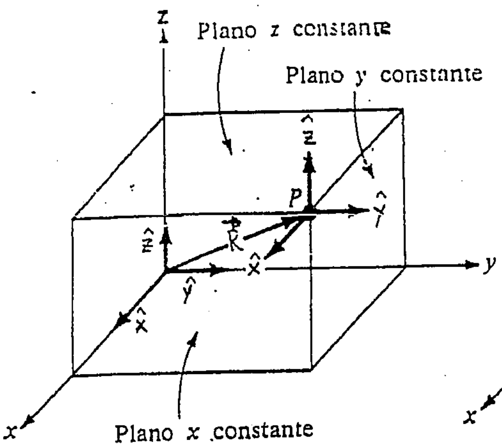
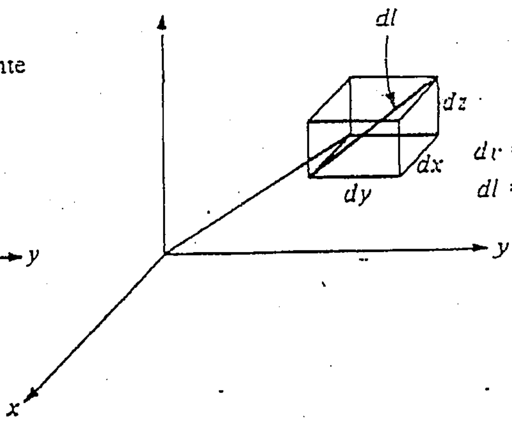
COORDENADAS CARTESIANAS





$dv = dx\,dy\,dz$

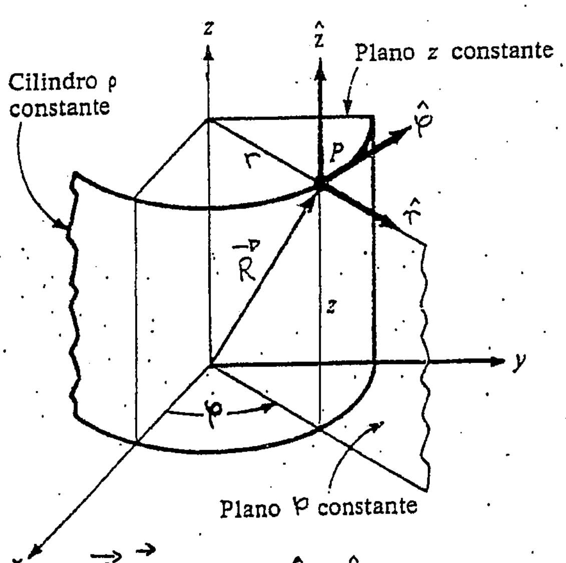
$dl = (dx^2 + dy^2 + dz^2)^{1/2}$

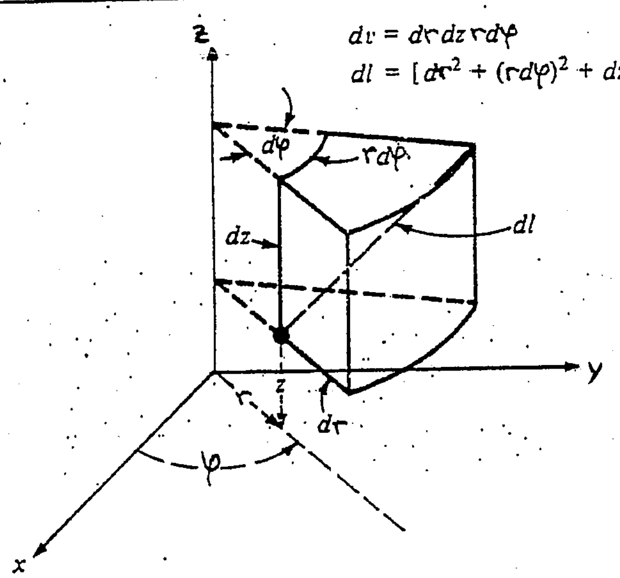
$\vec{R} = \vec{R}(x, y, z) = x\hat{x} + y\hat{y} + z\hat{z}$

$\mathbf{dl} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z} \equiv (dx, dy, dz)$

El punto P en el extremo del vector $\mathbf{r}$ se determina por la intersección de los planos x, y y z constantes.

