Problems

Problem 5.1 Calculate the following limits, simplifying the common factors that numerator and denominator may contain:

(i) $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a}$, $n \in \mathbb{N}$;

(iii) $\lim\limits_{x \to 64} \dfrac{\sqrt{x} - 8}{\sqrt[3]{x} - 4}$;

(v) $\lim\limits_{x \to 0} \dfrac{\frac{1}{(1-x)^3} - 1}{x}$;

(ii) $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$;

(iv) $\lim\limits_{x \to 0} \dfrac{1 - \sqrt{1 - x^2}}{x^2}$;

(vi) $\lim\limits_{x \to 1} \left(\dfrac{1}{\sqrt{x} - 1} - \dfrac{2}{x - 1} \right)$.

Problem 5.2 Calculate the following limits:

(i) $\lim\limits_{x \to 0} \dfrac{(\sin 2x^3)^2}{x^6}$;

(v) $\lim\limits_{x \to 0} \dfrac{\log(1 - 2x)}{\sin x}$;

(ix) $\lim\limits_{x \to 0} \left(\dfrac{x}{\sin x} \right)^{\frac{\sin x}{\sin x - x}}$;

(ii) $\lim\limits_{x \to 0} \dfrac{\tan x^2 + 2x}{x + x^2}$;

(vi) $\lim\limits_{x \to 0} (1 + \sin x)^{2/x}$;

(x) $\lim\limits_{x \to 0} (\cos x)^{1/x^2}$;

(iii) $\lim\limits_{x \to 0} \dfrac{\sin(x + a) - \sin a}{x}$;

(vii) $\lim\limits_{x \to 0} \dfrac{e^x - e^{\sin x}}{x - \sin x}$;

(xi) $\lim\limits_{x \to \pi} \dfrac{1 - \sin(x/2)}{(x - \pi)^2}$;

(iv) $\lim\limits_{x \to 0} (1 + x)^{1/x}$;

(viii) $\lim\limits_{x \to 0} \dfrac{\tan x - \sin x}{x^3}$;

(xii) $\lim\limits_{x \to 0} \dfrac{a^x - b^x}{x}$.

Problem 5.3 Calculate the following limits:

(i) $\lim\limits_{x \to \infty} \dfrac{x^3 + 4x - 7}{7x^2 - \sqrt{2x^6 + x^5}}$;

(iv) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 4x} - x \right)$;

(vii) $\lim\limits_{x \to \pm\infty} \tanh x$;

(ii) $\lim\limits_{x \to \infty} \dfrac{x + \sin x^3}{5x + 6}$;

(v) $\lim\limits_{x \to \pm\infty} \dfrac{e^x}{e^x - 1}$;

(viii) $\lim\limits_{x \to \pm\infty} \dfrac{e^x}{\sinh x}$;

(iii) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}$;

(vi) $\lim\limits_{x \to \pm\infty} \dfrac{x - 2}{\sqrt{4x^2 + 1}}$;

(ix) $\lim\limits_{x \to \pm\infty} \left(\dfrac{2x + 7}{2x - 6} \right)^{\sqrt{4x^2 + x - 3}}$.

Problem 5.4 Calculate the one-sided limits:

(i) $\lim\limits_{x \to 0^\pm} \left(\dfrac{1}{x} \right)^{\lfloor x \rfloor}$;

(ii) $\lim\limits_{x \to 0^\pm} e^{1/x}$;

(iii) $\lim\limits_{x \to 0^\pm} \dfrac{1 - e^{1/x}}{1 + e^{1/x}}$.