Linear momentum

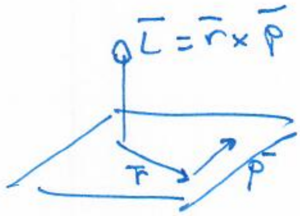
$$\bar{p} \equiv m\bar{v}$$

$$\bar{F} = m\bar{a} = m\frac{d\bar{v}}{dt} = \frac{d(m\bar{v})}{dt} = \frac{d\bar{p}}{dt}$$

Angular momentum

= torque of $\bar{p}$ ≡ moment of $\bar{p}$

$$\bar{L} \equiv \bar{r} \times \bar{p}$$



$$\frac{d\bar{L}}{dt} = \left(\bar{r} \times \frac{d\bar{p}}{dt}\right) + \underbrace{\left(\frac{d\bar{r}}{dt} \times \bar{p}\right)}_{=0}$$

$$\frac{d\bar{L}}{dt} = \bar{r} \times m\bar{a}$$

$=0$ because $\frac{d\bar{r}}{dt} = \bar{v}$ ⇒ parallel

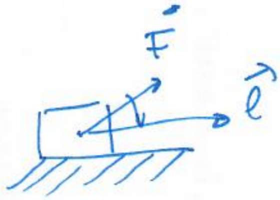
$$\frac{d\bar{L}}{dt} = \bar{r} \times \vec{F} \equiv \vec{M} \equiv \vec{\tau}$$ torque of a force

$$\boxed{\bar{\tau} = \frac{d\bar{L}}{dt}}$$

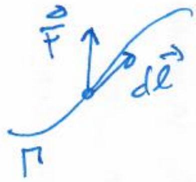
ENERGY AND WORK

Work

$dW = \vec{F} \cdot d\vec{l}$



$W = \vec{F} \cdot \vec{l}$



1 Jule = 1 · N m

1 eV = $1.6 \cdot 10^{-19}$ Jules

Additive principle $W_T = W_1 + W_2$

$W_T = \int_\Gamma \bar{F} \cdot \bar{dl}$ ≡ "work done by the vector field $\bar{F}$"

Energy

Capacity of bodies to perform a work.-

≡ 'something' that is observed is conserved →

it is 'stored somehow' and allows to do work.-

$$\Delta W = \int dW = \int F dl = \int m \frac{dv}{dt} dl = \int m v \frac{dv}{dl} dl =$$

$$\frac{dv}{dt} = \frac{dv}{dl} \cdot \frac{dl}{dt} = v \cdot \frac{dv}{dl}$$

v_f

energy.

theorem work-energy $\Delta W = \Delta T$

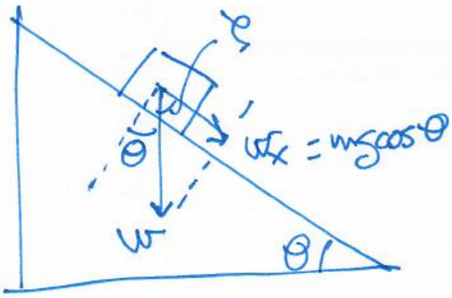
the work done on a particle is equal to the increase in kinetic energy.

Power

$P = \frac{dW}{dt}$ $\quad 1\,\text{watt} = 1\,\frac{\text{Jule}}{\text{sec}}$.

$P = \bar{F} \cdot \frac{d\bar{l}}{dt} = \underbrace{\bar{F}}_{m\bar{a}} \cdot \underbrace{\bar{v}}_{\bar{v}}$ "how fast the energy is released"

Example: work done by weight



$W = \text{work} = \overline{W} \cdot \bar{l}$ (weight)

$W = mg\, l \cos\theta$

$= mg\cos\theta \cdot l$

$= mg\cos(90-\xi) \cdot l$

$= mg\sin\xi \cdot l$

$= mgh$

$\boxed{mgh = \frac{1}{2}mv^2}$

a) $v = \sqrt{2gh}$ does not depend on mass

b) Work does not depend on $\theta \Rightarrow$

$\Rightarrow$ does not depend on trajectory

$\Rightarrow$ conservation of energy ...

Conservative Forces

Mathematical definitions for conservative field

- $\int_A^B \bar{F} d\bar{l}$ does not depend on trajectory
- $\oint \bar{F} d\bar{l} = 0$
- $\bar{\nabla} \times \bar{F} = 0$
- $\exists\, u_p$ such as $\vec{F} = -\vec{\nabla} u_p$

Potential energy

u_p escalar función.

u_p + constant returns the same $\vec{F}$

$$\Delta u_{A \to B} = u_B - u_A = -\int_A^B \bar{F} d\bar{l} \equiv \text{'Potential energy'}$$

$\Delta u_{A \to B}$ = work done by external agent for moving a mass 'm' from $A \to B$

the unit mass from $A \to B$.

