

## Problems

**Problem 8.1** Write the Taylor polynomial  $P_{5,0}(x)$  for these functions:

- (i)  $e^x \sin x$ ; (iii)  $\sin x \cos 2x$ ; (v)  $\sin^2 x$ ;  
 (ii)  $e^{-x^2} \cos 2x$ ; (iv)  $e^x \log(1-x)$ ; (vi)  $\frac{1}{1-x^3}$ .

**Problem 8.2** Write the polynomial  $x^4 - 5x^3 + x^2 - 3x + 4$  in powers of  $x - 4$ .

**Problem 8.3** Write the Taylor polynomial  $P_{n,a}(x)$  for these functions around the specified  $a$ :

- (i)  $f(x) = 1/x$  around  $a = -1$ ; (iii)  $f(x) = (1 + e^x)^2$  around  $a = 0$ ;  
 (ii)  $f(x) = xe^{-2x}$  around  $a = 0$ ; (iv)  $f(x) = \sin x$  around  $a = \pi$ .

**Problem 8.4** Write the Taylor polynomial  $P_{n,0}(x)$  of the function

$$f(x) = \begin{cases} e^{-1/x^2}, & x \neq 0, \\ 0 & x = 0. \end{cases}$$

**Problem 8.6** Calculate the following limits using Taylor's theorem:

- (i)  $\lim_{x \rightarrow 0} \frac{e^x - \sin x - 1}{x^2}$ ; (vi)  $\lim_{x \rightarrow 0} \frac{\cos x + e^x - x - 2}{x^3}$ ;  
 (ii)  $\lim_{x \rightarrow 0} \frac{\sin x - x + x^3/6}{x^5}$ ; (vii)  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{\sin x} \right)$ ;  
 (iii)  $\lim_{x \rightarrow 0} \frac{\cos x - \sqrt{1-x}}{\sin x}$ ; (viii)  $\lim_{x \rightarrow 0} \frac{1}{x} \left( \frac{1}{x} - \cot x \right)$ ;  
 (iv)  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ ; (ix)  $\lim_{x \rightarrow \infty} x^{3/2} \left( \sqrt{x+1} + \sqrt{x-1} - 2\sqrt{x} \right)$ ;  
 (v)  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x(1 - \cos 3x)}$ ; (x)  $\lim_{x \rightarrow \infty} \left[ x - x^2 \log \left( 1 + \frac{1}{x} \right) \right]$ .

**Problem 8.8** Determine the first nonzero order in the Taylor expansion of the following functions:

- (i)  $f(x) = \tan(\sin x) - \sin(\tan x)$ ;  
 (ii)  $f(x) = \frac{1}{R^2} - \frac{1}{(R+x)^2}$ ;  
 (iii)  $f(x) = \sqrt[3]{\frac{1+x}{1-x}} - \sqrt[3]{\frac{1-x}{1+x}}$ .

**Problem 8.15** Given the function  $f(x) = \cos x + e^x$ ,

- (i) find its Taylor polynomial  $P_{3,0}(x)$ ;

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