFT^{(N)}} X_2[k]$$

$$a\,x_1[n] + b\,x_2[n] \xrightarrow{DFT^{(N)}} a\,X_1[k] + b\,X_2[k]$$

- Desplazamiento circular:

$$\underset{0 \le n \le N-1}{x[((n-m))_N]} \longrightarrow e^{-j\frac{2\pi k}{N}m} \cdot X[k]$$

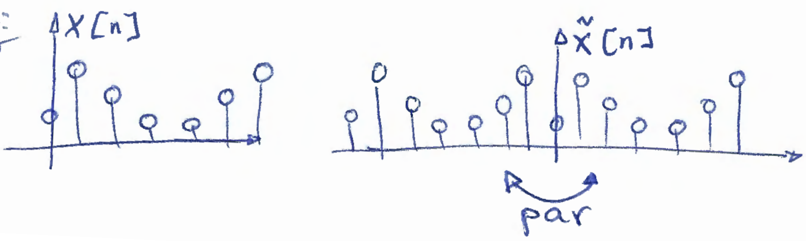
Ej:



- Simetría:

$x[n]$ es par periódica si $\tilde{x}[n] = x[((n))_N]$ es par

Ej:



ídem con impar periódica pero $x[0] = 0$

Siendo $x_{ep}[n]$ par periódica y $x_{op}[n]$ impar periódica, toda secuencia causal y de dimensión N:

$$x[n] = x_{ep}[n] + x_{op}[n] \quad 0 \leq n \leq N-1$$

$$x_{ep}[n] = \frac{1}{2}\left(x[n] + x[((-n))_N]\right) \quad 0 \leq n \leq N-1$$

$$x_{op}[n] = \frac{1}{2}\left(x[n] - x[((-n))_N]\right) \quad 0 \leq n \leq N-1$$

- Si $x[n] \in \mathbb{R}$:

$\hookrightarrow X[k] = X^*[((-k))_N] \quad 0 \leq k \leq N-1 \Rightarrow \begin{cases} \mathrm{Re}\{X[k]\} \text{ es par periódica} \\ \mathrm{Im}\{X[k]\} \text{ es impar periódica} \\ |X[k]| \text{ es par periódico} \\ \measuredangle X[k] \text{ es impar periódico} \end{cases}$

$\hookrightarrow x[n] = x_{ep}[n] + x_{op}[n]$

$$x_{ep}[n] \xrightarrow{DFT^{(N)}} X[k] = \mathrm{Re}\{X[k]\} \quad 0 \leq k \leq N-1$$

$$x_{op}[n] \xrightarrow{DFT^{(N)}} X[k] = \mathrm{Im}\{X[k]\} \quad 0 \leq k \leq N-1$$

- Convolución circular:

$$x_1[n] \longrightarrow X_1[k] \quad 0 \leq k \leq N-1$$

$$x_2[n] \longrightarrow X_2[k] \quad 0 \leq k \leq N-1$$

$$x_3[n] = x_1[n] \circledast^N x_2[n] \longrightarrow X_1[k] \cdot X_2[k], \quad 0 \leq k \leq N-1$$

$$= \sum_{m=0}^{N-1} x_1[m] \cdot x_2[((m-n))_N]$$

Sea $x_1[n]$ de longitud L, $x_2[n]$ de longitud P, $x_3[n] = x_1[n] \circledast x_2[n]$

$\hookrightarrow x_{cl}[n] = x_1[n] * x_2[n]$ de longitud $L+P-1$

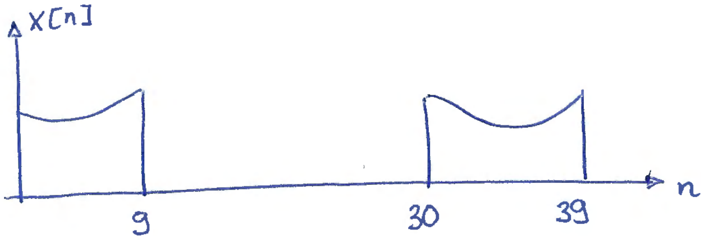
$\hookrightarrow \tilde{x}_3[n] = \sum_{k=-\infty}^{+\infty} x_{cl}[n-kN]$ (extensión periódica)

