Electrical Systems

Lecture 6: General methods for the analysis of electric circuits

Arnau Dòria-Cerezo

arnau.doria@upc.edu

Dept. Electrical Engineering, and
Inst. of Industrial and Control Engineering

Last revised: April 6, 2021

Outline

Branch Current Method

Goal

The goal of this method is to determine the currents through each essential branch of the circuit.

Steps

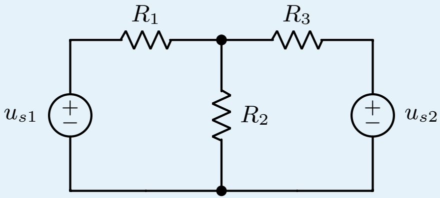
1. Identify and label all the (essential) nodes, (essential branches) and meshes of the circuit.
2. Assign directions of currents of each essential branch and voltage polarities of each element.
3. Write the KCL for each node of the circuit.
4. Write the KVL for each mesh of the circuit, using the element equations.
5. Solve the obtained system equation.

   Note: the number of equations must be equal to the number of essential branches.

Branch Current Method

Exercise 1

Write the set of equations of applying the Branch Current Method that solves the circuit below.



Outline

The Mesh Current Method

The goal of this method is to determine the unknown currents in a circuit.

This method...

- is also known as the Loop Current Method,
- it does not use Kirchhoff's Current Law,
- it is usually able to solve a circuit with less unknown variables and less simultaneous equations.
- Assumption: the circuit is planar.

Steps

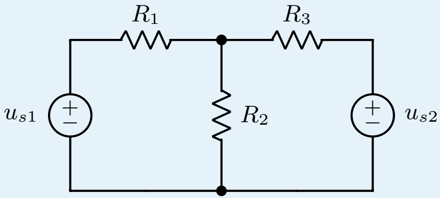
1. Identify and label all meshes (or loops) within the circuit encompassing all components.
2. Assign a mesh current for each mesh of the circuit (usually clockwise).
3. Write the KVL for each mesh of the circuit, using the element equations.
4. Solve the obtained system equation.

   Note: the number of equations must be equal to the number of meshes.

The Mesh Current Method

Exercise 2

Write the set of equations of applying the Mesh Current Method that solves the circuit below.



MCM in a matrix form

$$\underbrace{\begin{pmatrix} R_{AA} & -R_{AB} & \dots & -R_{A,N-1} & -R_{AN} \\ -R_{BA} & \ddots & \vdots & \iddots & -R_{BN} \\ \dots & \vdots & R_{KK} & \dots & \vdots \\ -R_{N-1,A} & \iddots & \vdots & \ddots & -R_{N-1,N} \\ -R_{NA} & -R_{NB} & \dots & -R_{N,N-1} & R_{NN} \end{pmatrix}}_{\boldsymbol{R}} \underbrace{\begin{pmatrix} i_A \\ \vdots \\ i_K \\ \vdots \\ i_N \end{pmatrix}}_{\boldsymbol{I}} = \underbrace{\begin{pmatrix} u_{sA} \\ \vdots \\ u_{sK} \\ \vdots \\ u_{sN} \end{pmatrix}}_{\boldsymbol{V}}$$

where

- u_K: Sum of the voltage sources across the K mesh.
- R_{KK}: Sum of the resistances in the K mesh.
- R_{KL}: Sum of the resistances of the common resistors in the K and L meshes.

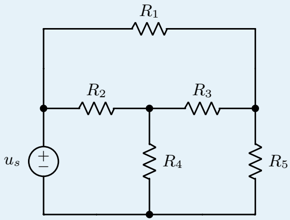
For a circuit without controlled sources...

- The resistance matrix is symmetric.
- The diagonal terms of the resistance matrix are positive.
- The non-diagonal terms of the resistance matrix are negative.

MCM in a matrix form

Exercise 3

Write, in a matrix form, the set of equations of applying the Mesh Current Method that solves the circuit below.



with $R_1 = 4\Omega$, $R_2 = 2\Omega$, $R_3 = 6\Omega$, $R_4 = 2\Omega$, $R_5 = 8\Omega$, *and* $u_s = 10V$.

