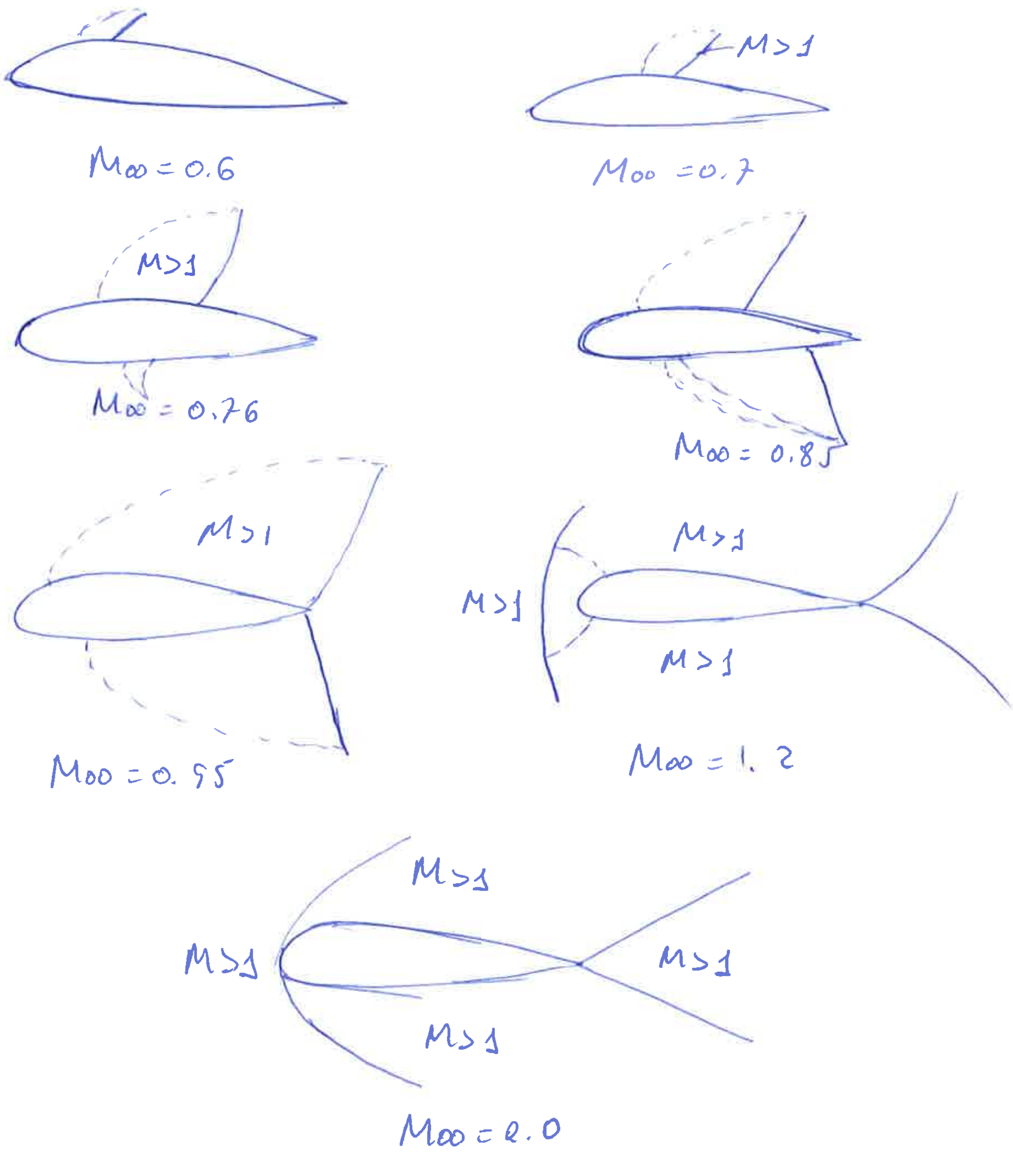
Wings In Compressible flow

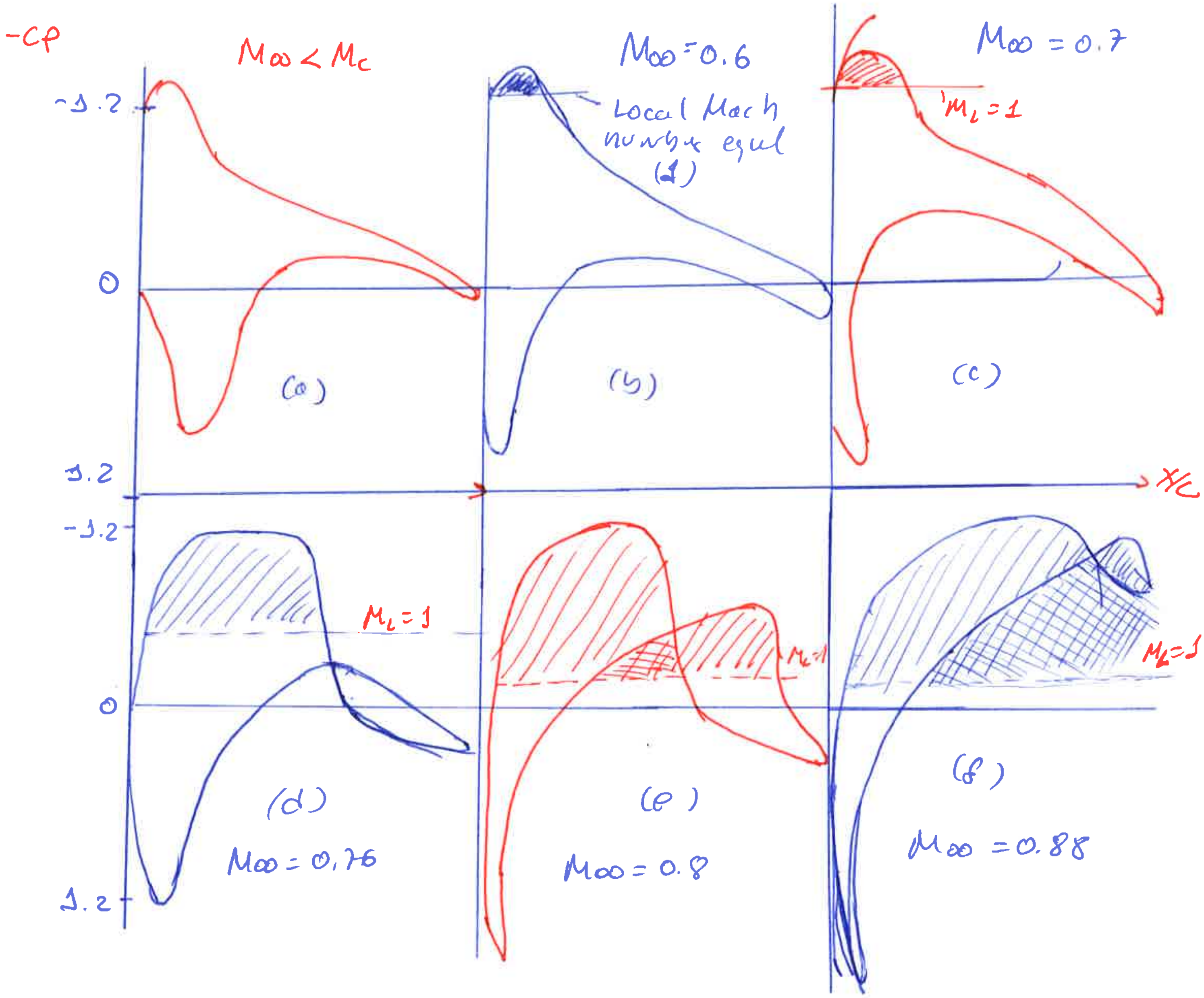
When the air flows past a body, or vice-versa a Symmetrical aerofoil section at low incidence. The local airspeed adjacent to the surface outside the boundary layer is higher or lower then the free stream speed depending on whether the local static pressure is less or greater than the ambient. In this situation the value of the Velocity Somewhere on the airfoil exceeds that of the free stream. So the freestream flow speed rises the Mach number at a point somewhere adjacent to the surface reaches sonic conditions before the free-stream. Usually this point is the minimum pressure which in this case is on the upper surface.

Free-stream Mach number (M_∞) when reaches the value of 1 is defined as the critical Mach number M_c. Typically for slender wing section at low incidence M_c may be about 0.75. Below that critical Mach number the flow is subsonic throughout.