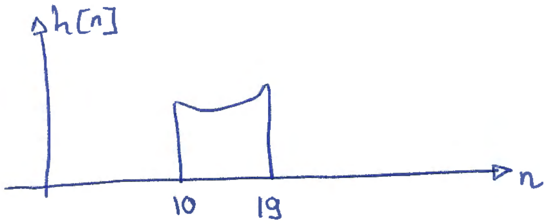
$\hookrightarrow x_3[n] = \tilde{x}_3[n] \quad 0 \leq n \leq N-1$ de dimensión N

si $N \geq L+P-1$:
conv. circular = conv. lineal

si $N < L+P-1$
conv. circular = conv. lineal con solapamiento temporal

sep05

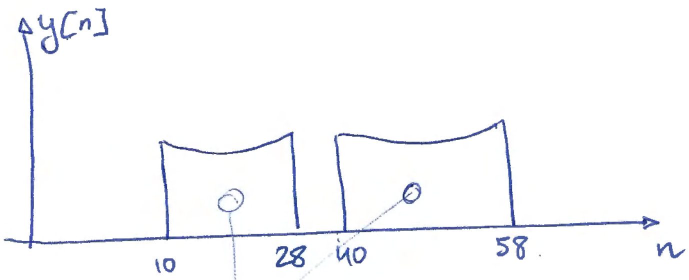


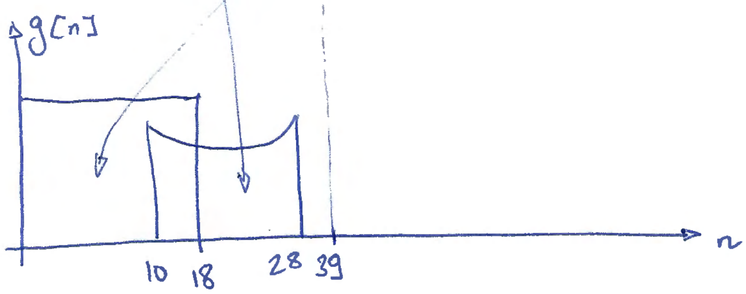


$y[n] = x[n] * h[n]$

$g[n] = x[n] \circledast^{40} h[n]$

$\hookrightarrow (N=40) < (L+P-1 = 59) \Rightarrow$ solapamiento temporal





Podemos recuperar todas las muestras de $y[n]$ a partir de $g[n]$ excepto las solapadas (de 10 a 18)

4. Realización de filtrados con la DFT

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot h[n-k]$$

$$\begin{cases} x[n] \Rightarrow L \\ h[n] \Rightarrow P \end{cases}$$

1) $DFT^{(L+P-1)}\{x[n]\} = X[k]$

$DFT^{(L+P-1)}\{h[n]\} = H[k]$

2) $Y[k] = X[k] \cdot H[k]$

3) $y[n] = IDFT^{(L+P-1)}\{Y[k]\}$

Ej 8.4:

$$x[n] = \alpha^n u[n] \quad ; \quad |\alpha|<1$$

$$\tilde{x}[n] = \sum_{r=-\infty}^{+\infty} x[n+rN]$$

a) $\mathcal{F}\{x[n]\} = X(e^{jw})$

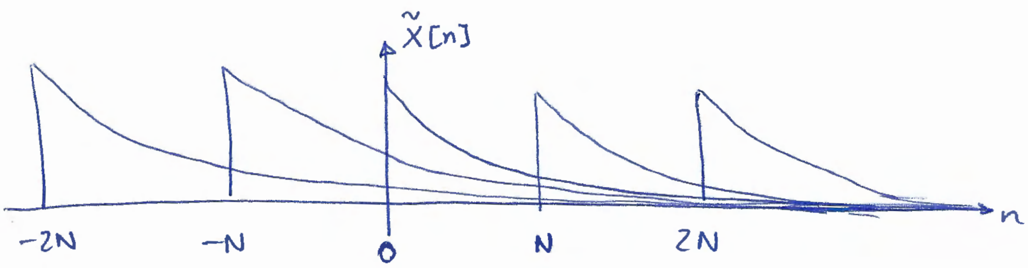
b) $\mathrm{DFS}\{\tilde{x}[n]\} = \tilde{X}[k]$