MCM in a matrix form for AC circuits

$$\underbrace{\begin{pmatrix} \underline{Z}_{AA} & -\underline{Z}_{AB} & \cdots & -\underline{Z}_{A,N-1} & -\underline{Z}_{AN} \\ -\underline{Z}_{BA} & \ddots & \vdots & \iddots & -\underline{Z}_{BN} \\ \cdots & \vdots & \underline{Z}_{KK} & \cdots & \vdots \\ -\underline{Z}_{N-1,A} & \iddots & \vdots & \ddots & -\underline{Z}_{N-1,N} \\ -\underline{Z}_{NA} & -\underline{Z}_{NB} & \cdots & -\underline{Z}_{N,N-1} & \underline{Z}_{NN} \end{pmatrix}}_{\boldsymbol{Z}} \underbrace{\begin{pmatrix} \underline{I}_A \\ \vdots \\ \underline{I}_K \\ \vdots \\ \underline{I}_N \end{pmatrix}}_{\boldsymbol{I}} = \underbrace{\begin{pmatrix} \underline{U}_A \\ \vdots \\ \underline{U}_K \\ \vdots \\ \underline{U}_N \end{pmatrix}}_{\boldsymbol{U}}$$

where

- $\underline{U}_K$: Sum of the voltage sources across the K mesh.
- $\underline{Z}_{KK}$: Sum of the impedances in the K mesh.
- $\underline{Z}_{KL}$: Sum of the impedances of the common resistors in the K and L meshes.

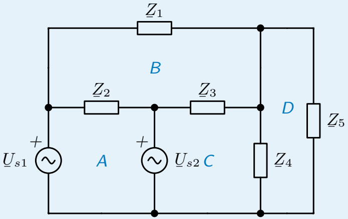
The impedance matrix, Z,...

- Is symmetric.
- The diagonal terms are positive.
- The non-diagonal terms are negative.

MCM in a matrix form for AC circuits

Exercise 4

Determine the mesh currents of the circuit below by using the general form of the Mesh Current Method.



with $\underline{Z}_1 = 4 + j\Omega$, $\underline{Z}_2 = -j2\Omega$, $\underline{Z}_3 = 1 + j6\Omega$, $\underline{Z}_4 = j2\Omega$, $\underline{Z}_5 = 8\Omega$, $\underline{U}_{s1} = 10V$, *and* $\underline{U}_{s2} = 10\angle 15^\circ V$.

MCM in a matrix form for AC circuits

Solution: Matlab code

```
% Example MCM
clear all; close all; clc

% Parameters
Z1=4+1i;
Z2=-1i*2;
Z3=1+1i*6;
Z4=1i*2;
Z5=8;
U1=10*exp(1i*0);
U2=10*exp(1i*deg2rad(15));

% Impedance matrix
Z=[Z2 -Z2 0 0;
    -Z2 Z1+Z2+Z3 -Z3 0;
    0 -Z3 Z3+Z4 -Z4;
    0 0 -Z4 Z4+Z5];

% Voltage vector
U=[U1-U2;0;U2;0];

% Currents
I=Z\U;

disp(['I1=' num2str(abs(I(1))) ',' num2str(rad2deg(angle(I(1)))) 'V'])
disp(['I2=' num2str(abs(I(2))) ',' num2str(rad2deg(angle(I(2)))) 'V'])
disp(['I3=' num2str(abs(I(3))) ',' num2str(rad2deg(angle(I(3)))) 'V'])
disp(['I4=' num2str(abs(I(4))) ',' num2str(rad2deg(angle(I(4)))) 'V'])
```

Outline

The Node Voltage Method

Goal

To determine the voltage nodes by using the KCL.

This method...

- it does not use Kirchhoff's Voltage Law,
- use the conductance definition of the resistors.
- Assumption: the circuit is planar.