Above the critical Mach number the flow is mixed, part subsonic part supersonic as M_∞ is increased progressively from low numbers to M_c the aerodynamic characteristic of the airfoil section undergo progressive and generally smooth changes and for thin aerofoil shapes at low incidences these changes may be predicted by the small-perturbation or linearized theory due to Prandtl and Glauert outlined below.



The effect on the airfoil characteristics of the flow described above is severe. the sudden loss of lift, increase in drag and rapid movement in the center of pressure are similar in flight ~~to~~ to those experienced at the stall and this flight regime became known as the shock stall.



Small perturbation theory (Prandtl-Glauert rule).

In certain cases of compressible flow, exact solutions to the equations of motion may be found. (assuming inviscid flow). and these are applied to the flow in the vicinity of airfoils.

Transonic Region (Mixed supersonic and subsonic)
the mathematical analysis of this regime involves the solution of a set of non-linear differential equations. It is acceptable the transformation of the equations into soluble linear differential equations. (Linearized theory)

This theory takes no account of viscous drag or the onset of shock waves in localized regions of supersonic flow, the experimental results of the time did indicate the now well-investigated critical region of flight where theory breaks down. The suggestion is that the critical speed at which the lift off depends on the shape and incidence of the airfoil.

The equation of motion of a compressible fluid

- Continuity equation (Consider the 2D-flow).

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \qquad (1)$$

- Momentum equation.

$$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = \dfrac{1}{\rho}\left(X - \dfrac{\partial p}{\partial x}\right) \\[2ex] \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} = \dfrac{1}{\rho}\left(Y - \dfrac{\partial p}{\partial y}\right) \end{cases} \qquad (2)$$

X and Y (External Forces). Consider external forces equal zero. and steady flow. The following equation is obtained.

$$\begin{cases} -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} \\[2ex] -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} = u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} \end{cases} \qquad (3)$$

Consider a adiabatic compression and expansion, implies isentropic flow.

$$p = k\rho^{\gamma}; \qquad \frac{\partial p}{\partial \rho} = a^2 = \gamma\frac{p}{\rho} \qquad (4)$$

For steady flow the continuity equations becomes.

$$u \cdot \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = 0 \qquad (5)$$

But we know.

$$\left.\begin{aligned} \frac{\partial \rho}{\partial x} &= \frac{\partial p}{\partial x} \frac{\partial \rho}{\partial p} = \frac{1}{a^2} \cdot \frac{\partial p}{\partial x} \\ \frac{\partial \rho}{\partial y} &= \frac{\partial p}{\partial y} \cdot \frac{\partial \rho}{\partial p} = \frac{1}{a^2} \cdot \frac{\partial p}{\partial y} \end{aligned}\right\} \text{replacing into the continuity equations.}$$

we obtain:

$$\left.\frac{u}{a^2} \cdot \frac{\partial p}{\partial x} + \frac{v}{a^2} \cdot \frac{\partial p}{\partial y} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = 0 \right\}$$

Replacing $\frac{\partial p}{\partial x}$ and $\frac{\partial p}{\partial y}$ from momentum equation

we obtain:

$$-\frac{u^2}{a^2} \frac{\partial u}{\partial x} - \frac{u \cdot v}{a^2} \cdot \frac{\partial u}{\partial y} - \frac{vu}{a^2} \cdot \frac{\partial v}{\partial x} - \frac{v^2}{a^2} \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6)$$

Collecting terms:

$$\left(1 - \frac{u^2}{a^2}\right) \frac{\partial u}{\partial x} - \frac{uv}{a^2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \left(1 - \frac{v^2}{a^2}\right) \frac{\partial v}{\partial y} = 0$$

For irrotational flow $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$ and velocity potential exists.

$$\left(1 - \frac{u^2}{a^2}\right) \frac{\partial u}{\partial x} - 2 \frac{uv}{a^2} \cdot \frac{\partial u}{\partial y} + \left(1 - \frac{v^2}{a^2}\right) \frac{\partial v}{\partial y} = 0 \qquad (7)$$

Potential velocity $u = \frac{\partial \phi_1}{\partial x}$ $v = \frac{\partial \phi_1}{\partial y}$

$$\left(1 - \frac{u^2}{a^2}\right)\frac{\partial^2 \phi_1}{\partial x^2} - \frac{2uv}{a^2}\cdot\frac{\partial^2 \phi_1}{\partial x \partial y} + \left(1 - \frac{v^2}{a^2}\right)\frac{\partial^2 \phi_1}{\partial y^2} = 0 \quad (8)$$

