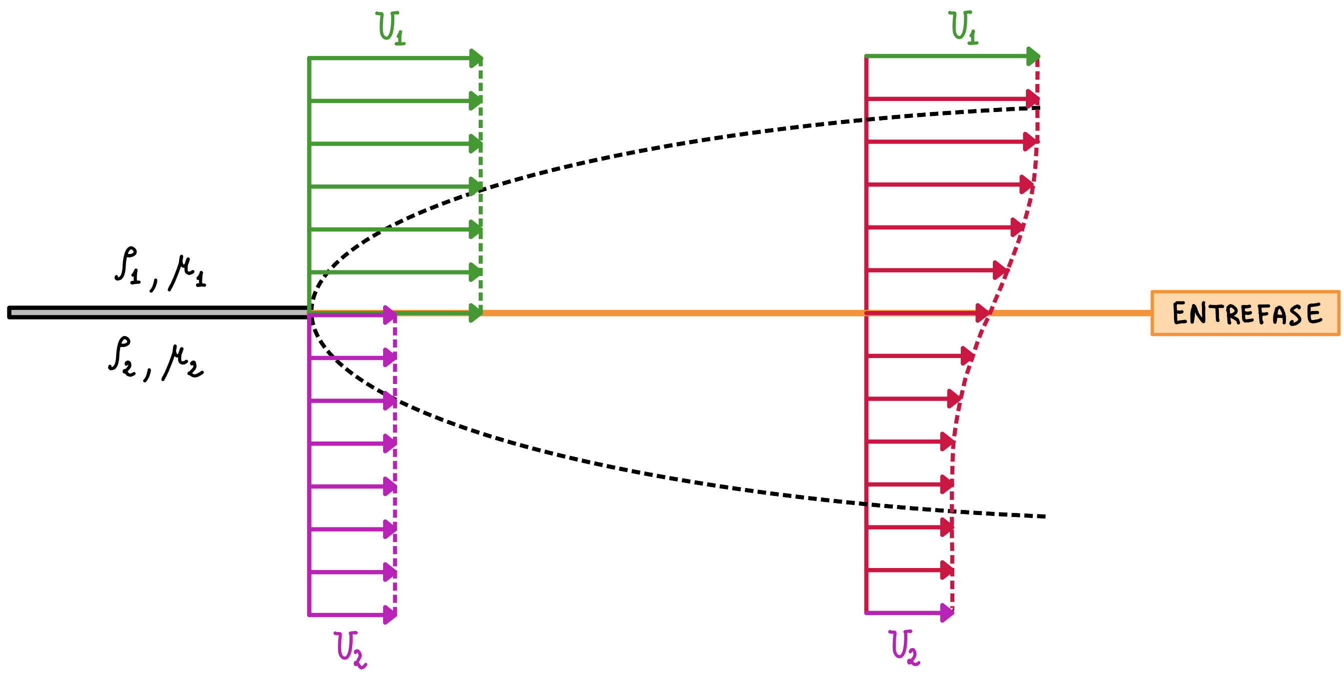
Capa de Mezcla



Fluido 1 : ρ_1, μ_1, U_1

Fluido 2 : $\rho_2, \mu_2, U_2 < U_1$

$\longrightarrow$ SE PONEN EN CONTACTO EN $x = 0$ $\longrightarrow$ CAPA DE MEZCLA

Ecuaciones del movimiento

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0$$

$$u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} = \nu_i \frac{\partial^2 u_i}{\partial y^2} \qquad (i = 1,2)$$

Condiciones de contorno

$y \to \infty : \quad u_1 = U_1$

$y \to -\infty : \quad u_2 = U_2$

$y = 0 : \; u_1 = u_2 \; ; \; v_1 = v_2 = 0 \; ; \; \mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y}$

Eliminamos $\nu_i \longrightarrow x , \frac{y}{\sqrt{\nu_i}} , u_i , \frac{v_i}{\sqrt{\nu_i}}$:

$u_i \frac{\partial u_i}{\partial x} + \frac{v_i}{\sqrt{\nu_i}} \frac{\partial u_i}{\partial\left(\frac{y}{\sqrt{\nu_i}}\right)} = \nu_i \frac{\partial^2 u_i}{\partial\left(\frac{y}{\sqrt{\nu_i}}\right)^2}$; $\frac{y}{\sqrt{\nu_i}} = 0 : u_1 = u_2 ; \frac{v_1}{\sqrt{\nu_1}} = \frac{v_2}{\sqrt{\nu_2}} = 0 ; \frac{\partial u_1}{\partial\left(\frac{y}{\sqrt{\nu_1}}\right)} = \frac{\rho_2}{\rho_1} \frac{\mu_2}{\mu_1} \frac{\partial u_2}{\partial\left(\frac{y}{\sqrt{\nu_2}}\right)}$

