Fluid dynamics Lubrication.

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UEM.

Lubrication Theory.

The lubrication or friction reduction of two bodies in near contact is generally accomplished by a viscous fluid Motion through a narrow but variable gap between the two bodies.
One or both bodies may be moving. The theory was developed by Reynolds (1886).

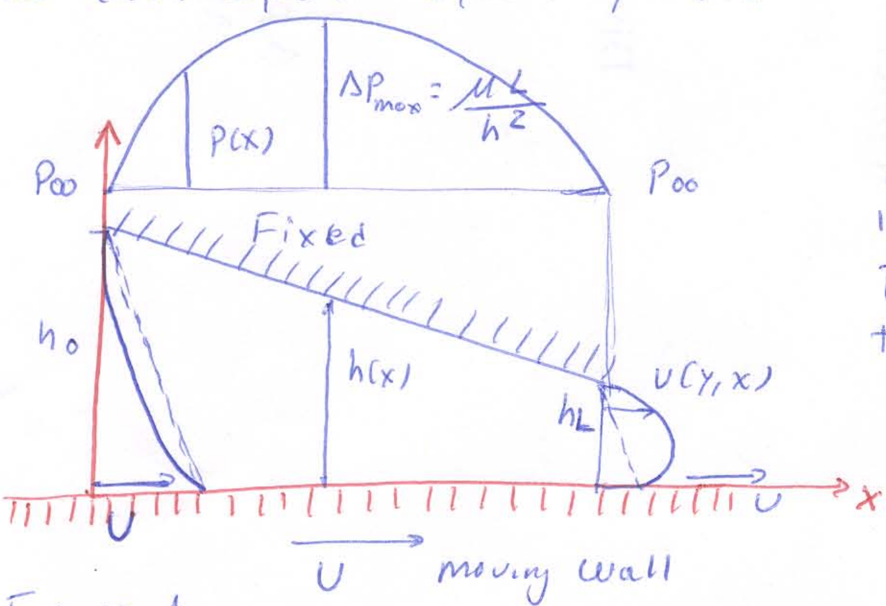

Figure 1

Low Reynolds number Couette flow in a Varing gap.
To maintain continuity the gap pressure rises to a maximum and superimposes.
Poiseville flow toward both ends of the gap.

An Idealization of the problem is the slipper-pad bearing

- The bottom wall moves at velocity U and creates a Couette flow in the gap.
- Upper block is fixed.
- Gap is very narrow $h(x) \lll L$ (h_0 to h_L) $h_0 > h_L$
- How we can find the pressure and velocity distribution

2.

- The flow is two-dimensional. $\frac{\partial}{\partial z} = 0$
- Negligible inertia.

$$\begin{cases} \rho . u \frac{\partial u}{\partial x} <<< \mu . \frac{\partial^2 u}{\partial y^2} \\ \rho U . \frac{U}{L} <<< \mu . \left(\frac{U}{h}\right)^2 \\ \frac{\rho . U . L}{\mu} \frac{h^2}{L^2} << 1 \end{cases}$$

Reynolds number Re_L can be large if the gap is very small.

Example: $U = 30 \, m/s$ $L = 4 cm$ $h = 0.1 \, mm$
SAE 50 Oil. $\nu = 7 \times 10^{-4} \, m^2/s$

$Re_L = 570$ but $Re_L \frac{h^2}{L^2} \cong 0.004$

Now examine ~~the~~ figure 1. The simple linear Couette flow profiles, shown as dashed lines at the entrance and exit, are impossible for this geometry because continuity is violated.

The mass flow at the entrance would exceed the exit flow.
To relieve this difficulty, high pressure develops in the gap and causes Poiseuille (parabolic) flow toward both ends of the gap, just sufficient to make the mass flow constant at every section x.

③.