Then the energy equation provides the relation between a, u, v.

$$\frac{u^2 + v^2}{2} + \frac{a^2}{\gamma - 1} = \text{Constant.}$$ or we can rearrange

$$\left(\frac{\partial \phi_1}{\partial x}\right)^2 + \left(\frac{\partial \phi_1}{\partial y}\right)^2 + \frac{2}{\gamma - 1}\cdot a^2 = \text{constant} \quad (9)$$

Combining these two equations, yields an expression in terms of the local velocity potential.

Small perturbations

We have assumed that the flow is steady at infinity and local velocity components u and v, parallel to coordinate axes x and y respectively. Assuming that the velocity changes only slightly from steady value at infinity and velocity gradients themselves are small.

$$u = U_\infty + u' \; ; \quad v = v'$$

and then

$$\frac{u'}{U_\infty} \ll 1 \quad \text{and} \quad \frac{v'}{V_\infty} \ll 1$$

Similarly $\frac{\partial u'}{\partial x}$ and $\frac{\partial v'}{\partial y}$ are small.

Replacing into the equation

$$\frac{u^2 + v^2}{2} + \frac{a^2}{\gamma-1} = \text{constant}$$

we obtain

$$\frac{(U_\infty + u')^2 + v'^2}{2} + \frac{a^2}{\gamma-1} = \frac{U_\infty^2}{2} + \frac{a_\infty^2}{\gamma-1}$$

when squares of small quantities are neglected, this equation is simplified to

$$\left[U_\infty u' = \frac{a_\infty^2 - a^2}{\gamma-1} \right] \quad (10)$$

$$a^2 = a_\infty^2 - (\gamma-1) U_\infty u'$$

Thus the coefficients terms of eqn. (7) become.

$$1 - \frac{u^2}{a^2} = 1 - \left(\frac{U_\infty + u'}{a}\right)^2$$

$$1 - \frac{U_\infty^2 + 2U_\infty u'}{a_\infty^2 - (\gamma-1).U_\infty u'}$$

Putting $\frac{U_\infty}{a_\infty} = M_\infty$ the free stream Mach number

$$1-\left(\frac{u}{a}\right)^2 = 1 - M_\infty^2\left(\frac{\left(1+\frac{2u'}{U_\infty}\right)}{1-(\gamma-1).M_\infty^2\,(u'/U_\infty)}\right)$$

$$= 1 - M_\infty^2\left(1 - \frac{u'(2+(\gamma-1)M_\infty^2)}{U_\infty\left[1-(\gamma-1)M_\infty^2\,(u'/U_\infty)\right]}\right)$$

And

$$\frac{2uv}{a^2} = \frac{2(U_\infty+u').v'}{a^2} \cong \frac{2U_\infty v'}{a_\infty^2-(\gamma-1).U_\infty u'}$$

$$= M_\infty^2\,\frac{\frac{2v'}{U_\infty}}{1-\left[(\gamma-1).M_\infty^2\,(u'/U_\infty)\right]}$$

Also the term of $\left(1-\frac{v'^2}{U_\infty^2}\right) \cong 1$

Now if the velocity potential ϕ_1 is expressed as sum of a velocity potential due to the flow at infinity plus a velocity potential due to the disturbance.

$$\phi_1 = \phi_\infty + \phi$$

The equation (8) can be written

(11)

$$(1 - M_\infty^2)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \frac{M_\infty^2}{\left(1 - (\gamma-1)M_\infty^2 \cdot \frac{u'}{U_\infty}\right)}^{*}$$

$$\left[\left[2 + (\gamma-1)M_\infty^2\right] \cdot \frac{u'}{U_\infty} \cdot \frac{\partial^2\phi}{\partial x^2} + \frac{2v'}{U_\infty}\frac{\partial^2\phi}{\partial x \partial y}\right]$$

where ϕ is the disturbance potential and $u' = \frac{\partial\phi}{\partial x}$ and $v' = \frac{\partial\phi}{\partial v}$. For $M_\infty = 0$ we obtain the Laplace equation.

