

OSCILLATIONS AND WAVES

- They are produced when a system is perturbed and moved from a stable equilibrium point,
- They result from a balance of elasticity and inertia.

$$F = -Kx$$

$$-Kx = m \ddot{x}$$

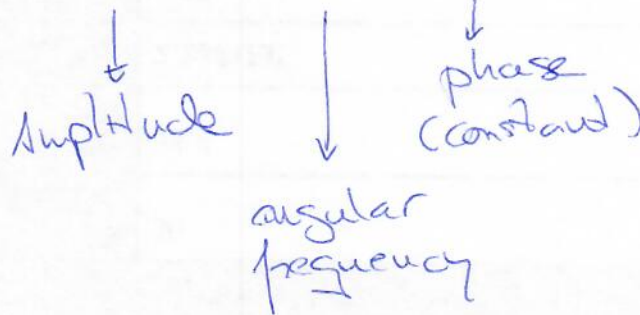
ODE $\left| \ddot{x} = -\frac{k}{m}x \right|$

simple harmonic oscillator

$$\ddot{x} = -\omega^2 x$$

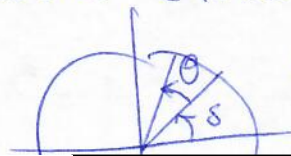
$\omega^2 = \frac{k}{m}$ — elasticity
— inertia

Solution: $x(t) = A \cos(\omega t + \phi)$



$x(t)$ is periodic in time $x(t) = x(t+T)$
 $T \equiv$ period.

Analogy with circular motion, particle rotating with ω .



$$\theta = \omega t$$

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A & S are determined from initial conditions

$$\begin{cases} x_0(t_0) = A \cos S \\ v_0(t) = -A\omega \sin(\omega t_0 + S) \end{cases}$$

Energy of harmonic oscillator

Potential

$$u = \frac{1}{2} K x^2$$

$$\boxed{u = \frac{1}{2} K A^2 \cos^2(\omega t + S)}$$

Kinetic

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{x}^2$$

$$\dot{x} = -A\omega \sin(\omega t + S)$$

$$\boxed{T = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + S)}$$

$$E_{\text{TOTAL}} = T + u$$

$$E_T = \frac{1}{2} K A^2 \cos^2(\omega t + S) + \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + S)$$

$$E_T = \frac{1}{2} K A^2 \cos^2(\quad) + \frac{1}{2} m A^2 \frac{K}{m} \sin^2(\quad)$$

$$\boxed{E_T = \frac{1}{2} K A^2}$$

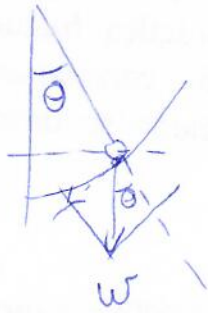
↑
u

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$$u = E_T$$

The pendulum



$$-mg \sin \theta = m \frac{d^2 s}{dt^2} \quad s \equiv \text{arc}$$

$$\theta = \frac{s}{l}, \quad s = \theta \cdot l$$
$$\ddot{s} = \ddot{\theta} l$$

$$-g \sin \theta = \frac{d^2 s}{dt^2}$$

$$-g \sin \theta = l \frac{d^2 \theta}{dt^2} = l \ddot{\theta}$$

$$\boxed{-\frac{g}{l} \sin \theta = \ddot{\theta}} \quad \text{Pendulum equation}$$

if small oscillations \downarrow

$$\sin \theta \approx \theta \quad \text{when } \theta \ll 1$$

$$\boxed{-\frac{g}{l} \theta = \ddot{\theta}} \quad \text{The harmonic oscillator}$$

$$\omega^2 = \frac{g}{l}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l}{g}}$$

- can be used for g measurements

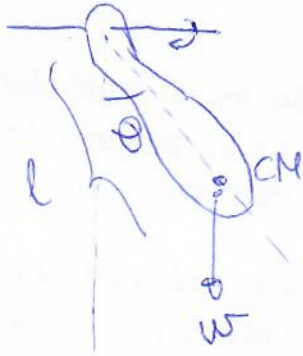
+ does not depend

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The physical pendulum



$$\tau = r \cdot F = I \cdot \alpha$$

τ because weight \rightarrow

$$-Mgl \sin \theta = I \ddot{\theta}$$

$$\ddot{\theta} = -\frac{Mgl}{I} \sin \theta \quad \left| \text{Physical Pendulum} \right.$$