c) Relación entre $\tilde{X}[k]$ y $X(e^{jw})$

a) $\alpha^n u[n] \xrightarrow{Z} \dfrac{1}{1-\alpha z^{-1}} \quad |z|>\alpha$

$$X(e^{jw}) = X(z)\Big|_{z=e^{jw}} = \frac{1}{1-\alpha e^{-jw}}$$

b) $\tilde{X}[k] = X(e^{jw})\Big|_{w=\frac{2k\pi}{N}}, \quad 0 \le k \le N-1 \Rightarrow \tilde{X}[k] = \dfrac{1}{1-\alpha e^{-j\frac{2\pi k}{N}}} \quad 0 \le k \le N-1$



$$\tilde{x}[0] = \alpha^0 + \alpha^N + \alpha^{2N} + \dots = \sum_{k=0}^{\infty} (\alpha^N)^k = \frac{1}{1-\alpha^N}$$

$$\tilde{x}[1] = \alpha^1 + \alpha^{N+1} + \alpha^{2N+1} + \dots = \alpha \cdot \tilde{x}[0]$$

$\vdots$

$$\tilde{x}[N-1] = \quad = \alpha^{N-1}\tilde{x}[0] = \frac{\alpha^{N-1}}{1-\alpha^N}$$

Luego: $\tilde{x}[n] = \dfrac{\alpha^n}{1-\alpha^N} \quad 0 \le n \le N-1$

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] \cdot e^{-j\frac{2\pi k}{N}n} = \frac{1}{1-\alpha^N} \sum_{n=0}^{N-1} \left(\alpha e^{-j\frac{2\pi k}{N}}\right)^n = \frac{1}{1-\alpha^N} \cdot \frac{1-\alpha^N e^{-j2\pi k}}{1-\alpha e^{-j\frac{2\pi k}{N}}}$$

$$= \frac{1}{1-\alpha e^{-j\frac{2\pi k}{N}}}$$

Ej 8.5:

a) $x[n] = \delta[n]$

b) $x[n] = \delta[n-n_0] \quad 0 \le n_0 \le N-1$

c) $x[n] = \begin{cases} 1 & 0 \le n \le \frac{N}{2}-1 \\ 0 & \frac{N}{2} \le n \le N-1 \end{cases}$; N par

d) $x[n] = \begin{cases} 1 & n \text{ par} \\ 0 & n \text{ impar} \end{cases} \quad 0 \le n \le N-1$

e) $x[n] = \begin{cases} a^n & 0 \le n \le N-1 \\ 0 & \text{resto} \end{cases}$

$$X[k] = X(e^{jw})\Big|_{w=\frac{2\pi k}{N}} = X(z)\Big|_{z=e^{jw}}$$

a) $\mathcal{F}\{\delta[n]\} = 1 = X(e^{jw})$

$X[k] = 1 \quad 0 \le k \le N-1$

b) $\mathcal{F}\{\delta[n-n_0]\} = e^{-jwn_0}$

$X[k] = X(e^{jw})\Big|_{w=\frac{2\pi k}{N}} = e^{-j\frac{2\pi k}{N}n_0} \quad 0 \le k \le N-1$

c) $X(z) = \sum_{n=0}^{\frac{N}{2}-1} 1 \cdot z^{-n} = \frac{1 - z^{-(\frac{N}{2}-1)}z^{-1}}{1-z^{-1}} = \frac{1-z^{-N/2}}{1-z^{-1}}$

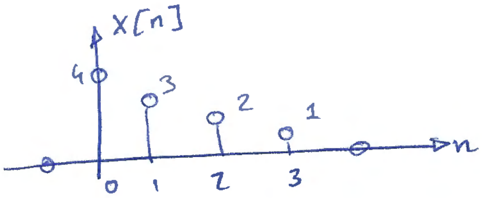
$X[k] = \frac{1-e^{-j\frac{2\pi k}{N}\frac{N}{2}}}{1-e^{-j\frac{2\pi k}{N}}} = \frac{1-e^{-j\pi k}}{1-e^{-j\frac{2\pi k}{N}}}$

d) $X(e^{jw}) = X'(e^{j2w}) \Rightarrow$ c) $x'[n] \rightarrow \boxed{\uparrow 2} \rightarrow x[n]$ d)

$X(z) = X'(z^2) = \frac{1-z^{-N/2 \cdot 2}}{1-z^{-2}} \Rightarrow X[k] = \frac{1-e^{-j\frac{2\pi k}{\cancel{N}}\cancel{N}}}{1-e^{-j\frac{2\pi k}{N}\cdot 2}}$

e) $X(z) = \sum_{n=0}^{N-1} a^n z^{-n} = \sum_{n=0}^{N-1} (az^{-1})^n = \frac{1-(az^{-1})^N}{1-az^{-1}}$

