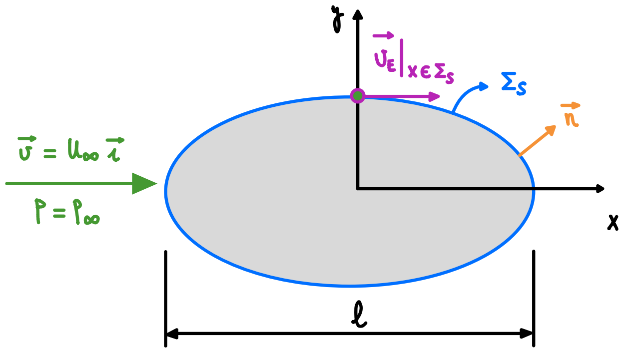
Capa Límite Laminar: Ecuaciones



Buscamos soluciones laminares, estacionarias e incompresibles.

$$\vec{v}' = \frac{\vec{v}}{U_\infty} \qquad \nabla' \blacksquare = \frac{1}{\ell}\nabla \blacksquare$$

$$\vec{x}' = \frac{\vec{x}}{\ell} \qquad \nabla' \cdot \blacksquare = \frac{1}{\ell}\nabla \cdot \blacksquare$$

$$p' = \frac{p - p_\infty}{\rho U_\infty^2} \qquad \Delta' \blacksquare = \frac{1}{\ell^2}\Delta \blacksquare$$

$$Re = \frac{U_\infty \ell}{\nu} >> 1$$

$$p - p_\infty = \frac{1}{2}\rho \vec{v}\vec{v} = \frac{1}{2}\rho U_\infty^2$$

$$\nabla \cdot \vec{v} = 0$$

$$\vec{v}\,\nabla\vec{v} = -\nabla\left(\frac{p - p_\infty}{\rho}\right) + \nu \Delta \vec{v}$$

$$\vec{x} \in \Sigma_s : \vec{v} = \vec{0}$$

$$|\vec{x}| \to \infty : \vec{v} \to U_\infty \vec{i} + O\left(\frac{\Gamma}{|\vec{x}|}\right)$$

$$\frac{p_E - p_\infty}{\rho} \to 0 + O\left(\frac{\Gamma}{|\vec{x}|}\right)$$

$$\nabla' \cdot \vec{v}' = 0$$

$$\vec{v}'\,\nabla'\vec{v}' = -\nabla' p' + Re^{-1}\Delta'\vec{v}'$$

$$\vec{x}' \in \Sigma_s : \vec{v}' = \vec{0}$$

$$|\vec{x}'| \to \infty : \vec{v}' \to \vec{i} + (\ )$$

$$p \to 0 + (\ )$$

Solución de Euler $(\epsilon = 0,\ Re \to \infty)$:

$$\left.\begin{array}{l} \nabla \cdot \vec{v}_E = 0 \\ \vec{v}_E\,\nabla\vec{v}_E = -\nabla\left(\dfrac{p_E - p_\infty}{\rho}\right) \\ \vec{x} \in \Sigma_s : \vec{v}_E \cdot \vec{n}_s = 0 + (\ ) \\ \vec{x} \to \infty : \dfrac{p - p_\infty}{\rho} \to 0 + (\ ) \end{array}\right\} \longrightarrow \vec{v}_E,\ \frac{p_E - p_\infty}{\rho}$$

EQUIVALENTE A U_E DEL EJEMPLO DE EXPANSIONES ASINTÓTICAS ACOPLADAS

La presión total es un invariante en todo el flujo ⟶ el campo de presiones viene dado por el de presiones por la relación: $p = cte = p_t = -\frac{1}{2}\rho U_\infty^2$

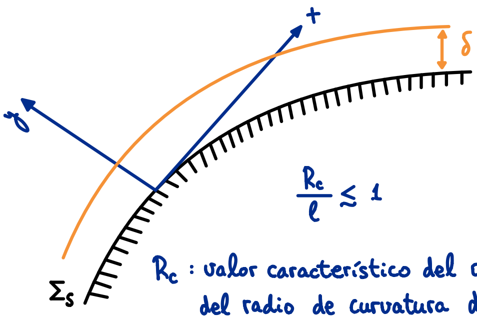
$$\sim \frac{U_\infty^2}{\ell} \qquad \sim \nu \frac{U_\infty}{\delta^2}$$

Comparando órdenes de magnitud de los términos convectivo y viscoso:

$$\frac{\delta}{\ell} \sim Re^{-1/2}$$

FLUJO ESBELTO EN LA REGIÓN INTERIOR (CAPA LÍMITE)

$$\frac{\Delta_y\left(\frac{p}{\rho}\right)}{\Delta_x\left(\frac{p}{\rho}\right)} \sim \left(\frac{\delta}{\ell}\right)^2 \sim Re^{-1} \ll 1$$



$$\frac{R_c}{\ell} \lesssim 1$$

R_c : valor característico del orden del radio de curvatura de Σ_S

Importante:

$$\frac{p_e}{\rho} + \frac{u_e^2}{2} = \frac{p_\infty}{\rho} + \frac{u_\infty^2}{2} \longrightarrow \frac{\partial}{\partial x}\left(\frac{p_E}{\rho}\right) = -\frac{d}{dx}\left(\frac{u_e^2}{2}\right) = -u_e \frac{du_e}{dx} \xrightarrow{p \sim p_E} \frac{\partial}{\partial x}\left(\frac{p}{\rho}\right) = -u_e\frac{du_e}{dx}$$

Por tanto:

SOLUCIÓN INTERIOR

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$0 \approx \frac{\partial}{\partial y}\left(\frac{p}{\rho}\right)$$

$y = 0$: $u = 0$; $v = v_p$ (SUCCIÓN SOPLADO)

$y \to \infty$: $u \to u_E\big|_{\vec{x}\to\Sigma_S} = u_e$

$x = x_0$: $u = u(x_0, y) = u_0(y)$

Llamando:

$$\tilde{x} = \frac{x}{\ell} \;;\; \tilde{y} = \frac{y}{\delta} = \frac{y}{\ell}Re^{-1/2} \;;\; \tilde{u} = \frac{u}{u_{ec}} \;;\; \tilde{v} = \frac{v}{v_c} = \frac{v}{u_{ec}}Re^{1/2} \;;\; \tilde{u}_e = \frac{u_e}{u_{ec}}$$

Obtenemos:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \;;\; \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \tilde{u}_e\frac{d\tilde{u}_e}{d\tilde{x}} + \nu\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}$$

$\tilde{y} = 0$: $\tilde{u} = 0$; $\tilde{v} = \tilde{v}_p$

$$\longrightarrow \quad \tilde{u} = \tilde{u}(\tilde{x}, \tilde{y} ; \tilde{u}_e, \tilde{v}_p, \tilde{x}_0, \tilde{u}_0)$$

$$\tilde{v} = \tilde{v}(\tilde{x}, \tilde{y} ; \tilde{u}_e, \tilde{v}_p, \tilde{x}_0, \tilde{u}_0)$$

Velocidad

$\delta(10^4)$ $\delta(10^3)$ $\delta(10^3)$