18 small oscillations

$$\theta \ll 1 \quad \sin \theta \approx \theta$$

$$\ddot{\theta} = -\frac{Mgl}{I} \theta \quad \left| \text{harmonic oscillations} \right.$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{Mgl}} \quad \text{period}$$

Damped oscillations

$$F_{\text{damp}} = -b\dot{x} \quad \leftarrow \text{typical, standard assumption}$$

$$-kx - b\dot{x} = m\ddot{x}$$

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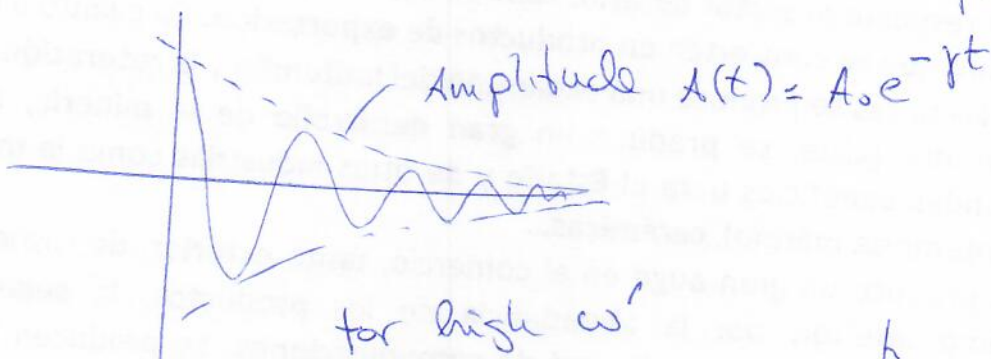
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$$x = A_0 e^{-\dots} \cos(\omega t + \phi)$$

$\tau \equiv$ 'extension time' \equiv 'time constant'
 time needed for reducing amplitudes
 a factor $1/e$.

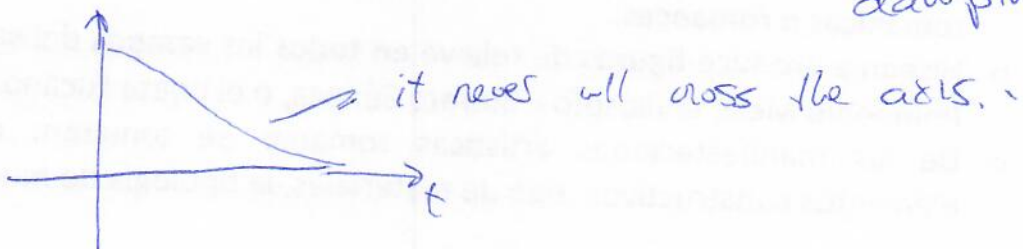
$\omega' = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$ with $\omega_0 = \sqrt{\frac{k}{m}} \equiv$ natural or undamped frequency.



b) when $b \gg$ is large.

ω' is even lower. -

when $b = 2m\omega_0 \rightarrow \omega' = 0 \equiv$ critical damping



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Forced oscillations (or 'driven' oscillations)

The damped oscillator does not move freely, but is subject to an external force $F(t)$

Typically, $F(t) = F_0 \cos(\omega t)$

Hence,

$$\left[m\ddot{x} + b\dot{x} + Kx = F_0 \cos(\omega t) \right] \text{ ODE}$$

\downarrow
 $m\omega_0^2$

driving freq.