Steps

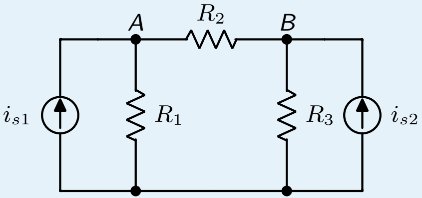
1. Identify and label all the essential nodes of the circuit.
2. Assign a node as a reference node (zero potential).
3. Assign a potential label to the other nodes of the circuit.
4. Write the KCL for each labeled node (the KCL at the reference node is not necessary), using the element equations.
5. Solve the obtained system equation.

   Note: the number of equations must be equal to the number of essential nodes minus one.

The Node Voltage Method

Exercise 5

Write the set of equations of applying the Node Voltage Method that solves the circuit below.



NVM in a matrix form

$$\underbrace{\begin{pmatrix} G_{1,1} & -G_{1,2} & \dots & -G_{1,N-1} & -G_{1,N} \\ -G_{1,2} & \ddots & \vdots & \iddots & -G_{2,N} \\ \dots & \vdots & G_{KK} & \dots & \vdots \\ -G_{N-1,1} & \iddots & \vdots & \ddots & -G_{N-1,N} \\ -G_{1,N} & -G_{2,N} & \dots & -G_{N-1,N} & G_{NN} \end{pmatrix}}_{\boldsymbol{G}} \underbrace{\begin{pmatrix} v_1 \\ \vdots \\ v_K \\ \vdots \\ v_N \end{pmatrix}}_{\boldsymbol{V}} = \underbrace{\begin{pmatrix} i_{s1} \\ \vdots \\ i_{sK} \\ \vdots \\ i_{sN} \end{pmatrix}}_{\boldsymbol{J}}$$

where

- i_{sK}: Sum of the currents sources connected to Node K.
- G_{KK}: Sum of the conductances of the resistors connected to Node K.
- G_{KL}: Sum of the conductances of the resistors between the nodes K and L.

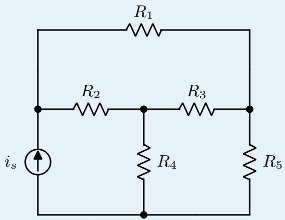
The conductance matrix, G,...

- Is symmetric.
- The diagonal terms are positive.
- The non-diagonal terms are negative.

NVM in a matrix form

Exercise 6

Write, in a matrix form, the set of equations of applying the Node Voltage Method that solves the circuit below, with the values $R_1 = 4\Omega$, $R_2 = 2\Omega$, $R_3 = 6\Omega$, $R_4 = 2\Omega$, $R_5 = 8\Omega$, *and* $i_s = 10A$.



NVM in a matrix form for AC circuits

$$\underbrace{\begin{pmatrix} \underline{Y}_{1,1} & -\underline{Y}_{1,2} & \cdots & -\underline{Y}_{1,N-1} & -\underline{Y}_{1,N} \\ -\underline{Y}_{1,2} & \ddots & \vdots & \iddots & -\underline{Y}_{2,N} \\ \cdots & \vdots & \underline{Y}_{KK} & \cdots & \vdots \\ -\underline{Y}_{1,N-1} & \iddots & \vdots & \ddots & -\underline{Y}_{N-1,N} \\ -\underline{Y}_{1,N} & -\underline{Y}_{2,N} & \cdots & -\underline{Y}_{N-1,N} & \underline{Y}_{NN} \end{pmatrix}}_{\boldsymbol{Y}} \underbrace{\begin{pmatrix} \underline{V}_1 \\ \vdots \\ \underline{V}_K \\ \vdots \\ \underline{V}_N \end{pmatrix}}_{\boldsymbol{V}} = \underbrace{\begin{pmatrix} \underline{I}_{s1} \\ \vdots \\ \underline{I}_{sK} \\ \vdots \\ \underline{I}_{sN} \end{pmatrix}}_{\boldsymbol{J}}$$

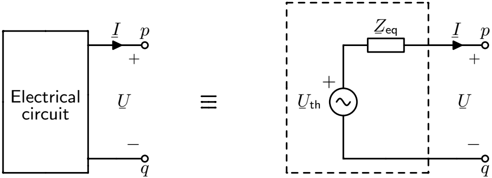
where

- $\underline{I}_{sK}$: Sum of the currents sources connected to Node K.
- $\underline{Y}_{KK}$: Sum of the admittances of the impedances connected to Node K.
- $\underline{Y}_{KL}$: Sum of the admittances of the impedances between the nodes K and L.