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

Since velocity components and their gradients are of the same small order their products can be neglected and the bracketted terms on equation (11) will be negligibly small.

If the quantity of $\frac{M_\infty^2}{\left[1 - (\gamma-1) M_\infty^2 \frac{u'}{U_\infty}\right]} \rightarrow \infty$

is when $(\gamma-1) \cdot M_\infty^2 \cdot \frac{u'}{U_\infty} \rightarrow 1$

This will be made for $M_\infty = 5$ and $\frac{u'}{U_\infty} = 0.1$ and $\gamma = 1.4$ $M_\infty^2 = 25$.

Within the limitations above the equation of motion reduces to the linear equation.

$$\left(1 - M_\infty^2\right) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (12)$$

A further limitation in the equation (11). occurs when $M_\infty = 1$; where the flow regime may be described as transonic.

As consequence of these restrictions the further application of the equations finds its most use in the high subsonic. where

$0.4 < M_\infty < 0.8$ and in the supersonic region where $1.2 < M_\infty < 5$.

Prandtl- Glauert rule. The application of linearized theories of subsonic flow.

Consider a two dimensional Subsonic flow. and applying equation (12).

$$\left(1 - M_\infty^2\right) . \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

For a given M_∞, this equation can be written as

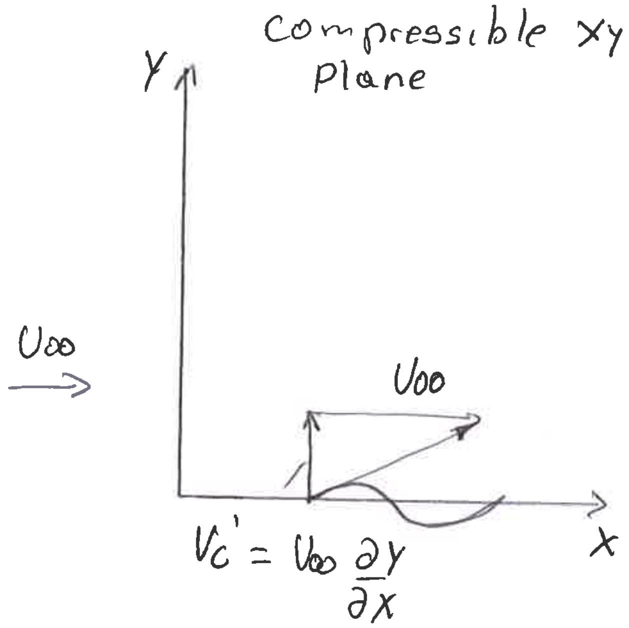
$$B^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (12')$$

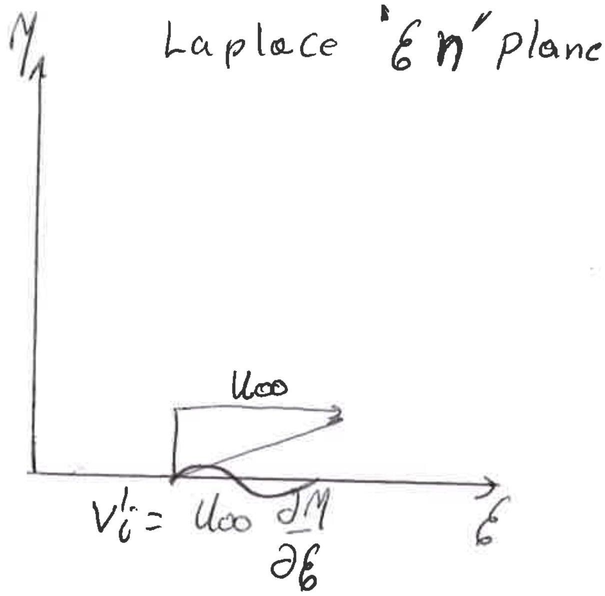
wher B is a constant. This bears a superficial resemblance to the Laplace Eq.

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \quad (13)$$

and If the problem expressed by equation (12), that of finding ϕ for the subsonic compressible flow round a thin airfoil, this can be transformed into an equation such as (13).