One can solve this ODE. There is a transient part that depends on initial condition, similar to the damped oscillator. But later on, it reaches a steady solution, the same for every initial condition,

$$x(t) = A \cos(\omega t - \delta)$$

\downarrow
driving frequency; $\omega' = \omega$ in steady solution

with

$$A = \frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}}$$

$$\omega_0 = \sqrt{\frac{K}{m}} \text{ natural}$$

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Notably, when $\omega = \omega_0 \Rightarrow \omega_{\text{resonant}}$, it is called
 \downarrow
 driving resonant case.

$$\left\{ \begin{array}{l} A \text{ is maximum} \\ \delta = \pi/2 \end{array} \right.$$

$$\left\{ \begin{array}{l} \delta \approx 0 \quad \omega \ll \omega_0 \\ \delta \approx \pi \quad \omega \gg \omega_0 \end{array} \right.$$

In this resonant case: -

$$v = \frac{dx}{dt} = \dot{x} = -A\omega \sin(\omega t - \delta)$$

$$\text{If } \delta = \pi/2; \quad v_r = -A\omega \sin(\omega t - \pi/2)$$

$$v_r = +A\omega \cos(\omega t)$$

$v_{\text{resonance}}$ is in phase with driving force \rightarrow
 the particle moves in the same direction
 than the force \rightarrow maximum energy transport

Averaged power

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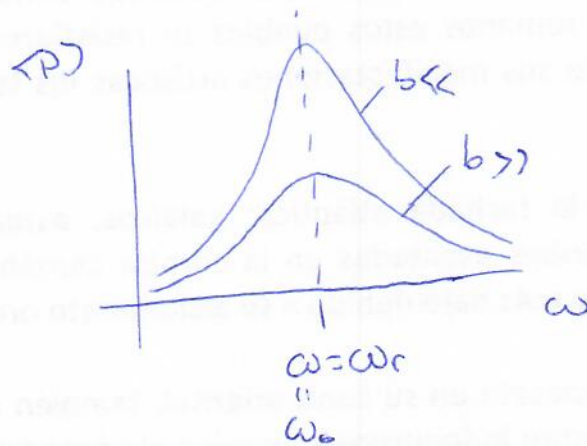
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$\mu = \frac{1}{\omega} \frac{d\omega}{dt} = \dots$

Averaging $\langle P \rangle = \frac{\int_0^T P(t) dt}{T} = \frac{1}{2} A \omega F_0 \sin \delta$

Replacing $\sin \delta = \frac{b \omega}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2 \omega^2}}$

$$\langle P \rangle = \frac{1}{2} \left[\frac{b \omega^3 F_0^2}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2 \omega^2}} \right]$$



The averaged power grows as $\omega \rightarrow \omega_0$

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WAVES

- A travelling perturbation that is periodic both in time and space.
- It transports energy and linear momentum without mass.

Mechanical waves: they travel through a medium
Electromagnetic waves: they are oscillating fields and they do not need any material medium

Transverse waves: the perturbation is perpendicular to the direction of propagation
Longitudinal waves: the perturbation travels parallel to the direction of propagation.

Waves are described by wave - functions. -

$$\Delta x = v \Delta t \quad v = \text{speed of the wave}$$

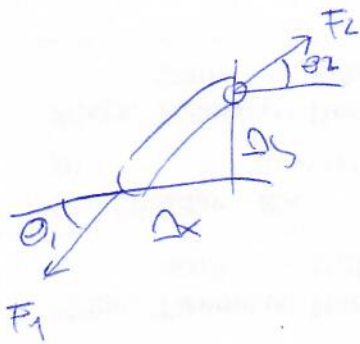
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Equation of a wave = wave function

in a particular case: an oscillating rope.



~~De~~ This segment of the rope will oscillate vertically.

$$\Sigma F = F \sin \theta_2 - F \sin \theta_1$$

$$\approx F (\tan \theta_2 - \tan \theta_1)$$

using $\tan \theta \approx \sin \theta$

$$\Sigma F \approx F \left(\frac{\partial y_2}{\partial x_2} - \frac{\partial y_1}{\partial x_1} \right) = ma$$

$$F \cdot \Delta \text{slope} = \text{"mass"} \cdot \frac{\partial^2 y}{\partial t^2}$$

linear density $\rightarrow \mu = \frac{m}{L} \Rightarrow \text{mass} = m = \mu \cdot L = \mu \Delta x$

$$F \Delta \text{slope} = \mu \Delta x \cdot \frac{\partial^2 y}{\partial t^2}$$

$$\neq \frac{\Delta \text{slope}}{\Delta x} = \mu \frac{\partial^2 y}{\partial t^2}$$

$$\neq \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial x} \right) = \mu \frac{\partial^2 y}{\partial t^2}$$

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relation: a wave travelling $v = \frac{1}{\mu}$

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The general wave equation is:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{wave equation.}$$

The solutions are of the form $f(x-vt)$

- Mechanical waves such as 'ropes' $\rightarrow v = \sqrt{\frac{F}{\mu}}$
- Sonic waves in a fluid: $v = \sqrt{\frac{B}{\rho}}$
 $B \equiv$ compressibility
 $\rho \equiv$ density
- in a gas for p & T $v = \sqrt{\frac{\gamma RT}{M}}$
- Electromagnetic waves $v = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$

The solutions can be diverse.