COORDENADAS CILÍNDRICAS





$dv = dr\,dz\,r\,d\varphi$

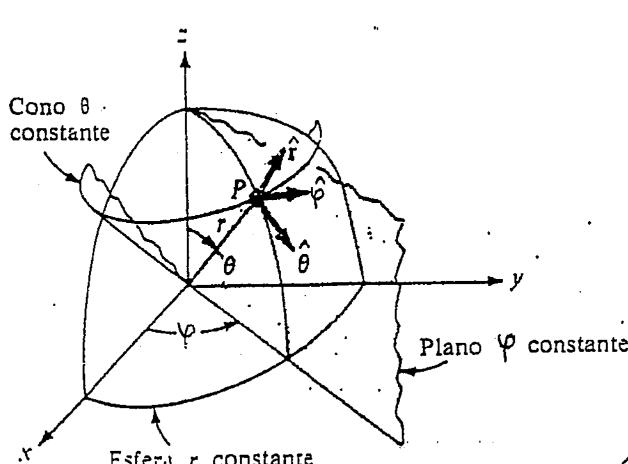
$dl = [dr^2 + (r\,d\varphi)^2 + dz^2]^{1/2}$

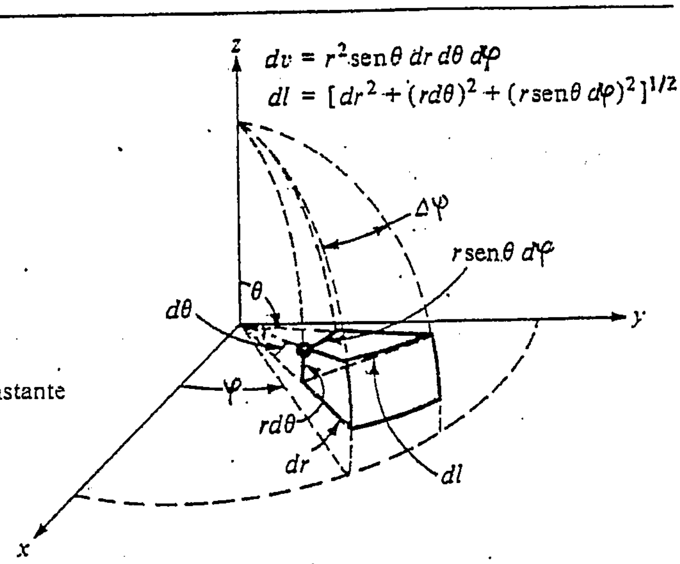
$\vec{R} = \vec{R}(r, \varphi, z) = r\,\hat{r} + z\,\hat{z}$

$\vec{dl} = dr\,\hat{r} + r\,d\varphi\,\hat{\varphi} + dz\,\hat{z}$

El punto P se determina por la intersección de las superficies z (plano), φ (semiplano) y r (cilindro) constantes.

COORDENADAS ESFÉRICAS





$dv = r^2 \operatorname{sen}\theta\, dr\, d\theta\, d\varphi$

$dl = [dr^2 + (r\,d\theta)^2 + (r \operatorname{sen}\theta\, d\varphi)^2]^{1/2}$

$\vec{R} \equiv \mathbf{r} = R(r, \theta, \varphi) = r\hat{r}$

$\mathbf{dl} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r \operatorname{sen}\theta\, d\varphi\,\hat{\varphi}$

El punto P se determina por la intersección de las superficies θ (cono), r (esfera) y φ (semiplano) constantes.

OPERADORES DIFERENCIALES

COORDENADAS CARTESIANAS (x, y, z)

$$\vec{\nabla}\psi = \frac{\partial\psi}{\partial x}\hat{x} + \frac{\partial\psi}{\partial y}\hat{y} + \frac{\partial\psi}{\partial z}\hat{z}$$

$$\vec{\nabla}\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$$

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

COORDENADAS ESFÉRICAS (r, θ, φ)

$$\vec{\nabla}\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial\psi}{\partial\varphi}\hat{\varphi}$$

$$\vec{\nabla}\cdot\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}(\mathrm{sen}\,\theta\, D_\theta) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial D_\varphi}{\partial\varphi}$$

$$\vec{\nabla}\times\vec{H} = \frac{1}{r\,\mathrm{sen}\,\theta}\left[\frac{\partial}{\partial\theta}(H_\varphi\,\mathrm{sen}\,\theta) - \frac{\partial H_\theta}{\partial\varphi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\mathrm{sen}\,\theta}\frac{\partial H_r}{\partial\varphi} - \frac{\partial}{\partial r}(rH_\varphi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r\,H_\theta) - \frac{\partial H_r}{\partial\theta}\right]\hat{\varphi}$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\,\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial^2\psi}{\partial\varphi^2}$$

