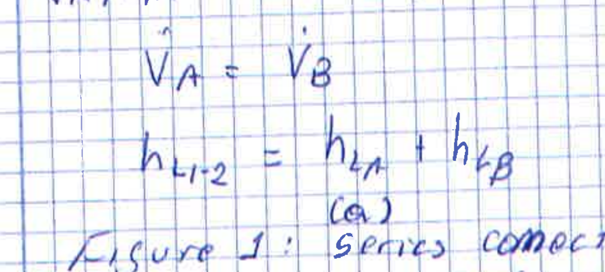
(1) # Fluid in Ducts

Piping Networks

Most systems (piping) encountered in practice such as the water distribution systems in cities or comercial or residential, fluid in airplane systems involve numerous parallel and series connections as well as several sources and loads (discharges of fluid from the systems)

The engineering objective in such projects is to design a piping system that will deliver the specified flow rates at specified pressures reliably at minimum total (operating and maintenance) cost.

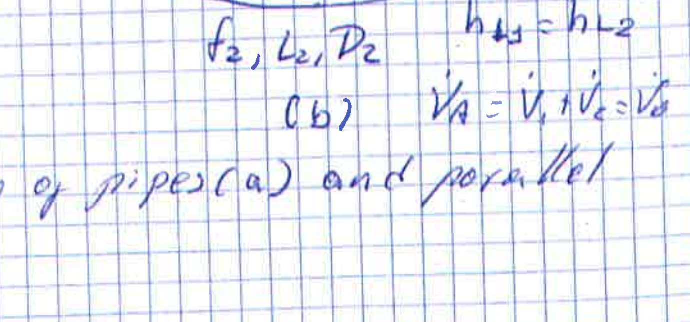
Piping systems typically involve several pipes connected to each other in series and/or in parallel,



$\dot{V}_A = \dot{V}_B$

$h_{L1-2} = h_{LA} + h_{LB}$

(a)



$h_{L1} = h_{L2}$

(b) $\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$

Figure 1: Series conection of pipes (a) and parallel piping connection (b)

In Series: The flow rate through the entire system remains constant regardless of the diameters of the individuals pipes system. The total head loss is equal to the sum of the head losses in individual pipes in the system, including the minor losses. Expansion and contraction losses at connections are considered to belong to the smaller-diameter pipe since the expansion and contraction loss coefficients are defined on the basis of the average velocity in the smaller-diameter pipe.

In Parallel: the total flow rate is the sum of the flow rates in the individuals pipes. The pressure drop (or head loss) in each individual pipe connected in parallel must be same since $\Delta P = P_A - P_B$ and the junction pressures P_A and P_B are the same for all the individual pipes.

$$h_{L1} = h_{L2} \Rightarrow \frac{f_1 L_1 V_1^2}{D_1 2g} = f_2 \frac{L_2 V_2^2}{D_2 2g}$$

Flow rates can be calculated by the ratio of the average velocities.

$$\frac{V_1}{V_2} = \left(\frac{f_2 L_2 D_1}{f_1 L_1 D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1} V_1}{A_{c,2} V_2}$$

$$\frac{\dot{V}_1}{\dot{V}_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2 L_2 D_1}{f_1 L_1 D_2}\right)^{1/2}$$

The relative flow rates in parallel pipes are established from the requirement that the head loss in each pipe be the same. This result can be extended to any number of pipes connected in parallel.

The analysis of piping network, no matter how complex they are based on two principles

1. Conservation of mass throughout the system must be satisfied.

2. Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions.

Piping systems with pumps and turbines

The steady-flow energy equation on unit mass can be expressed as.

$$\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + g z_1 + w_{pump} = \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + w_{Tur} + g h_L$$

or

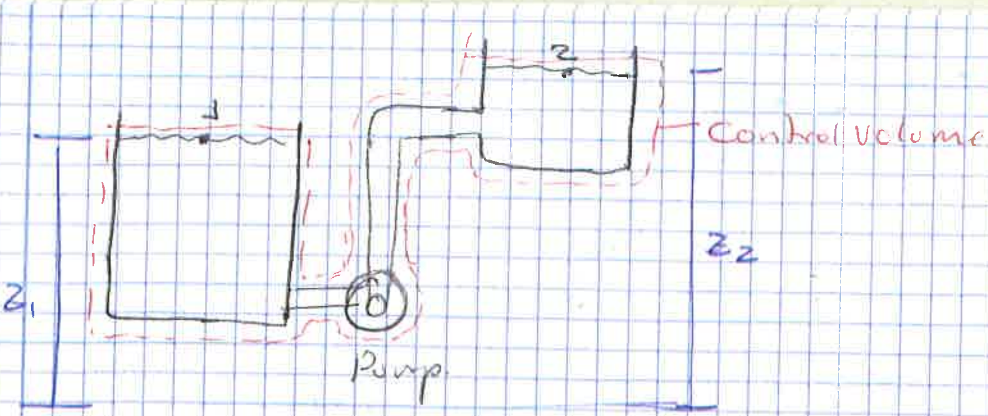
$$\frac{P_1}{g\rho} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{g\rho} + \alpha_2 \frac{V_2^2}{2g} + h_{Turb} + h_L$$