$$\bar{F} = -\vec{\nabla} U_p$$

$$\bar{E}_g = \bar{g} \equiv \frac{\bar{F}}{m} = -\vec{\nabla} V \qquad V = \frac{U_p}{m}$$

$$E_g = -\frac{dV}{dr}$$

$$dV = -g\,dr = -g\,dr = -E_g\,dr$$

$$\Delta V_{A\to B} = \int_A^B dV = -\int_A^B g\,dr$$

$$g = -\frac{GM}{r^2}$$

$$\Delta V_{A\to B} = V_B - V_A = -\int_{r_A}^{r_B}\left(-\frac{GM}{r^2}\right)dr$$

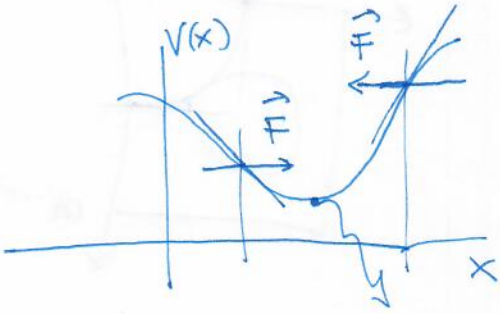
$$= \int_{r_A}^{r_B} \frac{GM}{r^2}\,dr = GM\left[-\frac{1}{r}\right]_{r_A}^{r_B}$$

$$\Delta V_{A\to B} = -\frac{GM}{r_B} + \frac{GM}{r_A}$$

$$V_B = V_A + \frac{GM}{r_A} - \frac{GM}{r_B}$$

$$\bar{F} = -\bar{\nabla} U_p \qquad \bar{F} = -\frac{dU_p}{dx}$$

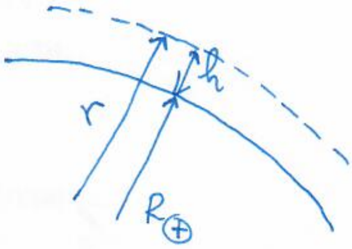
$$\bar{g} = -\vec{\nabla} V \qquad \bar{g} = -\frac{dV}{dx}$$



equilibrium point

Close to Earth, $\bar{g} \simeq$ constant

$$\bar{g} = \bar{E}_g = -\frac{GM}{r^2}$$

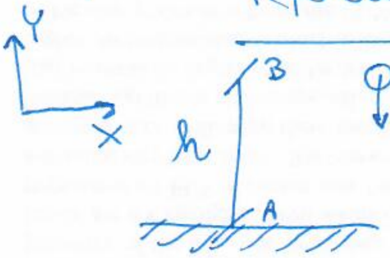


$$= -\frac{GM}{(r+R_\oplus)^2}$$

$$= -\frac{GM}{\left(1+\frac{r}{R_\oplus}\right)^2 R_\oplus^2}$$

$$= -\frac{GM}{R_\oplus^2}\left(\frac{1}{\left(1+\frac{h}{R_\oplus}\right)^2}\right)$$

Potential energy with respect given reference height



$$\Delta U_{A \to B} = U_B - U_A = -\int_{r_A}^{r_B} F\, dr$$

$$\Delta U_{A \to B} = -\int_{h_A}^{h_B} (-mg)\, dr$$

$$U_{h_B} - U_{h_A} = mgh_B - mgh_A$$

$$\boxed{U_{h_B} = mgh_B}$$ when $U_{h_A} = 0$ if $h_A = 0$

Poten. Energy $\Rightarrow$ external agent acting against $\vec{g}$

$\Rightarrow$ energy will be stored

$\Rightarrow$ can be released later.-

¿The field stores energy?

Potential energy for elastic bodies

$$F = -Kx$$

Not all forces are conservative.
check hooke's law forces are conservative.

$$\bar{\nabla} \times \bar{F} \overset{?}{=} 0$$

if so ... (they are)..

$\exists$ u_p such as $\bar{F} = -\bar{\nabla} u_p$

$$\Delta u_{A \to B} = -\int_A^B F\,dx =$$

$$= -\int_{x_A}^{x_B} (-Kx)\,dx = K \frac{x^2}{2} \Bigg]_{x_A}^{x_B}$$

$$u_B - u_A = K\frac{x_B^2}{2} - K\frac{x_A^2}{2}$$

Taking $x_A = 0 \Rightarrow u_A = 0$

$u_B = K\frac{x_B^2}{2}$ in general $\boxed{u_B = K\frac{x^2}{2}}$

the external agent acts against spring

Energy - work in conservative fields

In general, theorem energy work $\Delta W = \Delta T$
In conservative fields, work done is $-\Delta U_p$

Hence $W = -\Delta U_p$
$W = \Delta T$

So ... $\Delta T = -\Delta U_p$

$$T_f - T_0 = -(U_{pf} - U_{p0})$$

$$\underbrace{T_f + U_{pf}}_{\equiv \text{Mechanical energy} : E_M} = T_0 + U_{p0}$$

$$E_M \equiv T + U$$

$\boxed{\Delta U_M = 0 \text{ in conservative fields}}$