COORDENADAS CILINDRICAS (r, φ, z)

$$\vec{\nabla}\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\psi}{\partial\varphi}\hat{\varphi} + \frac{\partial\psi}{\partial z}\hat{z}$$

$$\vec{\nabla}\cdot\vec{D} = \frac{1}{r}\frac{\partial}{\partial r}(r\,D_r) + \frac{1}{r}\frac{\partial D_\varphi}{\partial\varphi} + \frac{\partial D_z}{\partial z}$$

$$\vec{\nabla}\times\vec{H} = \left(\frac{1}{r}\frac{\partial H_z}{\partial\varphi} - \frac{\partial H_\varphi}{\partial z}\right)\hat{r} + \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r}\right)\hat{\varphi} + \frac{1}{r}\left(\frac{\partial(rH_\varphi)}{\partial r} - \frac{\partial H_r}{\partial\varphi}\right)\hat{z}$$

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi^2} + \frac{\partial^2\psi}{\partial z^2}$$

IDENTIDADES VECTORIALES

1.- $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$

2.- $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{A}\cdot\vec{C}) - \vec{C}\cdot(\vec{A}\cdot\vec{B})$

3.- $\vec{\nabla}\cdot(\varphi\vec{A}) = \vec{A}\cdot\vec{\nabla}\varphi + \varphi\,\vec{\nabla}\cdot\vec{A}$

4.- $\vec{\nabla}\times(\varphi\vec{A}) = \varphi\,\vec{\nabla}\times\vec{A} - \vec{A}\times\vec{\nabla}\varphi$

5.- $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times B)$

6.- $\vec{\nabla}\times\vec{\nabla}\varphi = 0$

7.- $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

8.- $\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$ (Coordenadas cartesianas)

9.- $\vec{\nabla}\cdot\vec{\nabla}\varphi = \nabla^2\varphi$

10.- $\int_C \vec{A}\cdot\vec{dl} = \int_{S_C}(\vec{\nabla}\times\vec{A})\cdot\vec{ds}$ (Teorema de Stokes)

TRANSFORMACION DE COORDENADAS

* **CARTESIANAS - CILINDRICAS**

$$\mathbf{i} = \hat{x} = \cos\varphi\ \hat{r} - \operatorname{sen}\varphi\ \hat{\varphi}$$
$$\mathbf{j} = \hat{y} = \operatorname{sen}\varphi\ \hat{r} + \cos\varphi\ \hat{\varphi}$$
$$\mathbf{k} = \hat{z} = \hat{z}$$

$$x = r\cos\varphi$$
$$y = r\operatorname{sen}\varphi$$
$$z = z$$

* **CILINDRICAS - CARTESIANAS**

$$\hat{r} = \frac{x}{\sqrt{x^2+y^2}}\ \hat{x} + \frac{y}{\sqrt{x^2+y^2}}\ \varphi\ \hat{y}$$
$$\hat{\varphi} = -\frac{y}{\sqrt{x^2+y^2}}\ \hat{x} + \frac{x}{\sqrt{x^2+y^2}}\ \hat{y}$$
$$\hat{z} = \hat{z}$$

$$r = \sqrt{x^2+y^2}$$
$$\varphi = \text{arc tg }(y/x)$$
$$z = z$$

* **CARTESIANAS - ESFERICAS**

$$\hat{x} = \operatorname{sen}\theta\cos\varphi\ \hat{r} + \cos\theta\cos\varphi\ \hat{\theta} - \operatorname{sen}\varphi\ \hat{\varphi}$$
$$\hat{y} = \operatorname{sen}\theta\operatorname{sen}\varphi\ \hat{r} + \cos\theta\operatorname{sen}\varphi\ \hat{\theta} + \cos\varphi\ \hat{\varphi}$$
$$\hat{z} = \cos\theta\ \hat{r} - \operatorname{sen}\theta\ \hat{\theta}$$

$$x = r\operatorname{sen}\theta\cos\varphi$$
$$y = r\operatorname{sen}\theta\operatorname{sen}\varphi$$
$$z = r\cos\theta$$