Análisis dimensional :

$$u_i = F_i\left(x, \frac{y}{\sqrt{\nu_i}}, U_1, U_2, \frac{\rho_2\mu_2}{\rho_1\mu_1}\right) \quad ; \quad \frac{v_i}{\sqrt{\nu_i}} = G_i\left(x, \frac{y}{\sqrt{\nu_i}}, U_1, U_2, \frac{\rho_2\mu_2}{\rho_1\mu_1}\right) \qquad (i = 1,2)$$

Ecuaciones de dimensiones :

$$\left.\begin{array}{l} [x] = L \\ \left[\dfrac{y}{\sqrt{\nu_i}}\right] = \dfrac{L}{\left(L^2T^{-1}\right)^{1/2}} = T^{1/2} \\ [u_i] = LT^{-1} \end{array}\right\} \longrightarrow \frac{u_i}{U_1} = H_i\left(y\sqrt{\frac{U_1}{\nu_i\, x}}, \frac{U_2}{U_1}, \frac{\rho_2\mu_2}{\rho_1\mu_1}\right)$$

Introducimos la función de corriente ψ_i :

$$\left.\begin{array}{l} u_i = \dfrac{\partial \psi_i}{\partial y} \\ v_i = -\dfrac{\partial \psi_i}{\partial x} \end{array}\right\} \longrightarrow \frac{\psi_i}{\sqrt{\nu_i}} \longrightarrow \left[\frac{\psi_i}{\sqrt{\nu_i}}\right] = \frac{L^2T^{-1}L}{LT^{-1/2}} = LT^{-1/2}$$

Adimensionalizamos $\dfrac{\psi_i}{\sqrt{\nu_i}}, \dfrac{y}{\sqrt{\nu_i}}$:

$$\frac{\dfrac{\psi_i}{\sqrt{\nu_i}}}{x^\alpha U_1^\beta \dfrac{\rho_2\mu_2}{\rho_1\mu_1}} \longrightarrow \frac{LT^{-1/2}}{L^\alpha L^\beta T^{-\beta}} \longrightarrow \left\{\begin{array}{l} \frac{1}{2} = \beta \\ \alpha + \beta = 1 \end{array}\right. \longrightarrow \left\{\begin{array}{l} \alpha = \frac{1}{2} \\ \beta = \frac{1}{2} \end{array}\right. \longrightarrow \frac{\psi_i}{\sqrt{\nu_i\, x\, U_1}}$$

$$\frac{\dfrac{y}{\sqrt{\nu_i}}}{x^\alpha U_1^\beta \dfrac{\rho_2\mu_2}{\rho_1\mu_1}} \longrightarrow \frac{T^{1/2}}{L^\alpha L^\beta T^{-\beta}} \longrightarrow \left\{\begin{array}{l} -\frac{1}{2} = \beta \\ \alpha + \beta = 0 \end{array}\right. \longrightarrow \left\{\begin{array}{l} \alpha = \frac{1}{2} \\ \beta = -\frac{1}{2} \end{array}\right. \longrightarrow y\sqrt{\frac{U_1}{\nu_i\, x}}$$

Por tanto :

$$\frac{\psi_i}{\sqrt{\nu_i\, x\, U_1}} = f_i(\eta_i) \ , \ \text{donde} \ \ \eta_i = y\sqrt{\frac{U_1}{\nu_i\, x}}$$

$$\eta_i = y\sqrt{\frac{U_1}{\nu_i\, x}} \longrightarrow \frac{\partial \eta_i}{\partial x} = -\frac{\eta_i}{2x} \quad ; \quad \frac{\partial \eta_i}{\partial x} = \sqrt{\frac{U_1}{\nu_i\, x}}$$

Entonces: $\Psi_i = \sqrt{\nu_i\, x\, U_1}\, f_i(\eta_i)$

Las ECdM$_x$ eran: $u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} = \nu_i \frac{\partial^2 u_i}{\partial y^2} \quad (i = 1,2)$

u_i

$$u_i = \frac{\partial \Psi_i}{\partial y} = \frac{\partial}{\partial y}\left[\sqrt{\nu_i\, x\, U_1}\, f_i(\eta_i)\right] = \sqrt{\nu_i\, x\, U_1}\, f_i'\sqrt{\frac{U_1}{\nu_i\, x}} \longrightarrow \boxed{u_i = U_1\, f_i'}$$

v_i

$$v_i = -\frac{\partial \Psi_i}{\partial x} = -\frac{\partial}{\partial x}\left[\sqrt{\nu_i\, x\, U_1}\, f_i(\eta_i)\right] = -\frac{1}{2}\sqrt{\frac{U_1}{\nu_i\, x}}\, f_i + \sqrt{\nu_i\, x\, U_1}\, f_i'\, \frac{\eta_i}{2x} \longrightarrow$$

$$\longrightarrow \boxed{v_i = -\frac{1}{2}\sqrt{\frac{U_1}{\nu_i\, x}}\left(f_i - \eta_i f_i'\right)}$$