pump head delivered to the fluid

Turbine head extracted from the fluid.

Total head loss in piping (including minor losses)

Kinetic energy correction factor whose value is nearly 1 turbulent flow.



$$h_{pump} = (z_2 - z_1) + h_L$$

$$\dot{W}_{pump} = \rho \dot{V} g h_{pump}$$

$h_{Turb} = 0$
$V_2 = V_1 \cong 0$
$P_1 = P_2$
$\rho = constant$

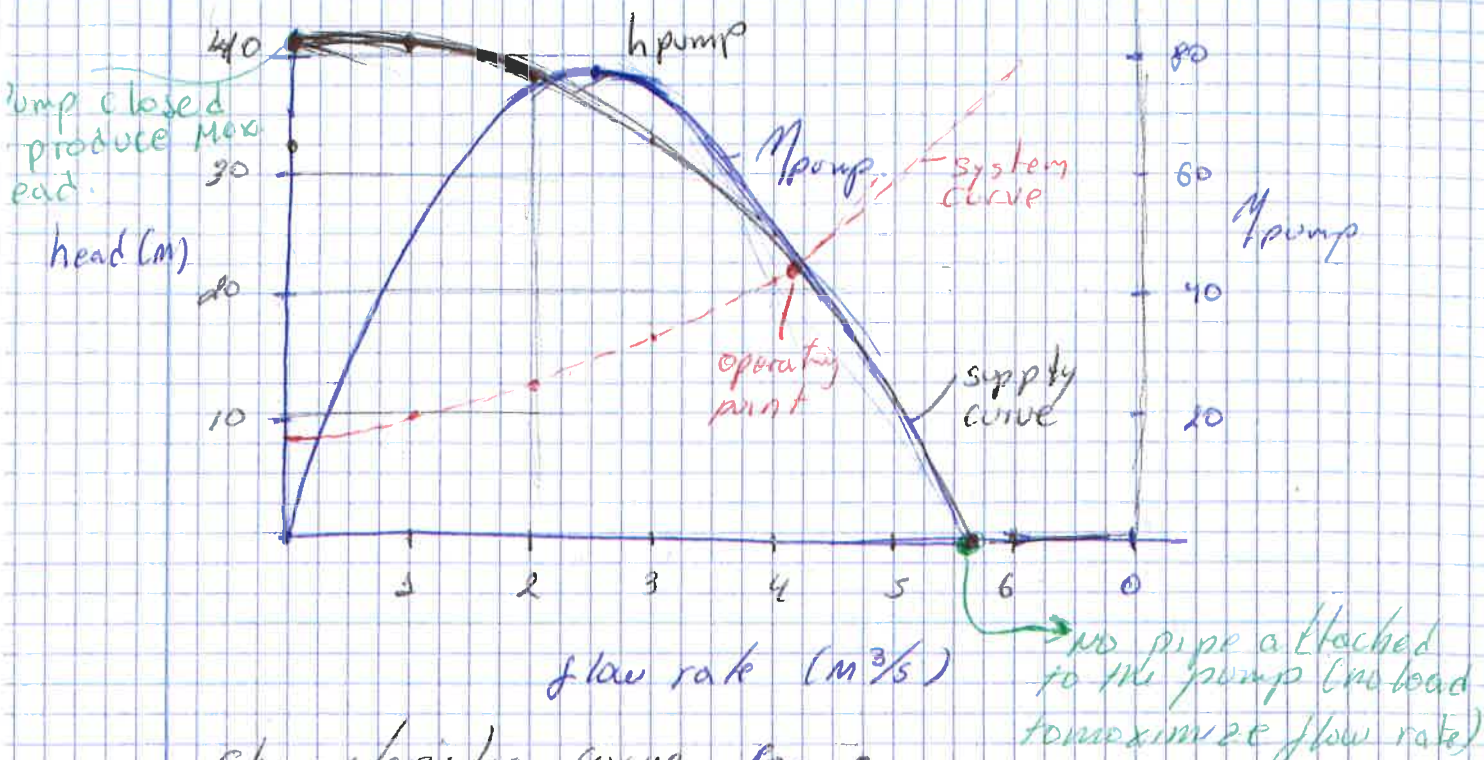
The equation is reduced to $h_{pump} = (z_2 - z_1) + h_L$