The admittance matrix, $\boldsymbol{Y}$,...

- Is symmetric.
- The diagonal terms are positive.
- The non-diagonal terms are negative.

NVM in a matrix form for AC circuits



Thévenin equivalent circuit from the NVM

From the NVM in the general matrix form

$$\boldsymbol{Y}\boldsymbol{V} = \boldsymbol{J},$$

the equivalent Thévenin circuit between two any terminals p and q is given by

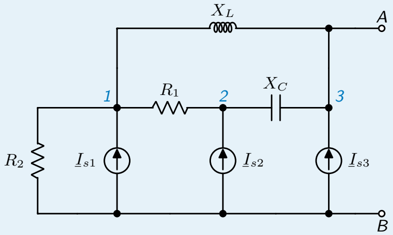
$$\underline{U}_{\mathsf{th}} = \underline{V}_p - \underline{V}_q$$
$$\underline{Z}_{\mathsf{th}} = \underline{Z}_{pp} + \underline{Z}_{qq} - \underline{Z}_{pq} - \underline{Z}_{qp}$$

where $\boldsymbol{Z} = \boldsymbol{Y}^{-1}$.

NVM in a matrix form for AC circuits

Exercise 7

Determine the node voltages of the circuit below using the general form of the Node Voltage Method, and find the equivalent Thévenin circuit between nodes A and B.



$$R_1 = 1\Omega$$
$$R_2 = 2\Omega$$
$$X_C = 2\Omega$$
$$X_L = 3\Omega$$
$$\underline{I}_{s1} = 1A$$
$$\underline{I}_{s2} = 1\angle 15^\circ A$$
$$\underline{I}_{s3} = 1\angle -15^\circ A$$

NVM in a matrix form for AC circuits

Solution: Matlab code

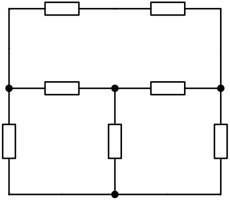
```
% Example NVM
clear all;close all;clc

% Parameters
R1=1;R2=2;
XC=2;XL=3;
I1=1*exp(1i*deg2rad(0));
I2=1*exp(1i*deg2rad(15));
I3=1*exp(1i*deg2rad(-15));

% Admittance matrix
Y=[1/R1+1/R2+1/(1i*XL) -1/R1 -1/(1i*XL);
    -1/R1 1/R1+1/(-1i*XC) -1/(-1i*XC);
    -1/(1i*XL) -1/(-1i*XC) 1/(1i*XL)+1/(-1i*XC)];

% Currents vector
J=[I1;I2;I3];

% Impedance matrix
Z=inv(Y);

% Voltages vector
V=Z*J;
disp(['V1=' num2str(abs(V(1))) ',' num2str(rad2deg(angle(V(1)))) 'V'])
disp(['V2=' num2str(abs(V(2))) ',' num2str(rad2deg(angle(V(2)))) 'V'])
disp(['V3=' num2str(abs(V(3))) ',' num2str(rad2deg(angle(V(3)))) 'V'])

% Thevenin equivalent circuit
disp(['Uth=' num2str(abs(V(3))) 'V'])
Zeq=Z(3,3);
disp(['Zeq=' num2str(Zeq) ' Ohm'])
```

Outline

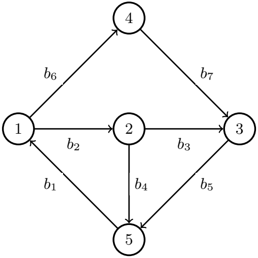
1. Branch Current Method
2. Mesh Current Method
3. Node Voltage Method
4. **Extended Node Voltage Method**
   - From circuits to graphs
   - The Extended Node Voltage Method
   - ENVM in a matrix form
5. Exercises and solutions

From circuits to graphs

The Kirchhoff laws can represented by a (directed) graph that retains all the information of the interconnection properties of the circuit (suppressing the information on the circuit elements).