The simplest solution (not the only one) is the so-called 'harmonic wave'

$$y(x,t) = A \sin(kx - \omega t) = A \sin[(k)(x - vt)]$$

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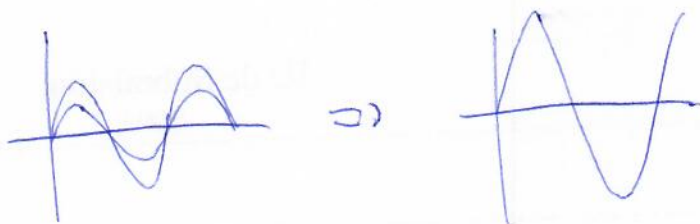
Principle of superposition

- A consequence of the linearity of the wave function:-

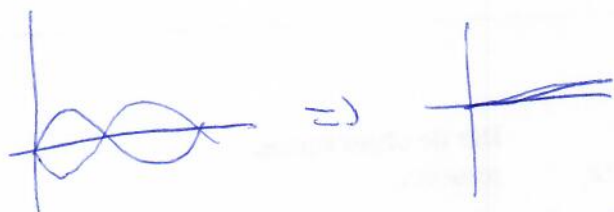
Given two solutions, $y_1(x,t)$ and $y_2(x,t)$

$y_3(x,t) = \alpha y_1 + \beta y_2$ is also solution
linear combination:-

Adding waves is called 'interference'



constructive
when in phase



destructive when
out of phase

But the general case \rightarrow we need to
add (mathematically) the two functions.

with similar frequencies and same

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the amplitude,

with phase constant

The group velocity is the velocity of the 'amplitude', of the 'envelope'.
It carries the information = signal velocity.

The phase velocity $v_p = \frac{\omega}{k}$ it is just the propagation of the crests, without information.

When the wave reaches a different medium than vacuum

$$v = \frac{c}{n}; \quad n = \frac{c}{v}$$

Vacuum $n=1$ $v=c$
Typically $n > 1$; v_{phase} decreases
air $n=1,0003$
water $n=1,33$
But it may happen $v_{\text{phase}} > c$! $n < 1$.

Water

$$v_{\text{light}} = 0,75c$$

Some e^- 's can travel faster than that with enough energy \rightarrow Cherenkov radiation.