Once we know the pump head, the mechanical power that needs to be delivered by the pump to the fluid and the electrical power consumed by the motor of the pump for a specific flow rate are determined from.

$$\dot{W}_{pump} = \frac{\rho \dot{V} g . h_{pump}}{\eta_{pump}} \quad \text{and} \quad \dot{W}_{elect} = \frac{\rho \dot{V} g \, h_{pump}}{\eta_{motor-pump}}$$

$\eta_{motor-pump}$ is the efficiency combination of motor and pump and it is just the product of $\eta_{pump} \cdot \eta_{motor}$.

A plot required useful pump head as a function of flow rate is called the "system or demand curve." The head produced by a pump is not constant. Both pump head and the pump efficiency vary with the flow rate and the manufacturers supply this variation ~~an~~ graphical or tabular. the graph of h_{pump} or η_{pump} versus flow rate $\dot{V}$ is called the "characteristic curves"

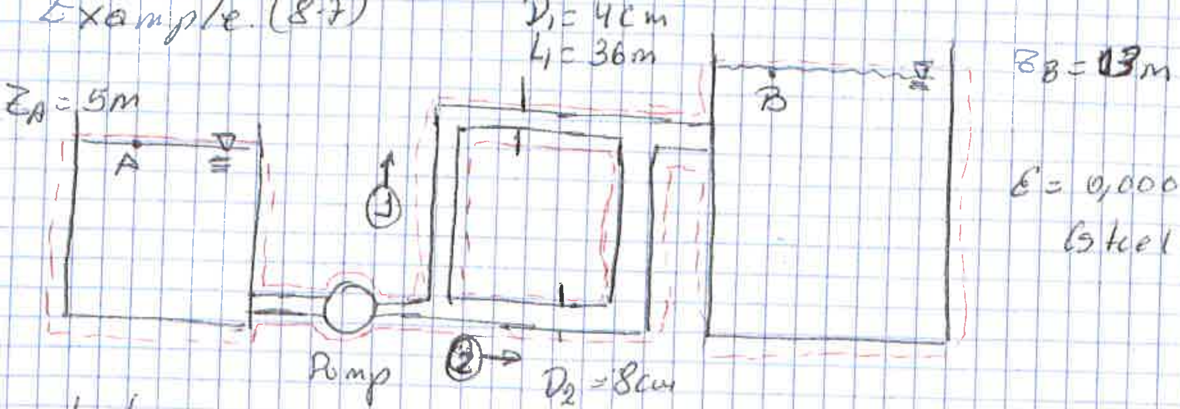


Characteristic curve for a Centrifugal Pump, the system pump and system curve for a piping system and the operating point

Note: - flow rate increases as η decreases and head decreases too.

- The efficiency of a pump is sufficiently high for a certain range of head and flow rate combination. Therefor, a pump that can supply the required head and flow rate is not necessarily a good choice
- The pump installed in a piping system will operate at the point where system curve and characteristic curve intersect. (operating system).

Example (8.7)



$\varepsilon = 0{,}00045\,m$ (steel pipe)

electric power $= 8\,KW$

- fluid water at $20°C$ $\rho = 998\ kg/m^3$ $\mu = 1.002\times10^{-3}\ kg/m\text{-}s$
- $\eta_{motor\text{-}pump}$ or $\eta_{pump\text{-}motor} = 70\%$
- No minor losses
- Find the total flow rate between the reservoirs and the flow rate through each of the parallel pipes

Solution:

This problem is not solved directly since the velocities or flow rates in the pipes are not known.
We use trial-error approach.

