$\frac{dx}{dt} \cdot \frac{dt}{dy}$

$m = 1 \cdot 10^{-11}\,Kg$

$\vec{F}r = -bv$

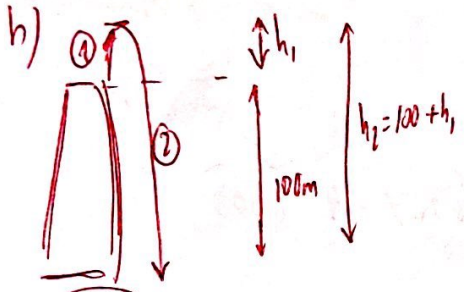
$b = 3 \cdot 10^{-9}\,N/(m/s)^2$

a) V_{lim}?

b) T al suelo si $h = 100m$ con $V_0 = 2\,m/s$

a) $\sum f \rightarrow mg - fr = 0$; $mg - bv_{lim} = 0$; $V_{lim} = \frac{mg}{b} = \frac{1 \cdot 10^{-11} \cdot 9'81}{3 \cdot 10^{-9}} =$

$= 0'0327\,m/s$

b) 

$\tau = \frac{m}{b} = \frac{1 \cdot 10^{-11}}{3 \cdot 10^{-9}} = 0'00\hat{3}$

① Subida

$\sum F = m \cdot a$

$m \cdot a = -mg - bv$

$m \cdot \frac{dv}{dt} = -b(v + V_{lim})$

$\frac{dv}{v + V_{lim}} = \frac{-b}{m} dt$

$\ln v + V_{lim} \Big]_{V_0}^{V_t} = \ln -t/\tau$

$\frac{V_t + V_{lim}}{V_0 + V_{lim}} = e^{-t/\tau}$

$V_t = (V_0 + V_{lim})(e^{-t/\tau}) - V_{lim}$

$\rightarrow$ Sacamos t.

$\ln \frac{V_t + V_{lim}}{V_0 + V_{lim}} = \frac{-b}{m} t$

$t = -\frac{m}{b} \ln \frac{V_t + V_{lim}}{V_0 + V_{lim}} = -0'00\hat{3} \ln \frac{0 + 0'0327}{2 + 0'0327}$

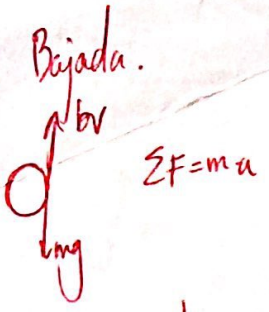
$t = 0'01375\,s$

$\frac{dy}{dt} = (V_0 + V_{lim})(e^{-t/\tau}) - V_{lim}$

$dy = \int (V_0 + V_{lim})(e^{-t/\tau})dt - \int V_{lim} dt$

$y = +\tau (V_0 + V_{lim})(1 - e^{-t/\tau}) - V_{lim} t$

② Bajada.



$\Sigma F = m\,a$

$m \cdot a = mg - bv$

$m \cdot \frac{dv}{dt} = -b(v - v_{lim})$

$\frac{dv}{v - v_{lim}} = \frac{-b}{m} t$

$\ln \frac{v_f - v_{lim}}{v_0 - v_{lim}} = -t/\tau$

$\frac{v_f - v_{lim}}{v_0 - v_{lim}} = e^{-t/\tau}$

$v_f = (v_0 - v_{lim})(e^{-t/\tau}) + v_{lim}$

$\frac{dy}{dt} = (v_0 - v_{lim})(e^{-t/\tau}) + v_{lim}$

$dy = \int (v_0 - v_{lim})(e^{-t/\tau})\,dt + \int v_{lim}\,dt$

$y = \tau(v_0 - v_{lim})(1 - e^{-t/\tau}) + v_{lim} t$

Alcanza v_{lim} en $t = 4\tau$

$y = 0'003(0 - 0'0327)(1 - e^{-4\tau/\tau}) + 0'0327 \cdot 4 \cdot 0'003$

$y = 0'000328$ m.

$y_{restante} = 100 + 0'00615 - 0'000328 = 100'00558$ m.