At any section in the gap, then, the local velocity profile Combined- Couette- Poiseuille flow:

$$\mu . \frac{d^2 u}{dy^2} = \frac{dP}{dx} = \text{const}$$

Boundary conditions
non-slip condition
$U(+h) = 0$ ~~the solution is~~
$U(0) = U$

$$u(y) = \frac{1}{2\mu}\frac{dP}{dx} y(y-h) + U\left(1 - \frac{y}{h}\right) \quad (1)$$

The correct distribution $P(x)$ is one which everywhere satisfies the continuity equation for two-dimensional flow in the gap.

$$\int_0^h \frac{\partial u}{\partial x} dy + \int_0^h \frac{\partial v}{\partial y} dy = 0 \qquad \int_0^h \frac{\partial u}{\partial x} dy = -\int_0^h \frac{\partial v}{\partial y} . dy \quad (2)$$

$$-\int_0^h \frac{\partial v}{\partial x} . dy = -V(h) + V(0)$$

Where in this particular case we are assuming that the vertical velocities $V(h)$ and $V(0)$ are zero at both walls.

Eq. (1) into Eq. (2)

We can obtain a second-order differential equation for the pressure:

From. eq. (1).

$$\frac{\partial U}{\partial x} = \frac{y^2}{2\mu} . \frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x}\right] - \frac{1}{2\mu} y . \frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x} . h\right] - Uy . \frac{\partial}{\partial x}\left[\frac{1}{h}\right]$$

④

$$\int_0^h \frac{\partial u}{\partial x}\, dy = \frac{1}{2\mu}\int_0^h \frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x}\right] y^2 dy - \frac{1}{2\mu}\frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x} h\right]\int_0^h y\, dy$$

$$- U \frac{\partial}{\partial x}\left[\frac{1}{h}\right]\int_0^h y\, dy$$

$$\int_0^h \frac{\partial u}{\partial x}\, dy = \frac{1}{6\mu}\left[\frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x} h^3\right]\right] - \frac{1}{4\mu}\frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x} h^3\right] - \frac{U}{2}\frac{\partial h}{\partial x} = 0$$

$$= \frac{-1}{12\mu}\frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x} h^3\right] = -\frac{U}{2}\frac{\partial h}{\partial x}$$

$$\left[\frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x} h^3\right] = 6\mu . U \frac{\partial h}{\partial x}\right] \quad (3)$$

Assuming U constant and we can determine the gap variation $h(x)$. we can find $P(x)$ subject to the conditions $P(0) = P_L = P_\infty$

Solution of the equation (3). may be integrated numerically for any gap variation $h(x)$. we assume a linear gap.

$$h = h_0 + (h_L - h_0)\frac{x}{L} \qquad \begin{matrix} h(0) = h_0 \\ h(L) = h_L \end{matrix}$$

We may substitute in eq. (3). and carry out the double integration. the final solution is

$$\frac{P - P_\infty}{\mu . \frac{UL}{h_0^2}} = \frac{6\,(x/L)\,(1 - x/L)\,(1 - h_L/h_0)}{\left(1 + \frac{h_L}{h_0}\right)\left[1 - \left(1 - \frac{h_L}{h_0}\right)\frac{x}{L}\right]^2} \quad (4)$$