The useful head supplied by the pump to the fluid is determined from.

$$\dot{W}_{elec} = \frac{\rho \dot{V} g\, h_{pump}}{\eta_{pump\text{-}motor}}$$

$$8000\,W = \frac{998 \times \dot{V} \cdot 9.8 \times h_{pump}}{0.70} \quad (1)$$

From energy equation applying in A and B we obtain.

$$\cancel{\frac{P_A}{\rho g}} + \alpha_A \underbrace{\cancel{\frac{V_A^2}{2g}}}_{=0} + z_A + h_{pump} = \cancel{\frac{P_B}{\rho g}} + \alpha_B \underbrace{\cancel{\frac{V_B^2}{2g}}}_{=0} + z_B + \cancel{h_{turb}} + h_L$$

$$h_{pump} = (z_B - z_A) + h_L$$

$$h_{pump} = (13 - 5) + h_L = 8 + h_L \quad (2)$$

where we know that $h_{L_1} = h_{L_2} = h_L$

4 cm - diameter equal to pipe (1)

8 cm - diameter equal to pipe (2)

Average velocity, Reynolds number and friction factor and head loss in each pipe are expressed as

$$V_1 = \frac{\dot{V}_1}{A_{c1}} = \frac{\dot{V}_1}{\frac{\pi D_1^2}{4}} \Rightarrow V_1 = \frac{\dot{V}_1}{\frac{\pi (0.04m)^2}{4}}$$

$$V_2 = \frac{\dot{V}_2}{A_{c2}} = \frac{\dot{V}_2}{\frac{\pi D_2^2}{4}} \Rightarrow V_2 = \frac{\dot{V}_2}{\frac{\pi (0.08)^2}{4}}$$

$$Re_1 = \frac{\rho V_1 D_1}{\mu} \qquad Re_1 = \frac{998 \times 0.04 \cdot V_1 \,(m) \left(\frac{kg}{m^3}\right)}{1.002 \times 10^{-3} \left(\frac{kg}{m \cdot s}\right)}$$

$$Re_2 = \frac{\rho V_2 D_2}{\mu} \qquad Re_2 = \frac{998 \times 0.08\, V_2}{1.002 \times 10^{-3}}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{\varepsilon / D_1}{3.7} + \frac{2.51}{Re_1 \sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{Re_1 \sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{Re_2 \sqrt{f_2}}\right)$$

$$h_{L_1} = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} \Rightarrow h_{L_1} = f_1 \frac{36}{0.04} \cdot \frac{V_1^2}{2 \times 9.8}$$

$$h_{L_2} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} \Rightarrow h_{L_2} = f_2 \frac{36}{0.08} \cdot \frac{V_2^2}{2 \times 9.8}$$

13 equations in 13 unknowns.

$$\dot{V} = \dot{V}_1 + \dot{V}_2$$

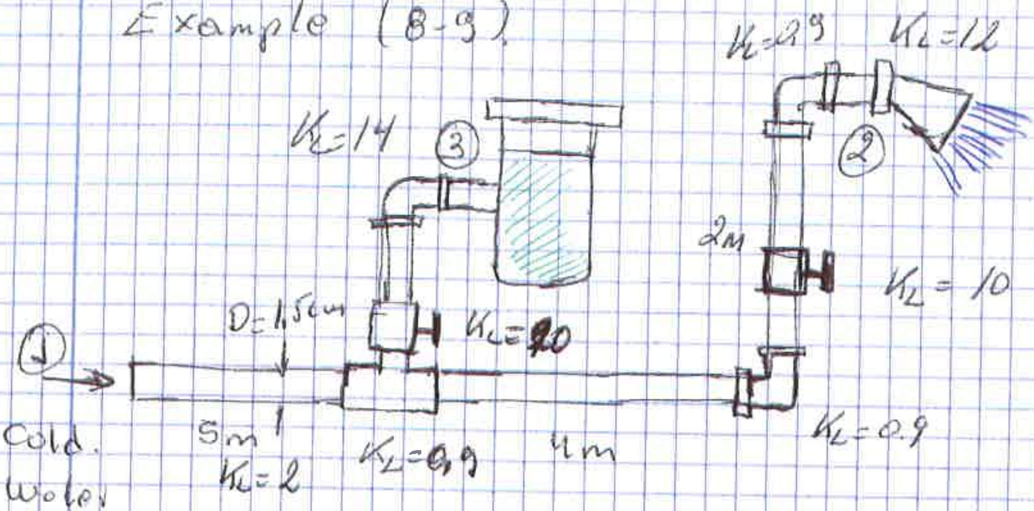
$\dot{V} = 0{,}0300\ \frac{m^3}{s}$ $\quad \dot{V}_1 = 0{,}00415\ \frac{m^3}{s}$ $\quad \dot{V}_2 = 0{,}0259\ m^3/s$

$V_1 = 3.3\ m/s$ $\quad V_2 = 5.15\ m/s$

$h_L = 11.1\ m$ $\quad h_{pump} = 19.3\ m$ $\quad Re_1 = 131.600$ $\quad Re_2 = 410000$

$f_1 = 0{,}0221$ $\quad f_2 = 0.0182$

Example (8-9)



Assumptions

(1) Steady state, incompressible

(2) Turbulent and fully developed

(3) Reservoir open to atmosphere

(4) Velocity heads are negligible.