Suppose that





The corresponding airfoil in Laplace or Incompressible (ξη) plane has a velocity Potential Φ.

$$\Phi = A\phi \qquad \xi = CX \quad \text{and} \quad \eta = DY$$

A, C, D constants

the boundary conditions on the airfoil surface demands that the flow be locally tangential to the surface so that

$$V_c' = U_\infty \frac{dy}{dx} = \left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

and

$$V_i = U_\infty \frac{d\eta}{d\xi} = \left(\frac{\partial \Phi}{\partial \eta}\right)_{\eta=0}$$

Then $$\frac{\partial \Phi}{\partial \xi} = \frac{\partial (A\phi)}{\partial (cx)} = \frac{A}{c} \cdot \frac{\partial \phi}{\partial x}$$

$$\frac{\partial^2 \Phi}{\partial \xi^2} = \frac{A}{c^2} \cdot \frac{\partial^2 \phi}{\partial x^2}$$

And $$\frac{\partial \Phi}{\partial \eta} = \frac{A}{D} \frac{\partial \phi}{\partial y} \quad ; \quad \frac{\partial^2 \Phi}{\partial \eta^2} = \frac{A}{D^2} \frac{\partial^2 \phi}{\partial y^2}$$

Substitution the above relations into eqn. (13).

$$\frac{\partial^2 \Phi}{\partial \xi^2} + \frac{\partial^2 \Phi}{\partial^2 \eta} = \frac{A}{D^2} \left[\frac{D^2}{c^2} \cdot \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right] = 0$$

observing equation (12'). we note that B must be equal to $\frac{D}{c}$ then $\frac{D}{c} = \sqrt{1 - M_\infty^2}$

If the airfoil is thin, and by definition this must be so for the small-disturbance conditions of the theory from eqn. (12) is derived, the implication of this restriction is that the airfoils are similar shape in both planes.

First case $C = 1$; the $\xi = x$. This gives that $D = B = \sqrt{1 - M_\infty^2}$ then $\eta = \sqrt{1 - M_\infty^2}\, y$ and $\Phi = \sqrt{1 - M_\infty^2}\, \phi$.

then $$V_c' = U_\infty \frac{dy}{dx} = \left[\frac{\partial \phi}{\partial y}\right]_{y=0}$$

$$V_c' = \frac{B}{A} \cdot \left[\frac{\partial \Phi}{\partial \eta}\right]_{\eta=0} = \frac{B}{A} \cdot U_\infty \frac{d\eta}{d\xi}$$

but $D = B$ since $\frac{D}{C} = B$ and $C = 1$

For similar airfoils, it is required that $\frac{dy}{dx} = \frac{\partial \eta}{d\xi}$ at corresponding points. $A = B$

the we obtain that $\Phi = \sqrt{1 - M_\infty^2} \cdot \phi$

the horizontal flow perturbations are now found as.

$$u'_c = \frac{\partial \phi}{\partial x} = \frac{1}{\sqrt{1-M_\infty^2}} \cdot \frac{\partial \Phi}{\partial x} \qquad (14)$$

$$u'_c = \frac{u'_i}{\sqrt{1-M_\infty^2}}$$

and $$C_{p_c} = \frac{-2u'_c}{U_\infty} = \frac{-1}{\sqrt{1-M_\infty^2}} \cdot \frac{2u'_i}{U_\infty} \qquad (15)$$

$$C_{p_c} = \frac{C_{p_i}}{\sqrt{1-M_\infty^2}} \qquad (16)$$

Since $$C_L = \frac{C_{L_i}}{\sqrt{1-M_\infty^2}} \qquad (17)$$

This simple use of the factor $\sqrt{1-M_\infty^2}$ is known as the Prandtl-Glauert rule or Law and $\sqrt{1-M_\infty^2}$ is known as Glauert's factor.

For the case $D=1$ and then $y=Y$. we obtain the relation $\varepsilon = \frac{1}{B} = \frac{1}{\sqrt{1-M_\infty^2}}$

By applying the transformation we get

$$\sigma = \frac{x}{\sqrt{1-M_\infty^2}}$$

Applying the equation $V_c' = U_\infty \frac{dy}{dx}$ and

$$V_c' = \left.\frac{\partial \phi}{\partial y}\right|_{y=0} = \frac{1}{A}\left[\frac{\partial \Phi}{\partial \eta}\right]_{\eta=0}$$

