Partial Exam

Mathematical Methods of Bioengineering
Ingeniería Biomédica - INGLÉS

12 of April 2020

Problems

1. Some astronauts landed in Mars at the point $A = (2, 2, 0)$ km. After planting the base camp, decide to go for a space walk with their spaceship to observe the surroundings. The spaceship is moving with constant velocity 60 km/h. First, the spaceship moves from the station at A to the point $C = (0, 4, 4)$ km, passing through the point $B = (0, 4, 0)$ km and moving always in a straight path. Then, they move in the direction of the projection of $\vec{OA}$ onto $\vec{OB}$ 2 minutes. Finally, the astronauts move in the direction of $\vec{OB} \times \vec{OC}$ another 2 minute.

   (a) (**0.5 points**) Sketch the path followed by the spaceship.

   (b) (**1 point**) Describe the movement of the spaceship using parametric equations[1].

   (c) (**1 point**) Find the plane containing A, B and C. What is the distance between a object at $D = (10, 0, 0)$ an the plane?

2. (**1.25 points**) Consider the Mexican "sombrero" surface in figure 1, described by the function

$$f(x, y) = \frac{\cos\left(\frac{1}{4}(x^2 + y^2)\right)}{x^2 + y^2 + 3}.$$

   Find the tangent plane at the point $(0, 0)$ and at the point $(\sqrt{2\pi}, 0)$. Does them intersect?

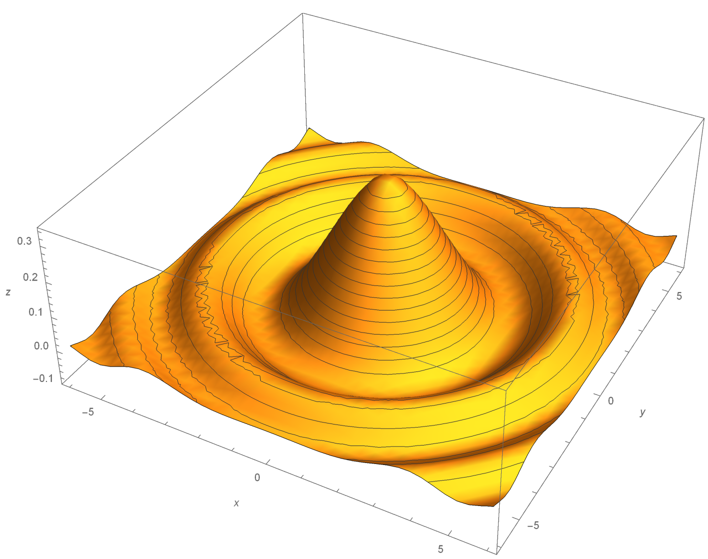

Figure 1: Mexican "sombrero" surface.

[1]Since the movement is determined by straight segments you can use the parametric equation of a line $\mathbf{l}(t) = \mathbf{x_0} + \vec{\mathbf{d}} \cdot t$, for appropriate values of $\mathbf{x_0}$, $\vec{\mathbf{d}}$ and $t \in [t_0, t_1]$.

3. Suppose that the following function is used to model the monthly demand for bicycles:

$$P(x, y) = 200 + 20\sqrt{0.1x + 10} - 12\sqrt[3]{y}$$

In this formula, x represents the price (in euros per gallon) of automobile gasoline and y represents the selling price (in euros) of each bicycle. Furthermore, suppose that the price of gasoline t months from now will be

$$x(t) = 1.5 + 0.1t - \cos\frac{\pi t}{6}$$

and the price of each bicycle will be:

$$y(t) = 200 + 2t\sin\frac{\pi t}{6}.$$

(a) (**0.5 points**) What is the actual demand for bicycles? And in six month?

(b) (**1.25 points**) Use the chain rule to compute the rate of change of the monthly demand for bicycles.

(c) (**0.5 point**) At what rate will the monthly demand for bicycles be changing six months from now? Will the demand increase?

4. The depth of a Lake can be represented by the expression $400 - 3x^2y^2$ in meters. If your calculus instructor is in the water at the point $(1, -2)$:

(a) (**0.5 points**) Find the direction he should swim so that the depth increases most rapidly. How much is the depth changing in that direction? Finally, in which direction should he swim so the depth remain constant?

(b) (**1.25 points**) Find the directional derivative in the direction $(-1, 0)$ by means of the definition and by using the gradient vector. Is the depth increasing or decreasing?

5. Consider the function $f(x, y) = ye^{-x^2-y^2}$.

(a) (**1 point**) Find the critical points of f.

(b) (**1.25 points**) Identify the nature of the critical point/s.