$\frac{\partial u_i}{\partial x}$

$$\frac{\partial u_i}{\partial x} = \frac{\partial}{\partial x}\left(U_1 f_i'\right) = -U_1 f_i''\, \frac{\eta_i}{2x} \longrightarrow \boxed{\frac{\partial u_i}{\partial x} = -\frac{U_1}{2x}\eta_i f_i''}$$

$\frac{\partial u_i}{\partial y}$

$$\frac{\partial u_i}{\partial y} = \frac{\partial}{\partial y}\left(U_1 f_i'\right) = U_1 f_i''\sqrt{\frac{U_1}{\nu_i\, x}} \longrightarrow \boxed{\frac{\partial u_i}{\partial y} = U_1\sqrt{\frac{U_1}{\nu_i\, x}}\, f_i''}$$

$\frac{\partial^2 u_i}{\partial y^2}$

$$\frac{\partial^2 u_i}{\partial y^2} = \frac{\partial}{\partial y}\left(U_1\sqrt{\frac{U_1}{\nu_i\, x}}\, f_i''\right) = U_1\sqrt{\frac{U_1}{\nu_i\, x}}\, f_i'''\sqrt{\frac{U_1}{\nu_i\, x}} \longrightarrow \boxed{\frac{\partial^2 u_i}{\partial y^2} = \frac{U_1^2}{\nu_i\, x}\, f_i'''}$$

Sustituyendo:

$$-U_1 f_i' \frac{U_1}{[illegible]}\eta\, f'' - \frac{1}{[illegible]}\sqrt{\frac{\nu_i U_1}{[illegible]}}\left(f - \eta f'\right) U \sqrt{\frac{U_1}{[illegible]}}\, f'' = \nu \frac{U_1^2}{[illegible]}\, f''' \; [illegible]$$

$$\boxed{2 f_i''' + f_i f_i'' = 0}$$

Convirtiendo las c.c. el problema queda:

ECUACIONES DEL MOVIMIENTO

$$2f_1''' + f_1 f_1'' = 0$$

$$2f_2''' + f_2 f_2'' = 0$$

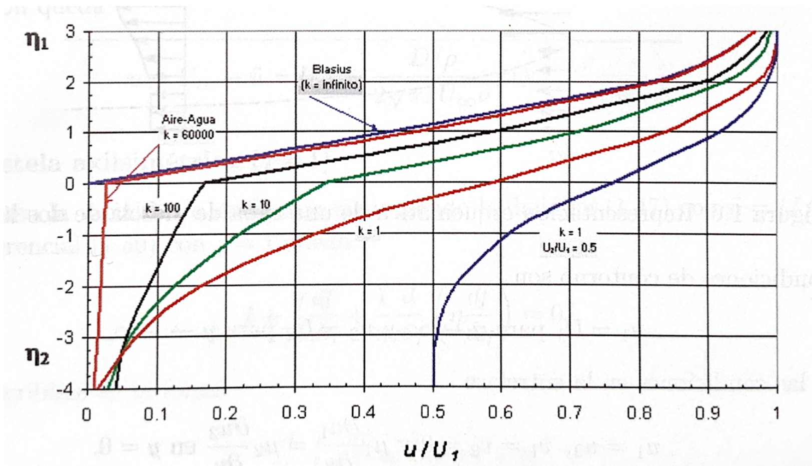
CONDICIONES DE CONTORNO

$$\eta_1 \to \infty : \; f_1' \to 1$$

$$\eta_1 \to -\infty : \; f_2' \to \frac{U_2}{U_1}$$

$$\eta_1 = \eta_2 = 0 : \; f_1 = f_2 = 0 \; ; \; f_1' = f_2' \; ; \; f_1'' = \frac{\rho_2 \mu_2}{\rho_1 \mu_1} f_2''$$

En la siguiente figura se muestran los resultados correspondientes a $\frac{\rho_2 \mu_2}{\rho_1 \mu_1} = 0$ (un solo líquido) y $\frac{U_2}{U_1} = 0$ y $\frac{U_2}{U_1} = 0'5$. También se muestran los resultados para $\frac{\rho_2 \mu_2}{\rho_1 \mu_1} = 5'97 \cdot 10^4$ y $\frac{U_2}{U_1} = 0$, correspondiente a una corriente de aire y agua, junto con los valores de $\frac{\rho_2 \mu_2}{\rho_1 \mu_1} = 10$ y $\frac{\rho_2 \mu_2}{\rho_1 \mu_1} = 100$.



Por último, en la siguiente figura se muestra la velocidad en la entrefase, $u(0)$, en función de $\sqrt{\frac{\rho_2 \mu_2}{\rho_1 \mu_1}}$ para $U_2 = 0$: