

Dimension of a Vector Space

The dimension of a Vector Space $V(\mathbb{R})$ is the maximum possible number of linearly independent vectors that can fit in one of its linear combinations.

Base of a Vector Space

A set of linearly independent vectors the size of the space's dimension.

e.g. : $\mathbb{R}^3(\mathbb{R}) \longrightarrow \dim(\mathbb{R}^3) = 3$

Linear combination

Any vector in a Vector space can be represented as a L.C. of any base in that space

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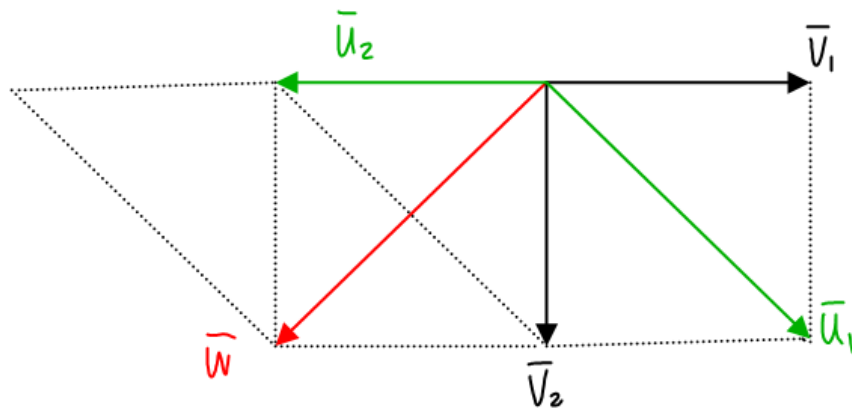
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Reference systems

 $\mathbb{R}^2(\mathbb{R})$

$$B_1 = \{\bar{u}_i\} = \{\bar{u}_1, \bar{u}_2\}$$

$$B_2 = \{\bar{v}_i\} = \{\bar{v}_1, \bar{v}_2\}$$



$$\bar{w} = -1\bar{v}_1 + \bar{v}_2 = (-1, 1)_{B_2}$$

$$\bar{w} = -1 \underbrace{(0\bar{u}_1, -1\bar{u}_2)}_{\bar{v}_2} + 1 \underbrace{(1\bar{u}_1 + 1\bar{u}_2)}_{\bar{v}_1} = 1\bar{u}_1 + 2\bar{u}_2 = (1, 2)_{B_1}$$

$$\bar{u}_1 = 1\bar{u}_1 + 0\bar{u}_2 = (1, 0)_{B_1}$$

$$\bar{u}_2 = 0\bar{u}_1 + 1\bar{u}_2 = (0, 1)_{B_1}$$

$$\bar{v}_1 = 1\bar{v}_1 + 0\bar{v}_2 = (1, 0)_{B_2}$$

$$\bar{v}_2 = 0\bar{v}_1 + 1\bar{v}_2 = (0, 1)_{B_2}$$

$$\bar{u}_1 = 1\bar{v}_1 + 1\bar{v}_2 = (1, 1)_{B_2}$$

$$\bar{u}_2 = -1\bar{v}_1 + 0\bar{v}_2 = (-1, 0)_{B_2}$$

$$\bar{v}_1 = 0\bar{u}_1 - 1\bar{u}_2 = (0, -1)_{B_1}$$

$$\bar{v}_2 = 1\bar{u}_1 + 1\bar{u}_2 = (1, 1)_{B_1}$$

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Change of Base

$$\begin{cases} \bar{u}_1 = 1\bar{v}_1 + 1\bar{v}_2 = (1, 1)_{B_2} \\ \bar{u}_2 = -1\bar{v}_1 + 0\bar{v}_2 = (-1, 0)_{B_2} \end{cases}$$

$$\begin{cases} \bar{v}_1 = 0\bar{u}_1 - 1\bar{u}_2 = (0, -1)_{B_1} \\ \bar{v}_2 = 1\bar{u}_1 + 1\bar{u}_2 = (1, 1)_{B_1} \end{cases}$$

$$\bar{w} = -1\bar{v}_1 + \bar{v}_2 = (-1, 1)_{B_2}$$

$$\bar{w} = -1 \underbrace{(0\bar{u}_1 - 1\bar{u}_2)}_{\bar{v}_1} + 1 \underbrace{(1\bar{u}_1 + 1\bar{u}_2)}_{\bar{v}_2} = 1\bar{u}_1 + 2\bar{u}_2 = (1, 2)_{B_1}$$

$$\bar{w} = 1\bar{u}_1 + 2\bar{u}_2 = (1, 2)_{B_1}$$

$$\begin{matrix} C \\ \left(\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array} \right) \begin{pmatrix} -1 \\ 1 \end{pmatrix}_{B_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}_{B_1} \\ \bar{v}_1 \quad \bar{v}_2 \\ \text{expressed in } B_1 \end{matrix}$$

$$\begin{matrix} C^{-1} \\ \left(\begin{array}{cc} 1 & -1 \\ 1 & 0 \end{array} \right) \begin{pmatrix} 1 \\ 2 \end{pmatrix}_{B_1} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}_{B_2} \\ \bar{u}_1 \quad \bar{u}_2 \\ \text{expressed in } B_2 \end{matrix}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}^{-1} = \frac{\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}^t}{1} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$| \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} | = 1$$

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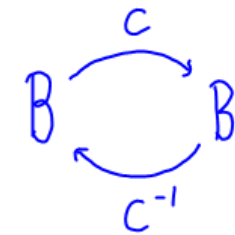
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Base change in \mathbb{R}^3

$$\mathbb{R}^3(\mathbb{R}) \quad B = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\}$$

$$B' = \{\bar{u}_1, \bar{u}_2, \bar{u}_3\}$$

$$\bar{x} \in \mathbb{R}^3 \begin{cases} \bar{x} = x^1 \bar{e}_1 + x^2 \bar{e}_2 + x^3 \bar{e}_3 = (x^1, x^2, x^3)_B \\ \bar{x} = \alpha \bar{u}_1 + \beta \bar{u}_2 + \gamma \bar{u}_3 = (\alpha, \beta, \gamma)_{B'} \end{cases}$$



$$\begin{cases} \bar{u}_1 = a^1 \bar{e}_1 + a^2 \bar{e}_2 + a^3 \bar{e}_3 = (a^1, a^2, a^3)_B \\ \bar{u}_2 = b^1 \bar{e}_1 + b^2 \bar{e}_2 + b^3 \bar{e}_3 = (b^1, b^2, b^3)_B \\ \bar{u}_3 = c^1 \bar{e}_1 + c^2 \bar{e}_2 + c^3 \bar{e}_3 = (c^1, c^2, c^3)_B \end{cases}$$

$$C = \begin{pmatrix} a^1 & b^1 & c^1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{pmatrix}$$

$\bar{u}_1 \quad \bar{u}_2 \quad \bar{u}_3$
expressed in B

The columns of C are the vectors of B' expressed in B.

How the Base Change works:

$$(C) \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

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Inverting a matrix

$$A^{-1} = \frac{\text{Adj}(A)^T}{|A|}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$A^t = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

$$\text{Adj}(A) = \begin{pmatrix} +A_{11} & -A_{12} & +A_{13} \\ -A_{21} & +A_{22} & -A_{23} \\ +A_{31} & -A_{32} & +A_{33} \end{pmatrix}$$

$$A_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = +a_{11}A_{11} - a_{12}A_{12} + a_{13}A_{13}$$

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