$$+ \frac{1.0}{y_{1n} - y_m}\left[1.0 + \frac{x_m - x_{1n}}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}}\right.$$

$$- \frac{1.0}{y_{2n} - y_m}\left[1.0 + \frac{x_m - x_{2n}}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}}\right.$$

Summing the contributions of all the vortices to the downwash at the
the mth panel,

$$w_m = \sum_{n=1}^{2N} w_{m,n}$$

Let us now apply the tangency requirement defined by equations (
Since we are considering a planar wing in this section, $(dz/dx)_m = 0$
$\phi = 0$. The component of the free-stream velocity perpendicular to the
at any point on the wing. Thus, the resultant flow will be tangent to the
vortex-induced downwash at the control point of the mth panel, whi
using equation (7.45) balances the normal component of the free-strea

$$w_m + U_\infty \sin \alpha = 0$$

For small angles of attack,

$$w_m = -U_\infty \alpha$$

In Example 7.2, we will solve for the aerodynamic coefficients for
a relatively simple planform and an uncambered section. The vortex latt
be applied using only a single lattice element in the chordwise directio
wise subdivision of the wing. Applying the boundary condition that t
through the wing at only one point in the chordwise direction is reasona
plate wing. However, it would not be adequate for a wing with camber
wing with deflected flaps.

EXAMPLE 7.2: Use the vortex lattice method (VLM) to calculate the aerodynamic coefficients for a swept wing

Let us use the relations developed in this section to calculat
cient for a swept wing. So that the calculation procedures c
lowed, let us consider a wing that has a relatively simple geo
illustrated in Fig. 7.31). The wing has an aspect ratio of 5,
unity (i.e., $c_r = c_t$), and an uncambered section (i.e., it is a f
the taper ratio is unity, the leading edge, the quarter-chord
quarter-chord line, and the trailing edge all have the same s

$$AR = 5 = \frac{b^2}{S}$$

and since for a swept, untapered wing

$$S = bc$$

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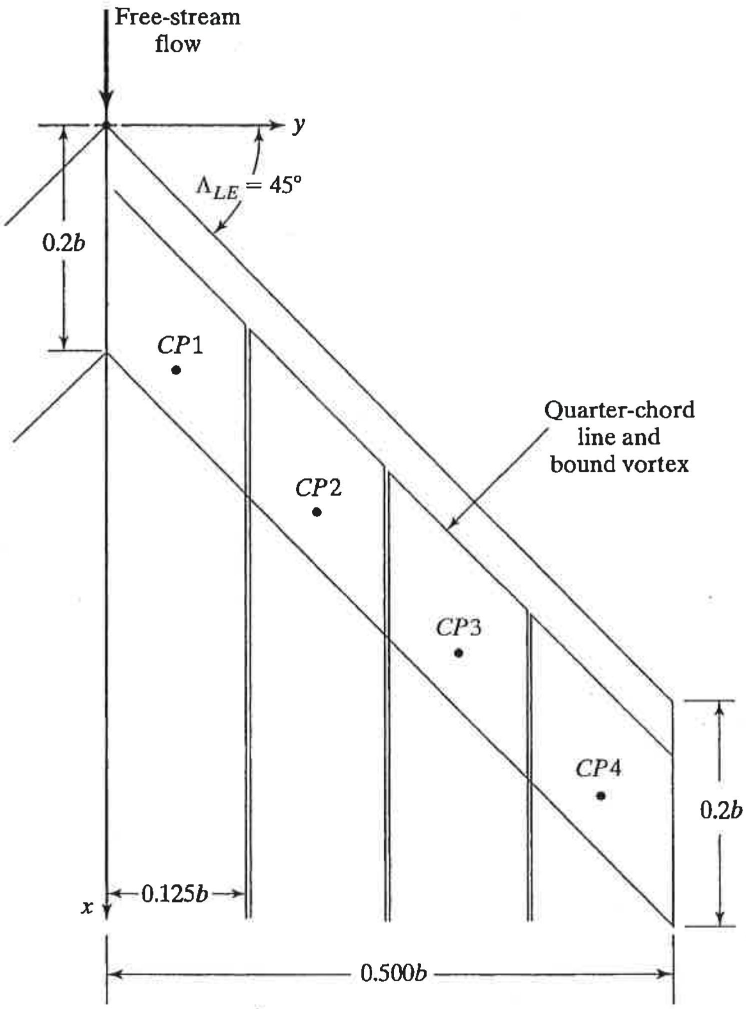

Figure 7.31 Four-panel representation of a swept planar win taper ratio of unity, AR = 5, Λ = 45°.

it is clear that $b = 5c$. Using this relation, it is possible to calcu necessary coordinates in terms of the parameter b. Therefore, does not require that we know the physical dimensions of the c

Solution: The flow field under consideration is symmetric with respect to th (xz plane); that is, there is no yaw. Thus, the lift force acting at a poi board wing ($+y$) is equal to that at the corresponding point on ($-y$). Because of symmetry, we need only to solve for the strengt tices of the starboard wing. Furthermore, we need to apply the ta tion [i.e., equation (7.47)] only at the control points of the sta However, we must remember to include the contributions of t vortices of the port wing to the velocities induced at these control starboard wing). Thus, for this planar symmetric flow, equation (7

$$w_m = \sum_{n=1}^{N} w_{m,ns} + \sum_{n=1}^{N} w_{m,np}$$

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where the symbols s and p represent the starboard a
respectively.

The planform of the starboard wing is divided into fo
panel extending from the leading edge to the trailing edge. I
selves to only four spanwise panels, we can calculate the
horseshoe vortices using only a pocket electronic calculato
more easily see how the terms are to be evaluated. As bef
portion of each horseshoe vortex coincides with the quarter-
panel and the trailing vortices are in the plane of the wing, p
axis. The control points are designated by the solid symb
Recall that $(x_m, y_m, 0)$ are the coordinates of a given contro
$(x_{1n}, y_{1n}, 0)$ and $(x_{2n}, y_{2n}, 0)$ are the coordinates of the
bound-vortex filament AB. The coordinates for a 4×1 lat
wise divisions and one chordwise division) for the starboa
are summarized in Table 7.2.

Using equation (7.44) to calculate the downwash veloc
panel 1 (of the starboard wing) induced by the horseshoe vo
of the starboard wing,

$$w_{1,1s} = \frac{\Gamma_1}{4\pi}\left\{\frac{1.0}{(0.1625b)(-0.0625b) - (0.0375b)(0.0 \ldots}\right.$$

$$\left[\frac{(0.1250b)(0.1625b) + (0.1250b)(0.0625b \ldots}{\sqrt{(0.1625b)^2 + (0.0625b)^2}}\right.$$

$$- \frac{(0.1250b)(0.0375b) + (0.1250b)(-0.062 \ldots}{\sqrt{(0.0375b)^2 + (-0.0625b)^2}}$$

$$+ \frac{1.0}{-0.0625b}\left[1.0 + \frac{0.1625b}{\sqrt{(0.1625b)^2 + (0.062 \ldots}}\right.$$

$$- \frac{1.0}{0.0625b}\left[1.0 + \frac{0.0375b}{\sqrt{(0.0375b)^2 + (-0.062 \ldots}}\right.$$

$$= \frac{\Gamma_1}{4\pi b}(-16.3533 - 30.9335 - 24.2319)$$

TABLE 7.2 Coordinates of the Bound Vortices and of the Control Poin
of the Starboard (Right) Wing

Panel	x_m	y_m	x_{1n}	y_{1n}	x_{2n}
1	0.2125b	0.0625b	0.0500b	0.0000b	0.1750b
2	0.3375b	0.1875b	0.1750b	0.1250b	0.3000b
3	0.4625b	0.3125b	0.3000b	0.2500b	0.4250b
4	0.5875b	0.4375b	0.4250b	0.3750b	0.5500b

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Note that, as one would expect, each of the vortex elements in
tive (downward) component of velocity at the control point
should visualize the flow induced by each segment of the hors
to verify that a negative value for each of the components is in
rect. In addition, the velocity induced by the vortex trailing fr
greatest in magnitude. Adding the components together, we fi

$$w_{1,1s} = \frac{\Gamma_1}{4\pi b}(-71.5187)$$

The downwash velocity at the CP of panel 1 (of the sta
induced by the horseshoe vortex of panel 1 of the port wing is

$$w_{1,1p} = \frac{\Gamma_1}{4\pi}\left\{\frac{1.0}{(0.0375b)(0.0625b) - (0.1625b)(0.1875\ldots}\right.$$

$$\left[\frac{(-0.1250b)(0.0375b) + (0.1250b)(0.1875b\ldots}{\sqrt{(0.0375b)^2 + (0.1875b)^2}}\right.$$

$$-\frac{(-0.1250b)(0.1625b) + (0.1250b)(0.0625\ldots}{\sqrt{(0.1625b)^2 + (0.0625b)^2}}$$

$$+\frac{1.0}{-0.1875b}\left[1.0 + \frac{0.0375b}{\sqrt{(0.0375b)^2 + (0.1875\ldots}}\right.$$

$$-\frac{1.0}{-0.0625b}\left[1.0 + \frac{0.1625b}{\sqrt{(0.1625b)^2 + (0.0625\ldots}}\right.$$

$$= \frac{\Gamma_1}{4\pi b}[-6.0392 - 6.3793 + 30.9335]$$

$$= \frac{\Gamma_1}{4\pi b}(18.5150)$$

Similarly, using equation (7.44) to calculate the downwa
the CP of panel 2 induced by the horseshoe vortex of panel
board wing, we obtain

$$w_{2,4s} = \frac{\Gamma_4}{4\pi}\left\{\frac{1.0}{(-0.0875b)(-0.3125b) - (-0.2125b)(-0\ldots}\right.$$

$$\left[\frac{(0.1250b)(-0.0875b) + (0.1250b)(-0.1875b\ldots}{\sqrt{(-0.0875b)^2 + (-0.1875b)^2}}\right.$$

$$-\frac{(0.1250b)(-0.2125b) + (0.1250b)(-0.3125\ldots}{\sqrt{(-0.2125b)^2 + (-0.3125b)^2}}$$

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$$+\frac{1.0}{0.1875b}\left[1.0 + \frac{-0.0875b}{\sqrt{(-0.0875b)^2 + (-0.1875}}\right.$$

$$-\frac{1.0}{0.3125b}\left[1.0 + \frac{-0.2125b}{\sqrt{(-0.2125b)^2 + (-0.3125}}\right.$$

$$= \frac{\Gamma_4}{4\pi b}[-0.60167 + 3.07795 - 1.40061]$$

$$= \frac{\Gamma_4}{4\pi b}(1.0757)$$

Again, the student should visualize the flow induced by each segm
the signs and the relative magnitudes of the components are indi

Evaluating all of the various components (or influenc
we find that at control point 1

$$w_1 = \frac{1}{4\pi b}[(-71.5187\Gamma_1 + 11.2933\Gamma_2 + 1.0757\Gamma_3 + 0.3775\Gamma$$

$$+ (+18.5150\Gamma_1 + 2.0504\Gamma_2 + 0.5887\Gamma_3$$

At CP 2,

$$w_2 = \frac{1}{4\pi b}[(+20.2174\Gamma_1 - 71.5187\Gamma_2 + 11.2933\Gamma_3 + 1.0757$$

$$+ (+3.6144\Gamma_1 + 1.1742\Gamma_2 + 0.4903\Gamma_3$$

At CP 3,

$$w_3 = \frac{1}{4\pi b}[(+3.8792\Gamma_1 + 20.2174\Gamma_2 - 71.5187\Gamma_3 + 11.2933$$

$$+ (+1.5480\Gamma_1 + 0.7227\Gamma_2 + 0.3776\Gamma_3 +$$

At CP 4,

$$w_4 = \frac{1}{4\pi b}[(+1.6334\Gamma_1 + 3.8792\Gamma_2 + 20.2174\Gamma_3 - 71.5187\Gamma$$

$$+ (+0.8609\Gamma_1 + 0.4834\Gamma_2 + 0.2895\Gamma_3 +$$

Since it is a planar wing with no dihedral, the no-flow condit
(7.47) requires that

$$w_1 = w_2 = w_3 = w_4 = -U_\infty \alpha$$

Thus

$$-53.0037\Gamma_1 + 13.3437\Gamma_2 + 1.6644\Gamma_3 + 0.6434\Gamma_4 = -$$

$$+23.8318\Gamma_1 - 70.3445\Gamma_2 + 11.7836\Gamma_3 + 1.3260\Gamma_4 = -$$

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$$+5.4272\Gamma_1 + 20.9401\Gamma_2 - 71.1411\Gamma_3 + 11.5112\Gamma_4 = -4$$

$$+2.4943\Gamma_1 + 4.3626\Gamma_2 + 20.5069\Gamma_3 - 71.3351\Gamma_4 = -4$$

Solving for $\Gamma_1, \Gamma_2, \Gamma_3$, and Γ_4, we find that

$$\Gamma_1 = +0.0273(4\pi b U_\infty \alpha)$$

$$\Gamma_2 = +0.0287(4\pi b U_\infty \alpha)$$

$$\Gamma_3 = +0.0286(4\pi b U_\infty \alpha)$$

$$\Gamma_4 = +0.0250(4\pi b U_\infty \alpha)$$

Having determined the strength of each of the vortices
the boundary conditions that the flow is tangent to the surface a
control points, the lift of the wing may be calculated. For wings
dihedral over any portion of the wing, all the lift is generated
stream velocity crossing the spanwise vortex filament, since there
wash or backwash velocities. Furthermore, since the panels ext
leading edge to the trailing edge, the lift acting on the nth pane

$$l_n = \rho_\infty U_\infty \Gamma_n$$

which is also the lift per unit span. Since the flow is symmetric
for the wing is

$$L = 2\int_0^{0.5b} \rho_\infty U_\infty \Gamma(y)\, dy$$

or, in terms of the finite-element panels,

$$L = 2\rho_\infty U_\infty \sum_{n=1}^{4} \Gamma_n \Delta y_n$$

Since $\Delta y_n = 0.1250b$ for each panel,

$$L = 2\rho_\infty U_\infty 4\pi b U_\infty \alpha(0.0273 + 0.0287 + 0.0286 + 0.025$$
$$= \rho_\infty U_\infty^2 b^2 \pi\alpha(0.1096)$$