Ej 8.23:



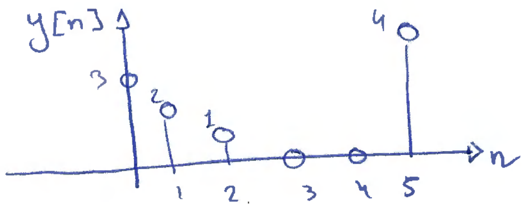
$X[k] = DFT^{(6)}\{x[n]\}$

a) Dibujar $y[n]$; $DFT^{(6)}\{y[n]\} = Y[k] = W_6^{5k} \cdot X[k]$

b) Dibujar $w[n]$; $DFT^{(6)}\{w[n]\} = W[k] = j\,\mathrm{Im}\{X[k]\}$

c) Dibujar $q[n]$; $DFT^{(3)}\{q[n]\} = Q[k] = X[2k]$
$0 \leq k \leq 2$

a) $y[n] = x[((n-5))_6] \quad 0 \leq n \leq 5$

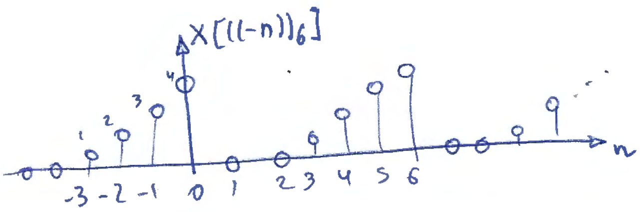


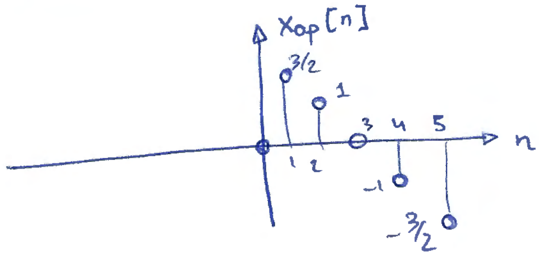
b) $x[n] = x_{ep}[n] + x_{op}[n]$

$\mathrm{Re}\{X[k]\} + j\,\mathrm{Im}\{X[k]\}$

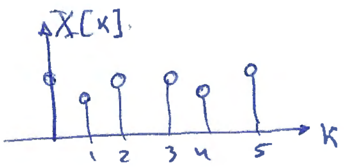
$W[n] = x_{op}[n]$

$x_{op}[n] = \dfrac{x[n] - x[((-n))_6]}{2} \quad 0 \leq n \leq T$

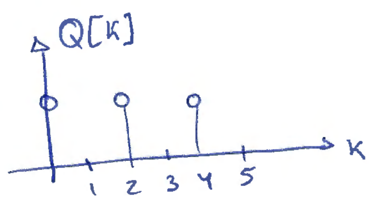




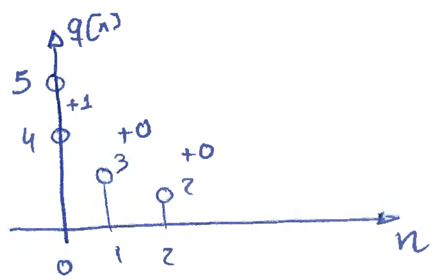
c)



$\Rightarrow$



$\Rightarrow q[n] = \sum_{k=0}^{1} x[n+k\cdot 3]$



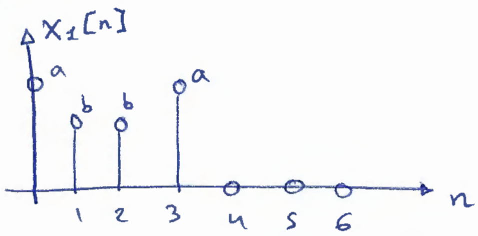
$Q[k] = X[2k] = \sum_{n=0}^{5} x[n]\, e^{-j\frac{2\pi 2k}{6}n} =$