By substituting $\Phi = A\phi$ and we know that $y = \eta$. we get:

$$V_c' = \frac{U_\infty}{A}\frac{d\eta}{d\xi} = \frac{U_\infty}{A}\frac{dy}{dx}\cdot\sqrt{1-M_\infty^2}$$

To preserve the identity, $A = \sqrt{1-M_\infty^2}$ and then the transform potential is

$$\Phi = \sqrt{1-M_\infty^2}\,\phi$$

Critical pressure coefficient

The pressure coefficient of point of minimum pressure on an airfoil section.

$$C_{p_{min}} = \frac{P_{min} - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \qquad (16)$$

but $\frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\gamma P_\infty M_\infty^2$ then

$$C_{p_{min}} = \left(\frac{P_{min}}{P_\infty} - 1\right)\cdot\frac{2}{\gamma M_\infty^2} \qquad (17)$$

The critical condition is when P_{min} first reaches the sonic pressure P^* and M_∞ becomes M_{crit}. $C_{P_{min}}$ is then the critical pressure coefficient of the airfoil section.

$$(18) \qquad C_{P_{crit}} = \left[\left(\frac{P^*}{P_\infty}\right) - 1\right] \cdot \frac{2}{\gamma M_{crit}^2} = \left[\frac{P^*}{P_\infty} - 1\right] \cdot \frac{2}{\gamma M_{crit}^2}$$

Applying the isentropic flow along a streamline in the energy equation, we obtain.

$$\frac{V_\infty^2}{2} + \frac{a_\infty^2}{\gamma - 1} = \frac{V^2}{2} + \frac{a^2}{\gamma - 1}$$

Divide through by a_∞^2 for the condition whe $V = a = a^*$; $M_\infty = M_{crit}$.

$$\frac{M_{crit}^2}{2} + \frac{1}{\gamma - 1} = \left(\frac{a^*}{a_\infty}\right)^2 \left(1 + \frac{1}{\gamma - 1}\right) \qquad (19)$$

$$\left(\frac{a^*}{a_\infty}\right)^2 = M_{crit}^2 \cdot \frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1} \qquad (20)$$

and $$\frac{P^*}{P_\infty} = \left(\frac{a^*}{a_\infty}\right)^{\frac{2\gamma}{\gamma - 1}} \qquad (21)$$

$$C_{p_{crit}} = \left[\left(\frac{\gamma-1}{\gamma+1} M_{crit}^2 + \frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \frac{2}{\gamma M_{crit}^2}$$

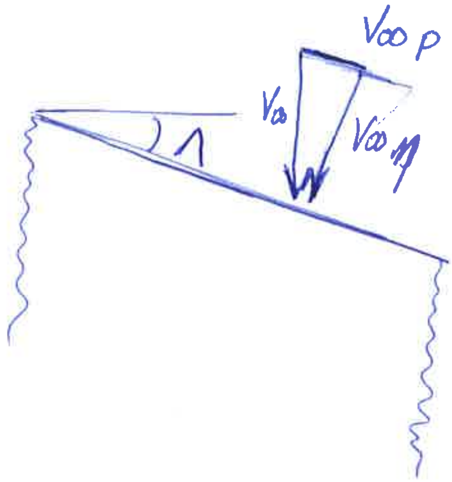
$M_{crit.}$ is the critical Mach number of the wing, and is the parameter that is often required to be found. $C_{p_{crit}}$ is the pressure coefficient at the point of maximum velocity on the wing when locally sonic conditions are just attained. This can be predicted from the corresponding minimum pressure coefficient in an incompressible flow (C_{p_i}). This can be obtained from pressure-distribution data from low-speed models or, from the solution of Laplace equation of a potential flow.

The Prandtl-Glauert law gives

$$C_{p_{crit}} = \frac{C_{p_i}}{\sqrt{1 - M_\infty^2}} \qquad (21)$$

Application to swept wings

this can be considered to be the superposition of two flows. One Component is the flow perpendicular to the swept leading edge. the other is the flow parallel to the leading edge.



$$(1 - M_\infty^2 \cos \Lambda) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}$$

only the perpendicular component affects the pressure so we obtain.