* **ESFERICAS - CARTESIANAS**

$$\hat{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}\ \hat{x} + \frac{y}{\sqrt{x^2+y^2+z^2}}\ \hat{y} + \frac{z}{\sqrt{x^2+y^2+z^2}}\ \hat{z}$$
$$\hat{\theta} = \frac{z}{\sqrt{x^2+y^2+z^2}}\frac{x}{\sqrt{x^2+y^2}}\ \hat{x} + \frac{z}{\sqrt{x^2+y^2+z^2}}\frac{y}{\sqrt{x^2+y^2}}\ \hat{y} - \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\ \hat{z}$$
$$\hat{\varphi} = -\frac{y}{\sqrt{x^2+y^2}}\ \hat{x} + \frac{x}{\sqrt{x^2+y^2}}\ \hat{y}$$

$$r = \sqrt{x^2+y^2+z^2}$$
$$\theta = \text{arc sen}\frac{\sqrt{x^2+y^2}}{r} = \text{arc cos }(z/r)$$
$$\varphi = \text{arc sen}\frac{y}{\sqrt{x^2+y^2}}$$

11.- $\int_V \vec{\nabla}\cdot\vec{A}\, dv = \int_{Sc} \vec{A}\cdot d\vec{s}$ (Teorema de la divergencia)

12.- $\int_V \vec{\nabla}\times\vec{A}\, dv = -\int_{Sc} \vec{A}\times d\vec{s}$

13.- $\int_{Sc} \varphi\vec{\nabla}\psi\cdot d\vec{s} = \int_V \left(\varphi\nabla^2\psi + \vec{\nabla}\varphi\cdot\vec{\nabla}\psi\right) dv$

(Identidades de Green)

14 .- $\int_{Sc} \left(\varphi\vec{\nabla}\psi - \psi\vec{\nabla\varphi}\right)\cdot d\vec{s} = \int_V \left(\varphi\nabla^2\psi - \psi\nabla^2\varphi\right) dv$

ELEMENTOS DIFERENCIALES

* **COORDENADAS CARTESIANAS**

$$\vec{dl} = dx\ \hat{x} + dy\ \hat{y} + dz\ \hat{z}$$

$$dv = dx\ dy\ dz$$

* **COORDENADAS CILINDRICAS**

$$\vec{dl} = dr\ \hat{r} + r\ d\varphi\ \hat{\varphi} + dz\ \hat{z}$$

$$dv = r\ dr\ d\varphi\ dz$$

* **COORDENADAS ESFERICAS**

$$\vec{dl} = dr\ \hat{r} + r\ d\theta\ \hat{\theta} + r\operatorname{sen}\theta\ d\varphi\ \hat{\varphi}$$

$$dv = r^2\operatorname{sen}\theta\ dr\ d\theta\ d\varphi$$

POLINOMIOS DE LEGENDRE

ECUACIÓN DIFERENCIAL DE LEGENDRE: $(1-x^2)y'' - 2xy' + n(n+1)\,y = 0$

POLINOMIOS DE LEGENDRE: Si n = 0, 1, 2, ..., las soluciones de la ecuación anterior son polinomios de Legendre, $P_n(x)$, que se pueden hallar mediante la *fórmula de Rodrigues*:

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

EJEMPLOS DE POLINOMIOS DE LEGENDRE:

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = (3x^2 - 1) / 2$

$P_3(x) = (5x^3 - 3x) / 2$

$P_4(x) = (35x^4 - 30x^2 + 3) / 8$

$P_5(x) = (63x^5 - 70x^3 + 15x) / 8$

ORTOGONALIDAD DE LOS POLINOMIOS DE LEGENDRE:

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = 0 \qquad (m \neq n)$$

$$\int_{-1}^{1} \{P_n(x)\}^2\, dx = \frac{2}{2n+1}$$

DELTA DE DIRAC

$$\int_V F(\vec{r})\, \delta(\vec{r}-\vec{r}_o)\, dv = \begin{matrix} F(\vec{r}_o) & (\vec{r}_o \in V) \\ 0 & (\vec{r}_o \notin V) \end{matrix}$$

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(\vec{r})$$