Circuit example



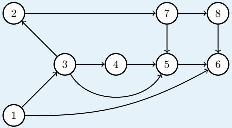
Equivalent digraph

From circuits to graphs

Brief notion on Graph Theory

A graph G is a pair $(\mathcal{N}, \mathcal{B})$ that consists of a set of nodes $\mathcal{N} = 1, 2 \ldots, n$ and m branches $\mathcal{B} \subseteq \{(i,j) : i,j \in \mathcal{N}\}$.

- The branch $(i,j) \in \mathcal{B}$ denotes that i (the *parent* node) is connected to j (the *child* node), and is indicated by an arrow from i to j.
- In a circuit graph
  - all weights are 1 for all $(i,j) \in \mathcal{B}$,
  - the graph is *strongly connected* (there is a directed path between every node to every node),
  - there is no self-loops (a branch starting and ending at the same node)



Example of a directed graph (digraph)

From circuits to graphs

Incidence matrix

Consider an arbitrary orientation of the branches. The (node-branch) *incidence matrix*, $\mathbf{A} \in \mathbb{R}^{n \times m}$, is defined by the (k,l)-th elements as

$$a_{kl} = \begin{cases} 1 & \text{if branch } l \text{ leaves node } k \\ -1 & \text{if branch } l \text{ enters node } k \\ 0 & \text{otherwise.} \end{cases}$$

Fact: The column sum of $\mathbf{A}$ is zero.

Reduced incidence matrix

As $\mathbf{A}$ contains exactly two nonzero elements (one 1, and one -1), we can delete any row of $\boldsymbol{A}$ without losing information. The matrix $\boldsymbol{A}_r$ obtained from deleting any row of $\boldsymbol{A}$ is called *the reduced matrix* of $\boldsymbol{A}$.

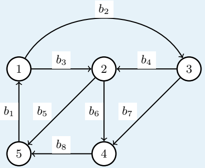
Incidence columns

Incidence columns of the reduced matrix $\mathbf{A}_r$ are the columns corresponding to a certain branches.

From circuits to graphs

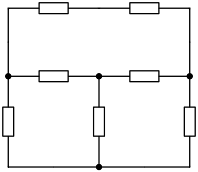
Exercise 8

Write the incidence matrix of the following circuit:



From circuits to graphs

Write the both KVL and KCL and the incidence matrix corresponding to the circuit



and check that

Kirchhoff laws from the incidence matrix

$$\mathbf{A}i = 0 \quad \Leftrightarrow \quad \text{(KCL)}$$
$$u = \mathbf{A}^T v \quad \Leftrightarrow \quad \text{(KVL)}$$

where $u \in \mathbb{R}^m$ is the branch voltage vector and $v \in \mathbb{R}^n$ is the node potential vector.

Any row of the incidence matrix contains redundant information and one can use a *reduced* Incidence matrix just removing one row.

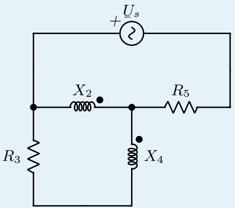
The Extended Node Voltage Method

The Extended Node Voltage Method

The aim of the Extended Node Voltage Method is to include ideal voltage sources and magnetic coupling in the general description using the matrix form.

Exercise 9

Analyse the circuit below applying the Extended Node Voltage Method.



ENVM in a matrix form

The ENVM results in the following matrix equation

$$\underbrace{\begin{pmatrix} \boldsymbol{Y} & \boldsymbol{A}_m & \boldsymbol{A}_s \\ \boldsymbol{A}_m^T & -\boldsymbol{Z}_m & \boldsymbol{0} \\ \boldsymbol{A}_s^T & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}}_{\boldsymbol{H}} \underbrace{\begin{pmatrix} \boldsymbol{V} \\ \boldsymbol{I}_m \\ \boldsymbol{I}_s \end{pmatrix}}_{\boldsymbol{X}} = \underbrace{\begin{pmatrix} \boldsymbol{J} \\ \boldsymbol{0} \\ \boldsymbol{U}_s \end{pmatrix}}_{\boldsymbol{W}}$$

where $\boldsymbol{Y}$, $\boldsymbol{V}$ and $\boldsymbol{J}$ are the admittance matrix, the potentials vector and the vector containing the current sources, respectively, and

