

Fundamental Concepts of Statistics

Exercise session 7

1. Let X_1, \dots, X_n be a random sample from a Poisson distribution with mean λ and $T = \sum_{i=1}^n X_i$. Show that the distribution of X_1, \dots, X_n given T is independent of λ so that T is sufficient for λ .

2. Let X_1, \dots, X_n be a random sample from the distribution with density

$$f(x; \theta) = \frac{\theta}{(1+x)^{1+\theta}}, \quad x > -1 \text{ and } \theta > 0.$$

Find a sufficient statistic for θ .

3. Suppose that X is binomially (n, p) distributed.

a) Show that the MLE of p is $\hat{p} = X/n$.

b) Show that this MLE attains the Cramr-Rao lower bound.

4. Suppose that X_1, \dots, X_n is a random sample from geometric distribution $\text{Geo}(p)$ with

$$P(X = x) = p(1-p)^{x-1}, \quad x = 1, 2, \dots$$

Expected value of geometric distribution is $1/p$.

a) Find the MLE of p .

b) Find the asymptotic variance of the MLE.

5. The Pareto distribution is defined through

$$f(x; \theta) = \theta x_0^\theta x^{-\theta-1}, \quad x > x_0.$$

Assume that x_0 is given.

Consider X_1, X_2, \dots, X_m iid sample from this distribution.

a) Find the MLE of θ

b) Find the asymptotic variance of the MLE.

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