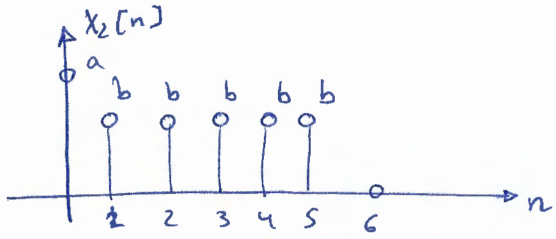
$= \sum_{n=0}^{2} x[n]\, e^{-j\frac{2\pi k}{3}n} + \sum_{n=3}^{5} x[n]\, e^{-j\frac{2\pi k}{3}n} =$

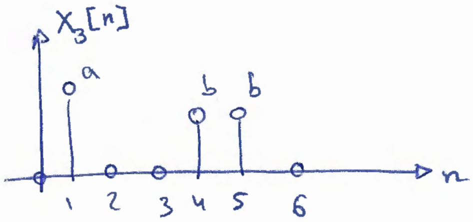
$= \sum_{r=0}^{1} DFT^{(3)}\{x[n+3\cdot r]\} = DFT^{(3)}\left\{\sum_{r=0}^{1} x[n+3\cdot r]\right\}$

$0 \leq n \leq 2$

Ej 8.40:







$$\left\{\begin{array}{l} DFT^{(7)}\{x_i[n]\} \overset{?}{\doteq} A_i[k]\, e^{-j\frac{2\pi k}{7}\alpha_i} \\ A_i[k] \in \mathbb{R} \\ 2\alpha_i \in \mathbb{Z} \end{array}\right.$$

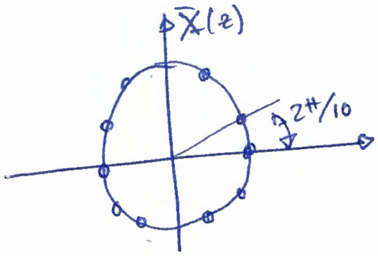
$x_1[n]$ sí cumple ; $\alpha_1 = -3/2$

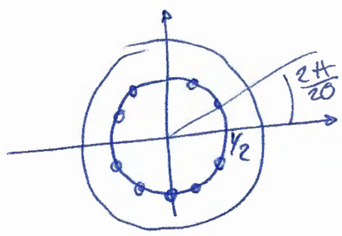
$x_2[n]$ no cumple PERO si $x_2[n=6]=b \Rightarrow$ sí cumple ; $\alpha_2 = 0$

$x_3[n]$ no cumple

Ej 8.50:

$$X(z)\Big|_{z=e^{j\frac{2\pi k}{10}}} = X(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{10}} = DFT^{(10)}\{x[n]\}$$



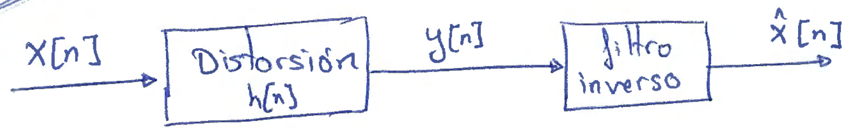


$$X(z)\Big|_{z=\frac{1}{2}e^{j\left[\frac{2\pi k}{10}+\frac{\pi}{10}\right]}} = \sum_{n=0}^{9} x[n]\cdot z^{-n}\Big|_{z=\frac{1}{2}e^{j\left[\frac{2\pi k}{10}+\frac{\pi}{10}\right]}} =$$

$$= \sum_{n=0}^{9} x[n]\, 2^n e^{-j\frac{2\pi k}{10}n} \cdot e^{-j\frac{\pi}{10}n} = \sum_{n=0}^{9} \Big(\underbrace{x[n]\, 2^n e^{-j\frac{\pi}{10}n}}_{x_1[n]}\Big) e^{-j\frac{2\pi k}{10}n} =$$

$$= DFT^{(10)}\left\{x[n]\, 2^n e^{-j\frac{\pi}{10}n}\right\}$$

Ej 8.67



$$h[n] = \delta[n] - \tfrac{1}{2}\delta[n-n_0]$$

a) $H(z)$?

$$H[k] = DFT^{(N)}\{h[n]\} \; ; \; N = 4n_0$$

b) si $H_i(z) = 1/H(z)$: $h_i[n]$?

