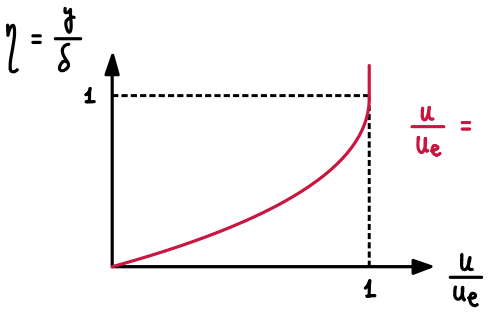
Métodos Aproximados de Capa Límite

Para $u_e(x)$ genérica queremos $\left\{ \tau_p,\ \delta_2(x),\ \text{Condiciones de separación} \right\} \longrightarrow$ MÉTODOS INTEGRALES (APROXIMADOS)

Estos métodos se basan en la ecuación de Karman : $\dfrac{d\delta_2}{dx} + (2 + H_{12}) \dfrac{1}{u_e} \dfrac{du_e}{dx} \delta_2 = \dfrac{C_f}{2} + \cancelto{0}{\dfrac{v_p}{u_e}}$ (Simplificación)

Necesitamos $H_{12}, C_f = f\left(u_e, \dfrac{du_e}{dx}, \nu, \delta_2 \ldots\right) \longrightarrow$ hacemos perfiles aproximados de la capa límite.



¡OJO! NO COINCIDE CON f DE BLASIUS Y FALKNER-SKAN

$$\frac{u}{u_e} = \begin{cases} f(\eta) & 0 < \eta < 1 \\ 1 & \eta > 1 \end{cases} \longrightarrow \text{Nos quitamos la asíntota} \star$$

Con $f(\eta)$ especificadada $\longrightarrow$ Karman $\longrightarrow$ $\delta_2(x)$

$y = \eta\delta\ ;\ \dfrac{u}{u_e} = f$ — Restringimos a una longitud finita ($\star$)

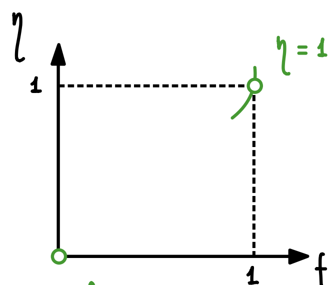
$$\delta_1 = \int_0^\infty \left(1 - \frac{u}{u_e}\right) dy = \delta \int_0^\infty (1-f)\, d\eta = \delta \int_0^1 (1-f)\, d\eta \longrightarrow \boxed{\delta_1 = \delta \int_0^1 (1-f)\, d\eta}$$

$$\delta_2 = \int_0^\infty \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dy = \delta \int_0^\infty f(1-f)\, d\eta \overset{\star}{=} \delta \int_0^1 f(1-f)\, d\eta \longrightarrow \boxed{\delta_2 = \delta \int_0^1 f(1-f)\, d\eta}$$

$$\boxed{H_{12} = \frac{\delta_1}{\delta_2}}$$

$$C_f = \frac{\tau_p}{\frac{1}{2}\rho u_e^2} \longrightarrow \frac{C_f}{2} = \frac{\mu \left.\frac{\partial u}{\partial y}\right|_{y=0}}{\rho u_e^2} = \frac{\nu}{u_e} \left.\frac{\partial\left(\frac{u}{u_e}\right)}{\partial y}\right|_{y=0} \xrightarrow{y=\eta\delta} \boxed{\frac{C_f}{2} = \frac{\nu}{u_e\delta} \left.\frac{df}{d\eta}\right|_{\eta=0}}$$

¿Qué propiedades tiene una $f(\eta)$ adecuada ?



$\eta = 1 : f = 1$

$f' = 0$

$f'' = 0$

$f''' = 0$

$\vdots$

Empalme lo más suave posible

$\eta = 0 : f = 0$

f_0' no se toca (es τ_p)

¿ f_0'', f_0''' ?

Para $f_0'' \longrightarrow$ ECdMx : $u \dfrac{\partial u}{\partial x} + v \dfrac{\partial u}{\partial y} = u_e \dfrac{du_e}{dx} + \nu \dfrac{\partial^2 u}{\partial y^2}$

En $y = 0$:

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = -\frac{u_e}{\nu}\frac{du_e}{dx} \longrightarrow \left.\frac{\partial^2 f}{\partial \eta^2}\right|_{\eta=0} = -\frac{\delta^2}{u_e}\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} =$$

$$= \underbrace{-\frac{\delta^2}{\nu}\frac{du_e}{dx}}_{\Lambda} \longrightarrow \boxed{f_0'' = -\frac{\delta^2}{\nu}\frac{du_e}{dx} = -\Lambda}$$

Para $f_0''' \longrightarrow \dfrac{\partial(\text{ECdM}_x)}{\partial y}$: $u\dfrac{\partial^2 u}{\partial x\,\partial y} + \cancel{\dfrac{\partial u}{\partial x}\dfrac{\partial u}{\partial y}} + v\dfrac{\partial^2 u}{\partial y^2} + \cancel{\dfrac{\partial v}{\partial y}\dfrac{\partial u}{\partial y}} = \nu\dfrac{\partial^3 u}{\partial y^3}$ (CONTINUIDAD)

$$u\frac{\partial^2 u}{\partial x\,\partial y} + v\frac{\partial^2 u}{\partial y^2} = \nu\frac{\partial^3 u}{\partial y^3}$$

En $y=0$: $u = v = 0 \longrightarrow \left.\dfrac{\partial^3 u}{\partial y^3}\right|_{y=0} = 0 \longrightarrow \left.\dfrac{d^3 f}{d\eta}\right|_{\eta=0} = 0 \longrightarrow \boxed{f_0''' = 0}$

Nos quedamos con :

	f	f'	f''	f'''
$\eta=0$	0		$-\Lambda$	0
$\eta=1$	1	0	0	0

... $\longrightarrow$ f SERÁ UN POLINOMIO DE GRADO 4