To calculate the lift coefficient, recall that $S = bc$ and b
wing. Therefore,

$$C_L = \frac{L}{q_\infty S} = 1.096\pi\alpha$$

Furthermore,

$$C_{L,\alpha} = \frac{dC_L}{d\alpha} = 3.443 \text{ per radian} = 0.0601 \text{ per degre}$$

Comparing this value $C_{L,\alpha}$ with that for an unswept wing (such as t
sented in Fig. 7.14), it is apparent that an effect of sweepback is
in the lift-curve slope.

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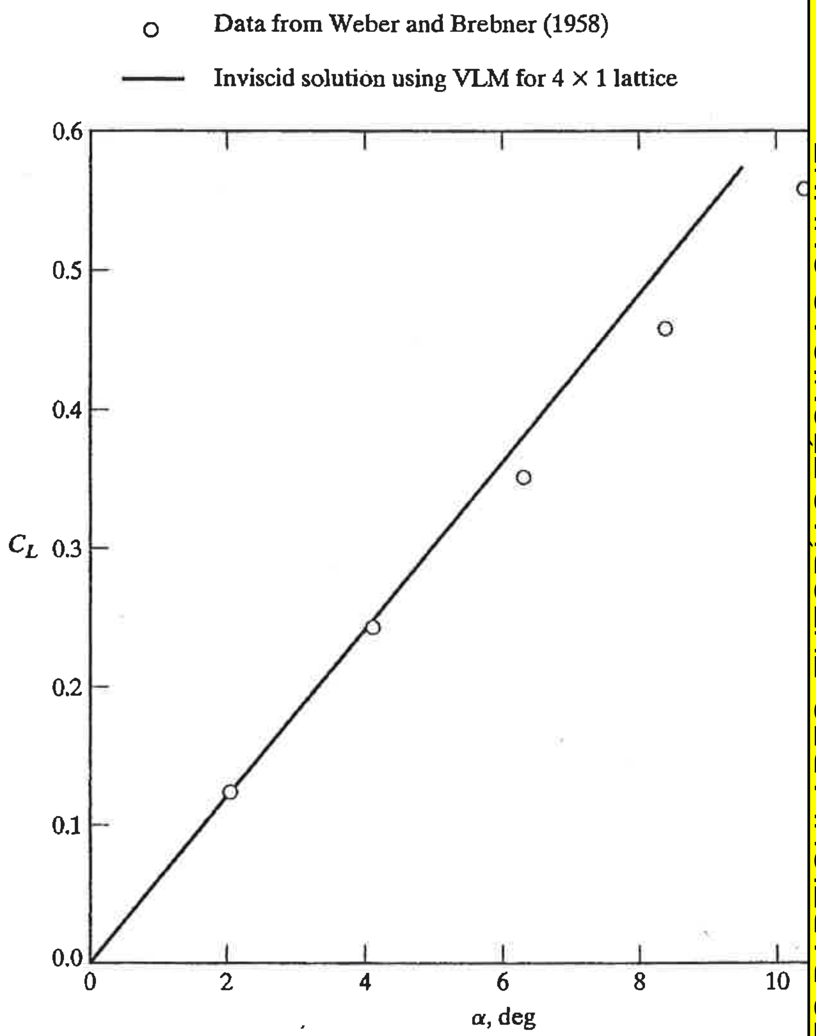

Figure 7.32 Comparison of the theoretical and the experim
lift coefficients for the swept wing of Fig. 7.31 in a subsonic st

The theoretical lift curve generated using the VLM is 7.32 with experimental results reported by Weber and Bre experimentally determined values of the lift coefficient are stant chord and of constant section, which was swept 45° a aspect ratio of 5. The theoretical lift coefficients are in goo the experimental values.

Since the lift per unit span is given by equation (7.49), the section lift c nth panel is

$$C_{l(n\text{th})} = \frac{l_n}{\frac{1}{2}\rho_\infty U_\infty^2 c_{\text{av}}} = \frac{2\Gamma}{U_\infty c_{\text{av}}}$$

When the panels extend from the leading edge to the trailing edge, s for the 4 × 1 lattice shown in Fig. 7.31, the value of Γ given in equat

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