a) $$H(z) = 1 - \tfrac{1}{2} z^{-n_0}$$

$$H[k] = H(z)\Big|_{z=e^{j\frac{2\pi k}{N}}} = 1 - \tfrac{1}{2} e^{-j\frac{2\pi k}{N} n_0} \qquad 0 \le k \le N-1$$

b) $$H_i(z) = \frac{1}{1-\frac{1}{2}z^{-n_0}}$$

$$\Big\downarrow z^{-1}$$

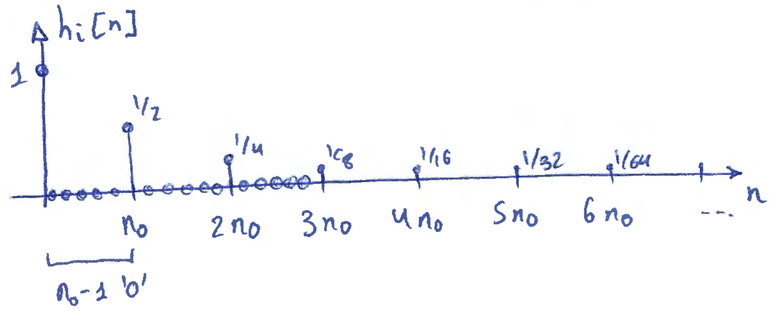
$$\sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^k \delta[n-kn_0]$$

$$h'[n] \longrightarrow \boxed{\uparrow n_0} \longrightarrow h_i[n]$$

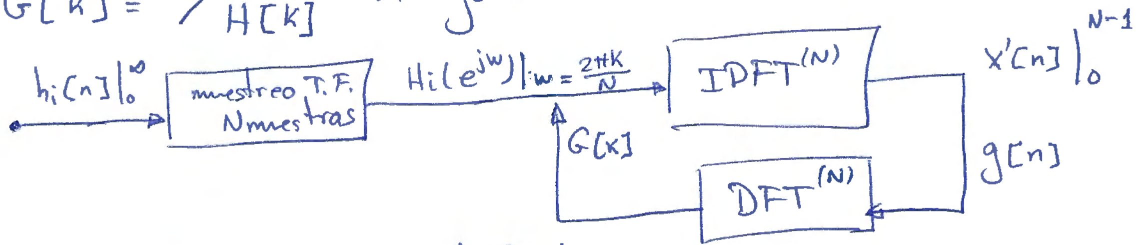
$$H(z) \qquad\qquad H(z^{n_0})$$

$$\frac{1}{1-\frac{1}{2}z^{-1}} \xrightarrow{z^{-1}} \left(\tfrac{1}{2}\right)^n u[n] \qquad \sum_{k=0}^{\infty}\left(\tfrac{1}{2}\right)^k \delta[n-kn_0] \longrightarrow \frac{1}{1-\frac{1}{2}z^{-n_0}}$$

$$|z| > 1/2$$

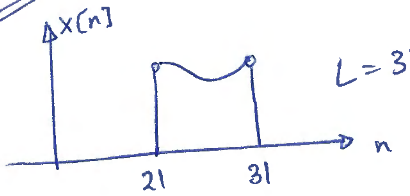


c) $G[k] = 1/H[k] \Rightarrow g[n]$?



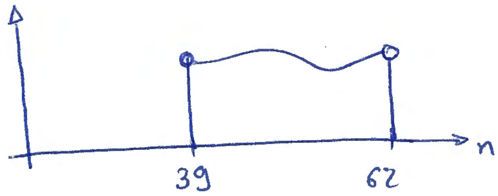
$$g[n] = \sum_{k=-\infty}^{+\infty} h_i[n-kN] = \sum_{k=0}^{NumSeq-1} h_i[n+kN] \qquad 0 \le n \le N-1$$

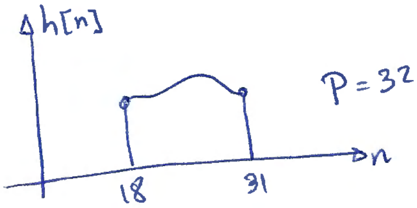
Ej 8.30:



$L = 32 \;\Big\}_{0}^{31}$

$y[n] = x[n] * h[n]$





$P = 32 \;\Big\}_{0}^{31}$

$L + P - 1 = 63 \;\Big\}_{0}^{62}$

b) $X_1[k] = DFT^{(32)}\{x[n]\}$

$H_1[k] = DFT^{(32)}\{h[n]\}$

si $Y_1[k] = X_1[k] \cdot H_1[k] \Rightarrow y_1[n] = x[n] \overset{32}{\circledast} h[n]$