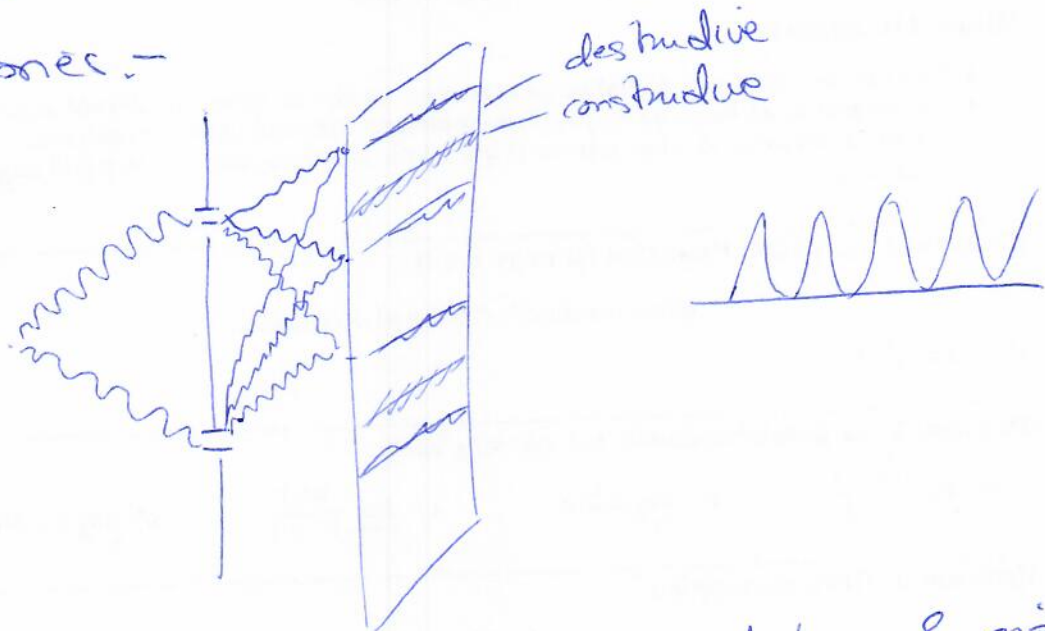
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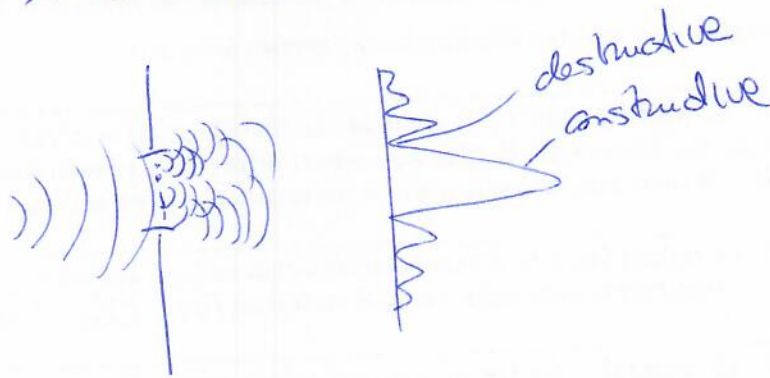
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Interference

Two waves originated in phase, can arrive at a given point with different phases because they travelled along different trajectories. -



When waves pass through a hole of size similar to $\lambda \rightarrow$ 'diffraction'



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Standing waves

- A wave travelling can find an obstacle, a wall and can be reflected.



bouncing back.

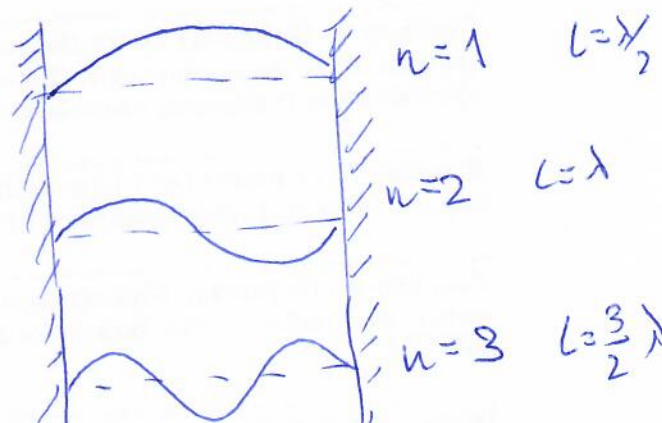
- If the wave gets confined between two fixed walls one can get a stationary oscillating schema = "standing waves".

Because we get a final schema resulting from the superposition principle, when waves travelling from left to right, and vice-versa interfere.

The resulting schema:

$$L = n \frac{\lambda_n}{2}$$

$$n = 1, 2, 3, \dots$$



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The resonance condition is reached when

$$\frac{2L}{v} = nT \Rightarrow \boxed{v = n \frac{v}{2L}} \quad \begin{array}{l} \text{natural freq's} \\ \text{or} \\ \text{resonant freq's} \end{array}$$

The final motion: a linear combination of the natural freq's:-

* A standing wave will have equation:

$$y_n(x, t) = A_n \sin(k_n x) \cos(\omega_n t + \phi_n)$$

$$k_n = \frac{2\pi}{\lambda_n}$$

$$A_n(x) = A_n \sin(k_n x)$$

Every point at position 'x' ~~moves~~ oscillates harmonically but it does not travel. ~

* The final motion: superposition of several standing waves:-

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