

Problem 3.2 Given the following recurrent sequences, find the general term and compute their limit:

(i) $a_{n+1} = \frac{a_n + 1}{2}$, with $a_0 = 0$;

Problem 3.3 Calculate the following limits:

(iii) $\lim_{n \rightarrow \infty} n(\sqrt{n^2 + 1} - n)$;

Problem 3.4 Calculate the following limits:

(i) $\lim_{n \rightarrow \infty} \frac{n}{\pi} \sin n\pi$;

(iv) $\lim_{n \rightarrow \infty} n^{-3/n}$;

Problem 3.8 Calculate the limit

$$\lim_{n \rightarrow \infty} \sum_{k=1}^{3n} \frac{1}{\sqrt{n^2 + k}}$$

using the sandwich rule.

HINT: Use the largest and smallest terms in the sum to bound the sum from above and from below, respectively.

Problem 3.9 Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive terms such that $\lim_{n \rightarrow \infty} (a_n - n) = \ell$.

(a) Prove that $\lim_{n \rightarrow \infty} \frac{a_n}{n} = 1$.

Problem 4.3 Discuss, depending on the value of the parameter a in the given range, whether the following series converge or diverge:

(ii) $\sum_{n=1}^{\infty} \frac{n^n}{a^n n!}$, for $a > 0$;

(iii) $\sum_{n=1}^{\infty} \frac{n! e^n}{n^{n+a}}$, for any $a \in \mathbb{R}$;

(iv) $\sum_{n=1}^{\infty} \frac{a^n}{(1+a)(1+a^2) \cdots (1+a^n)}$, for $a \geq 0$.

Problem 4.4 Determine whether the following series are absolutely convergent, and if not, whether they converge conditionally:

(i) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\log n}$;

(v) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2 - 1} - n)$;

(ii) $\sum_{n=1}^{\infty} \sin\left(\pi n + \frac{1}{n}\right)$;

(vi) $\sum_{n=1}^{\infty} (-1)^n \log\left(\frac{n}{n+1}\right)$;

(iii) $\sum_{n=1}^{\infty} (-1)^n \left(\arctan \frac{1}{n}\right)^2$;

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HINT: Expand the general term in elementary fractions.

$$\frac{1}{n-1} - \frac{1}{n} + \frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+3} - \frac{1}{n+4} + \frac{1}{n+5} - \frac{1}{n+6} + \frac{1}{n+7} - \frac{1}{n+8} + \frac{1}{n+9} - \frac{1}{n+10} + \frac{1}{n+11} - \frac{1}{n+12} + \frac{1}{n+13} - \frac{1}{n+14} + \frac{1}{n+15} - \frac{1}{n+16} + \frac{1}{n+17} - \frac{1}{n+18} + \frac{1}{n+19} - \frac{1}{n+20} + \frac{1}{n+21} - \frac{1}{n+22} + \frac{1}{n+23} - \frac{1}{n+24} + \frac{1}{n+25} - \frac{1}{n+26} + \frac{1}{n+27} - \frac{1}{n+28} + \frac{1}{n+29} - \frac{1}{n+30} + \frac{1}{n+31} - \frac{1}{n+32} + \frac{1}{n+33} - \frac{1}{n+34} + \frac{1}{n+35} - \frac{1}{n+36} + \frac{1}{n+37} - \frac{1}{n+38} + \frac{1}{n+39} - \frac{1}{n+40} + \frac{1}{n+41} - \frac{1}{n+42} + \frac{1}{n+43} - \frac{1}{n+44} + \frac{1}{n+45} - \frac{1}{n+46} + \frac{1}{n+47} - \frac{1}{n+48} + \frac{1}{n+49} - \frac{1}{n+50}$$