- $\boldsymbol{I}_m$, vector containing the currents in the magnetic coupling,
- $\boldsymbol{Z}_m$, magnetic coupling matrix. In the case of coupling of two inductances,

$$\boldsymbol{Z}_m = \begin{pmatrix} \underline{Z}_1 + jX_1 & jX_m \\ jX_m & \underline{Z}_2 + jX_2 \end{pmatrix}$$

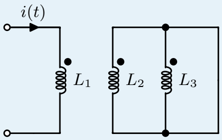
- $\boldsymbol{A}_m = (\boldsymbol{A}_1 \quad \boldsymbol{A}_2)$, incidence columns of branches where magnetic coupling are connected,
- $\boldsymbol{I}_s$, vector containing the currents of the voltage sources,
- $\boldsymbol{U}_s$, vector containing the values of the voltage sources,
- $\boldsymbol{A}_s$, incidence columns of branches where voltage sources are connected,

Outline

Exercises I

Exercise 10

Find the equivalent impedance of the circuit below

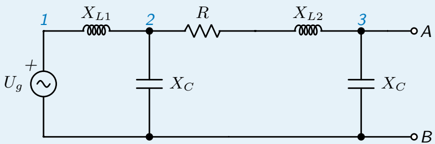


where the coupling matrix is defined by $\boldsymbol{Z}_m = j \begin{pmatrix} 20 & 5 & 8 \\ 5 & 13 & 2 \\ 8 & 2 & 20 \end{pmatrix} \Omega.$

Exercises II

Exercise 11

MN1: *Find the Extended Node Voltage Method to calculate the voltages in the circuit below, and find the Thévenin equivalent circuit between terminals A and B.*

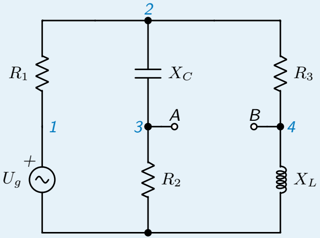


$$U_g = 15kV$$
$$X_{L1} = 2.5\Omega$$
$$X_{L2} = 3\Omega$$
$$X_C = 400\Omega$$
$$R = 1\Omega$$

Exercises III

Exercise 12

MN2: *Find the Extended Node Voltage Method to calculate the voltages in the circuit below, and find the Thévenin equivalent circuit between terminals A and B.*

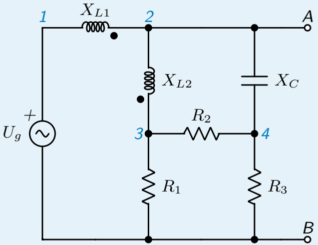


$$U_g = 100V$$
$$R_1 = 50\Omega$$
$$R_2 = 10\Omega$$
$$R_3 = 35\Omega$$
$$X_C = 15\Omega$$
$$X_L = 20\Omega$$

Exercises IV

Exercise 13

MN3: *Find the Extended Node Voltage Method to calculate the voltages in the circuit below, and find the Thévenin equivalent circuit between terminals A and B.*



$$U_g = 100V$$
$$R_1 = 20\Omega$$
$$R_2 = 50\Omega$$
$$R_3 = 25\Omega$$
$$X_C = 25\Omega$$
$$X_{L1} = 10\Omega$$
$$X_{L2} = 40\Omega$$
$$X_m = 15\Omega$$

Solutions I

Solution to Exercise 1

$$\begin{aligned} -i_1 + i_2 + i_3 &= 0 \\ u_{s1} - R_1 i_1 - R_2 i_2 &= 0 \\ R_2 i_2 - R_3 i_3 - u_{s2} &= 0 \end{aligned}$$

Solution to Exercise 2

$$\begin{aligned} u_{s1} - R_1 i_A - R_2(i_A - i_B) &= 0 \\ -R_2(i_B - i_A) - R_3 i_B - u_{s2} &= 0 \end{aligned}$$