$$(u'_c)_n = \frac{(u'_i)_n}{\sqrt{1 - M_\infty^2 . \cos^2 \Lambda}}$$

$$C_{p_c} = \frac{-2 . \cos \Lambda . (u'_i)_n}{U_\infty (\sqrt{1 - M_\infty^2 . \cos^2 \Lambda}}$$

$$C_L = \frac{C_{Li}}{\sqrt{1 - M_\infty^2 . \cos^2 \Lambda}}$$

Example: For NACA 4 digit series of symmetrical airfoil sections in incompressible flow the maximum disturbance velocity $\left(\frac{u'}{U_\infty}\right)_{max}$

Corresponding for $(C_p)_{min}$ varies in the following way with thickness-to-chord ratio (t/c).

	t/c	$(u'/U_\infty)_{max}$
NACA 0006	0.06	0.107
NACA 0008	0.08	0.133
NACA 0.010	0.10	0.158
NACA 0.012	0.12	0.188
NACA 0.015	0.15	0.233
NACA 0.018	0.18	0.278
NACA 0.021	0.21	0.325
NACA 0024	0.24	0.374

Use the above data to determine the Critical Mach number for.

(i) A straight wing of infinite span with NACA 0010

The NACA 0010 wing section at a given freestream Mach number M_∞ is equivalent to a 4-digit NACA series for in incompressible flow

$$(t/c)_i = 0.1\sqrt{1-M_\infty^2}$$

The maximum disturbance velocity $\left[\left(\frac{u'}{U_\infty}\right)_{max}\right]_i$ is obtained by using linear interpolation on the data in the table given above.

The Maximum perturbation velocity in actual compressible flow at M_∞ is given by

$$\left(\frac{u'}{U_\infty}\right)_{max} = \frac{1}{\sqrt{(1-M_\infty^2)}} \cdot \left(\frac{u'}{U_\infty}\right)_{max}\Big|_i$$

The maximum local Mach number is given by

$$M_{max} = \frac{U_\infty + (U_c)_{max}}{a_\infty}$$

$$M_{max} = \frac{U_\infty}{a_\infty}\left(1 + \left(\frac{U_c}{U_\infty}\right)_{max}\right) = M_\infty \cdot \left(1 + \frac{(U_c)_{max}}{U_\infty}\right)$$

M_∞	$\sqrt{1-M_\infty^2}$	$\left(\frac{u'}{U_\infty}\right)_{max}\Big\vert_i$	$\left(\frac{u'}{U_\infty}\right)_{max}\Big\vert_c$	M_{max}
0.5	0.866	0.141	0.188	0.594
0.6	0.80	0.133	0.207	0.725
0.7	0.714	0.120	0.2353	0.865
0.75	0.66	0.114	0.2606	0.945
0.8	0.6	0.107	0.2972	1.038

Linear interpolation between $M_\infty = 0.75$ and 0.8 gives the critical value of $M_\infty \cong 0.78$ Corresponding to $M_{max} = 1.0$

- If the wing is swept-back 45^0 determine the maximum Mach

Determining the relation of thickness and Chord.

$$(t/c)_i = 0.1\sqrt{1 - M_\infty^2 \cos^2\Lambda}$$

$$= 0.1\sqrt{1 - 0.5 M_\infty^2}$$

V_∞, must be replaced by ($V_{\infty n}$).

$V_{\infty n} = V_\infty \cos\Lambda$, so then the maximum disturbance velocity is given by

$$\left[\left(\frac{u'}{V_{\infty n}}\right)_{max}\right]_c = \frac{1}{1 + 0.5 M_\infty^2}\left[\left(\frac{u'}{V_{\infty n}}\right)_{max}\right]_i$$

The Maximum local Mach number is then obtained from.

$$M_{max} = M_\infty\left[1 + \left[\left[\frac{u'}{V_{\infty n}}\right]_{max}\right]_c\right]$$

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M_∞	$\sqrt{1-M_\infty^2}$	$\left[\left(\frac{u'}{V_{\infty n}}\right)_{max}\right]_i$	$\left[\frac{u'}{V_{\infty n}}\right]_{max}\Big\vert_c$	M_{max}
0.5	0.935	0.14654	0.128	0.565
0.6	0.906	0.143	0.123	0.674
0.7	0.869	0.141	0.132	0.792
0.8	0.825	0.136	0.141	0.913
0.85	0.799	0.133	0.147	0.975
0.9	0.771	0.128	0.152	1.037

Linear interpolation gives a critical Mach number of about (0.87).