⑤

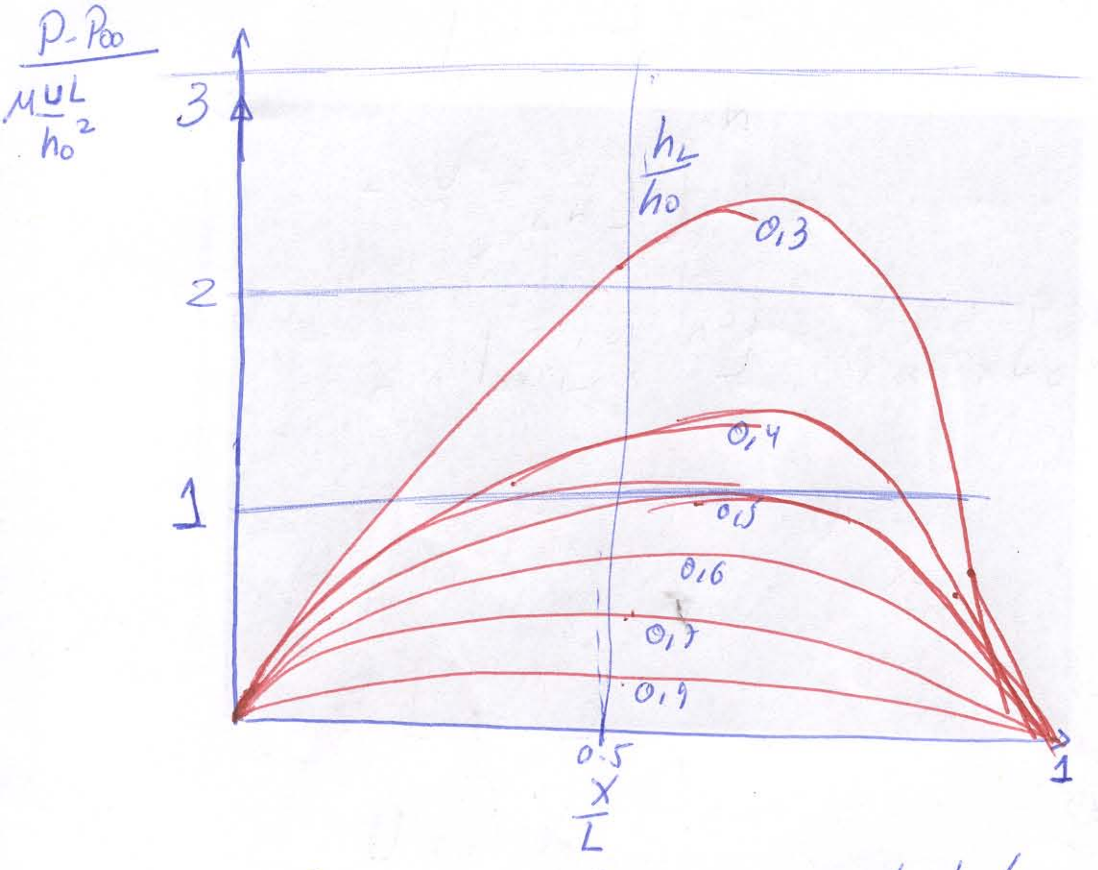

Figure 2. Pressure distribution in two-dimensional linear gap slipper-pad bearing.

When $\frac{h_L}{h_0}$ is only slight, the pressure distribution is nearly symmetric with P_{max} at $\frac{X}{L} = 0.5$

As the degree of contraction increases, P_{max} increases and moves toward the exit plane.

The maximum pressure rise is of the order of $\mu \frac{UL}{h_0^2}$ and can be amazingly high.

Previous example SAE 30 $U = 10\,m/s$ $L = 4\,cm$ $h_0 = 0.1$

We may compute $\mu \frac{UL}{h_0^2} = 2{,}5 \times 10^7$ Pa or 250 atm.

This provide a high force to the slipper block which can support a large load without the block touching the wall.

Theme case.

If we reverse the wall in fig 1 to move to the left, that is. $U<0$, then the pressure change in (4) is negative. the fluid will not actually develop a large negative pressure but rather will cavitate and form a vapor void in the gap. and for an expanding narrow gap may not generally bear much load or provide good lubrication. This effect is unavoidable in a rotating journal bearing, where the gap contracts and then expands and partial cavitation often occurs.

The General Reynolds Equation

In a general lubrication problem, both the upper and lower walls in Fig. 1 may be moving tangentially and normally, and the depth into the paper may be small, inducing a flow in the z. direction. We assume that there is no translation of the walls at z-direction The pressure now varies with both x and z and satisfies the following

$$\frac{\partial}{\partial x}\left[h^3 \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial z}\left[h^3 \frac{\partial P}{\partial z}\right] = 6\mu \frac{\partial}{\partial x}\left[h\{U(0) + U(h)\right] + 12\mu\left[V(h) - V(0)\right]$$

$h = h(x,z)$. This is the Three-dimensional Reynolds equation for Incompressible fluid Lubrication The Pressure must be known on all four open sides of the gap.