Solution to Exercise 3

$$\begin{pmatrix} 12 & -2 & -6 \\ -2 & 4 & -2 \\ -6 & -2 & 16 \end{pmatrix} \begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix}$$

Solutions II

Solution to Exercise 4

$\underline{I}_A = 2.67\angle -14.47°\,\text{A}$, $\underline{I}_B = 1.54\angle -32.98°\,\text{A}$, $\underline{I}_C = 2.32\angle -48.70°\,\text{A}$,
$\underline{I}_D = 0.56\angle 27.26°\,\text{A}$

Solution to Exercise 5

$$
\begin{aligned}
i_{s1} - \frac{1}{R_1}v_A + \frac{1}{R_2}(v_B - v_A) &= 0\\
-\frac{1}{R_3}v_B + \frac{1}{R_3}(v_A - v_B) + i_{s2} &= 0
\end{aligned}
$$

Solution to Exercise 6

$$
\begin{pmatrix} \frac{3}{4} & -\frac{1}{2} & -\frac{1}{4}\\ -\frac{1}{2} & \frac{7}{6} & -\frac{1}{6}\\ -\frac{1}{4} & -\frac{1}{6} & \frac{13}{24} \end{pmatrix} \begin{pmatrix} v_A\\ v_B\\ v_C \end{pmatrix} = \begin{pmatrix} 10\\ 0\\ 0 \end{pmatrix}
$$

Solutions III

Solution to Exercise 7

$$\begin{pmatrix} 1.5-j0.33 & -1 & j0.33 \\ -1 & 1+j0.5 & -j0.5 \\ j0.33 & -j0.5 & j0.17 \end{pmatrix} \begin{pmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1\angle 15^\circ \\ 1\angle -15^\circ \end{pmatrix}$$

- $\underline{V}_1 = 5.86\angle 0^\circ \mathrm{V}$, $\underline{V}_2 = 8.23\angle 11.73^\circ \mathrm{V}$, $\underline{V}_3 = 10.91\angle -4.08^\circ \mathrm{V}$
- $U_{\mathrm{th}} = 10.91\mathrm{V}, \quad \underline{Z}_{\mathrm{th}} = 6.5 - j1.5\Omega$

Solution to Exercise 8

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

Solutions IV

Solution to Exercise 9

$$\begin{pmatrix} G_2 & 0 & 0 & -1 & 0 & 1 \\ 0 & G_5 & -G_5 & 1 & 1 & 0 \\ 0 & -G_5 & G_5 & 0 & 0 & -1 \\ -1 & 1 & 0 & -jX_2 & jX_m & 0 \\ 0 & 1 & 0 & -jX_m & jX_4 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_3 \\ \underline{I}_2 \\ \underline{I}_4 \\ \underline{I}_s \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \underline{U}_s \end{pmatrix}$$

Solution to Exercise 10

$$X_{\text{eq}} = 15.42\Omega$$

Solution to Exercise 11

- $V_1 = 15\angle 0°\text{kV}$, $V_2 = 15.19\angle -0.001°\text{kV}$, $V_3 = 15.305\angle -0.145°\text{kV}$
- $U_{\text{th}} = 15.305\text{kV}$, $\underline{Z}_{\text{th}} = 1.03 + j5.59\Omega$

Solutions V

Solution to Exercise 12

- $V_1 = 100\angle 0°$V, $V_2 = 25.05\angle -25.08°$V, $V_3 = 13.89\angle 31.22°$V, $V_4 = 12.43\angle 35.17°$V
- $U_{th} = 1.72$V, $\underline{Z}_{th} = 15.59 + j10.49\Omega$

Solution to Exercise 13

- $V_1 = 100\angle 0°$V, $V_2 = 74.86\angle -6.77°$V, $V_3 = 18.73\angle -77.43°$V, $V_4 = 36.66\angle 46.85°$V
- $U_{th} = 74.85$V, $\underline{Z}_{th} = 1.569 + j2.623\Omega$

Electrical Systems

Lecture 6: General methods for the analysis of electric circuits

Arnau Dòria-Cerezo
arnau.doria@upc.edu

Dept. Electrical Engineering, and
Inst. of Industrial and Control Engineering

Last revised: April 6, 2021