Water at 20°C $\rho = 998\ \frac{kg}{m^3}$ $\quad \mu = 1.002\times10^{-3}\ \frac{kg}{m\text{-}s}$

roughness of copper pipe $\varepsilon = 1.5\times10^{-6}\ m$ and P_1 is 200 kPa during a shower and the toilet reservoir is full. Determine the flow rate at shower head.

Total length is 11m.

$\sum K_L = 0.9 + 2\times0.9 + 10 + 12 = 24.7$

Energy equation (1) and (2)

$$\frac{P_1}{\rho g} + \cancelto{\text{neglected}}{\alpha_1\frac{V_1^2}{2g}} + z_1 + h_{pump} = \cancelto{0}{\frac{P_2}{\rho g}} + \cancelto{\text{neglected}}{\alpha_2\frac{V_2^2}{2g}} + h_{turb} + h_L$$

$$\frac{P_{1,gage}}{\rho g} = (z_2 - z_1) + h_L$$

$$h_L = \frac{200000\ N/m^2}{9.8 \times 998\ \frac{kg}{m^3} \cdot \frac{m}{s^2}} - 2\ m = 18.4\ m.$$

$$h_L = \left(f \cdot \frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \Rightarrow 18.4 = \left(f \frac{11}{0.015} + 24.7\right)\frac{V^2}{2(9.81)}$$

The average velocity for a system with constant diameter is given by.

$$V = \frac{\dot{V}}{A_c} = \frac{4\dot{V}}{\pi(0.015)^2}$$

$$Re = \frac{\rho V D}{\mu} \Rightarrow Re = \frac{998 \times 0.015}{1.000 \times 10^{-3}} V$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

$\dot{V} = 0.00053\ \frac{m^3}{s}$ $\quad f = 0.0218$ $\quad V = 2.98\ m/s$ $\quad Re = 44550$

The flow rate at shower head is **0.53 L/s**

$0.53\ \frac{L}{s} \times 600\ s = 318\ L$

(b). Determine the effect of flushing of the toilet on the flow rate through the shower head. Take the loss coefficients of the shower head and the reservoir to be 12 and 14. respectively.

- when the toilet is flushed, float moves and open the valve. The discharged water starts to refill

$$h_{L3} = \frac{200000\ N/m^2}{998 \times 9.81} - 1\ m = 19.4\ m$$

$$\sum K_{L3} = 2 + 10 + 0.9 + 14 = 26.9$$

Relevant equations

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L2} = f_1 \frac{5m}{0.05m} \frac{V_1^2}{2(9.81)} + \left(f_2 \left(\frac{6m}{0.015m}\right) + 24.7\right) \cdot \frac{V_2^2}{2 \cdot (9.81)} = 18.4$$

$$h_{L3} = f_1 \frac{5m}{0.015m} \frac{V_1^2}{2 \cdot (9.81)} + \left(f_3 \frac{3m}{0.015m} + 26.9\right) \cdot \frac{V_3^2}{2(9.81)} = 19.4$$

$$V_1 = \frac{\dot{V}_1}{\frac{\pi (0.015)^2}{4}} \qquad V_2 = \frac{\dot{V}_2}{\frac{\pi (0.015)^2}{4}} \qquad V_3 = \frac{\dot{V}_3}{\frac{\pi (0.015)^2}{4}}$$

$$Re_1 = \frac{V_1 \rho D}{\mu} \qquad Re_2 = \frac{V_2 \rho D}{\mu} \qquad Re_3 = \frac{V_3 \rho D}{\mu}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log \left(\frac{1.5 \times 10^{-6}}{3.7(0.015)} + \frac{2.51}{Re_1 \sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log \left(\frac{1.5 \times 10^{-6}}{3.7(0.015)} + \frac{2.51}{Re_2 \sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log \left(\frac{1.5 \times 10^{-6}}{3.7(0.015)} + \frac{2.51}{Re_3 \sqrt{f_3}}\right)$$

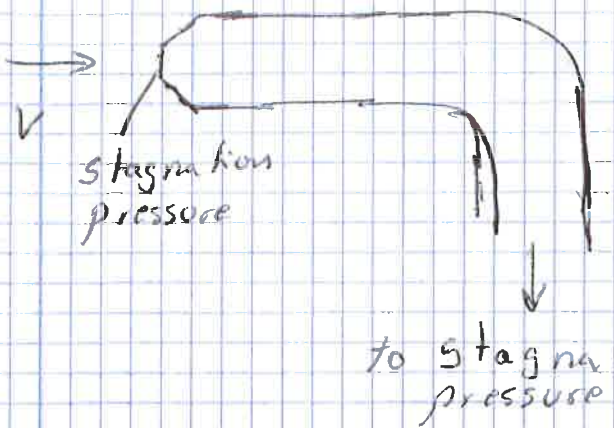
$\dot{V}_1 = 0.0009 \frac{m^3}{s}$ $\quad \dot{V}_2 = 0.00042\ m^3/s$ $\quad \dot{V}_3 = 0.00048\ m^3/s$

$\dot{V}_2 = 0.42\ L/s$

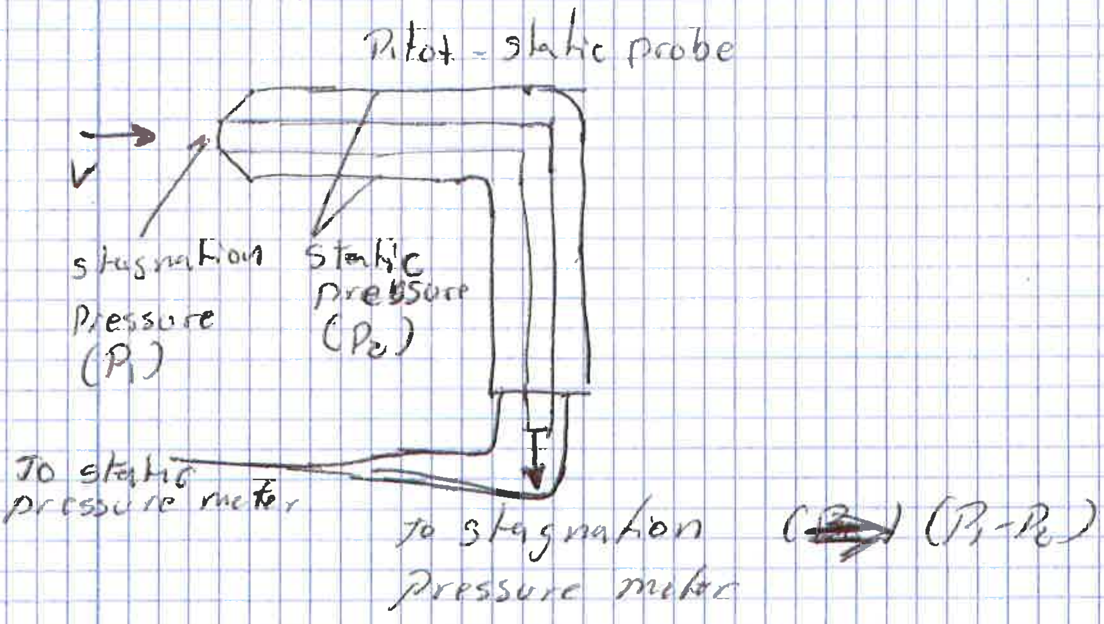
If we consider heads velocities, the flow rate through the shower would be $0.43\ L/s$

consider head velocities zero is a good assumption.

Pitot and Pitot-static Probes



(a) Pitot probe measures stagnation pressure at the nose.



(b). Pitot-static probe measures both stagnation pressure and static pressure from which the flow speed can be calculated

The pitot-static probe measures local velocity by measuring the pressure differences in conjuction with Bernoulli equation. It consists of a slender double-tube aligned with the flow and connected to a differential pressure meter

The inner tube is only open ~~to~~ flow at the nose, and thus it measures the stagnation pressure at location (point 1) the outer tube is sealed at the nose, but it has holes on the side of the outer wall (point 2) and thus measures the static pressure. For incompressible flow with sufficiently high velocities where the frictional effects are negligible

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \qquad z_2 = z_1$$

since the static pressure holes of the pitot-static probe are arranged circumferentially around the tube and $V_1 = 0$ because the stagnation conditions the flow velocity $V = V_2$

Pitot formula $V = \sqrt{\dfrac{2(P_1 - P_2)}{\rho}}$

If the velocity is measured at a location where the local velocity is equal to average flow velocity the volume flow rate can be determined from $\dot{V} = VA_c$