$v = \frac{y}{t}$; $t = \frac{y}{v} = \frac{100'00558}{0'0327} = 3058,28$ s.

$t_1 = 0'01375$ s

$t_2 = 3058'28 + 4\tau = 3058'29$ s

$t_{TOT} = t_1 + t_2 = 3058'307$ s

$t = 50'97$ min 57 s.

Problema 2. ⇒ Demostración movimiento cuadrático hacia abajo.

$\Sigma F = m \cdot a$

$m \cdot a = mg - cv^2$

$m \cdot \frac{dv}{dt} = mg - cv^2$

$\frac{dv}{dt} = \frac{mg}{m} - \frac{cv^2}{m}$

$\frac{dv}{dt} = g\left(1 - \frac{cv^2}{mg}\right)$

$\frac{dv}{dt} = g\left(1 - \frac{v^2}{Vlm^2}\right)$

$\frac{dv}{\left(1 - \frac{v^2}{Vlm^2}\right)} = g\,dt$

$\int \frac{dv}{\left(1 - \left[\frac{v}{Vlm}\right]^2\right)} = \int g\,dt$

cambio de variable:

$\frac{v}{Vlm} = u$

$dv = Vlm \cdot du$

$\int \frac{Vlm \cdot du}{1 - u^2} = \operatorname{arctgh}(u)\, Vlm$

$\operatorname{arctgh}\left(v(t)/Vlm\right) \cdot Vlm \Big]_0^{vt} = gt$

$\operatorname{arctgh}\left(v(t)/Vlm\right) = gt/Vlm$

$tgh\left(\frac{gt}{Vlm}\right) = \frac{v(t)}{Vlm}$

$$v(t) = Vlm \cdot tgh\left(gt/Vlm\right)$$

$\frac{dy}{dt} = v(t)$, $dy = v(t) \cdot dt$

Supongo $y_0 = 0$

$\int dy = \int_0^t Vlm \cdot tgh\left(gt/Vlm\right) dt$;

$y(t) = \int_0^t Vlm \cdot \frac{\operatorname{sen}h\left(gt/Vlm\right)}{\cos h\left(gt/Vlm\right)} \cdot dt$

$\begin{cases} gt/Vlm = u \\ dt = \frac{Vlm}{g}\, du \end{cases}$ $\int Vlm \cdot \frac{\operatorname{sen}h(u)}{\cos h(u)} \cdot \frac{Vlm}{g}\, du$

$y(t) = \frac{Vlm^2}{g} \cdot \int \frac{\operatorname{sen}h\, u}{\cos h\, u} \cdot du = \frac{Vlm^2}{g} \cdot \ln\left(\cos h\, u\right)$

$y(t) = \frac{Vlm^2}{g} \cdot \ln\left(\cos h\left(gt/Vlm\right)\right) \Big]_0^t$

$\frac{Vlm^2}{g} \left[\ln\left(\cos h\left(gt/Vlm\right)\right) - 0 \right]$

(*) Aprender.

Examen

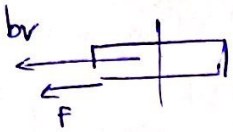
1/ FN LINEAL + F

$\mu = 0'2$

ρ (plomo)

β, γ

$V_0 = 10\ m/s$



$\sum F = m \cdot a$

$m \cdot a = -bv - F$

$m \cdot \frac{dv}{dt} = -bv - F$

$m \cdot \frac{dv}{dt} = -b\left(v + \frac{F}{b}\right)$

$$\int \frac{dv}{\left(v + \frac{F}{b}\right)} = \int \frac{-b}{m}\, dt$$

cambio variable: $F/b + v = u$, $dv = du$

$$\int \frac{du}{u} = \ln(u)$$

$$\Rightarrow \ln\left(F/b + v\right)\Big]_{V_0}^{Vt} = -t/\tau \; ; \quad \frac{F/b + vt}{F/b + V_0} = e^{-t/\tau} \; ;$$

$$\therefore \frac{F}{b} + vt = \left(\frac{F}{b} + V_0\right)\left(e^{-t/\tau}\right) \; ; \quad Vt = \left(\frac{F}{b} + V_0\right)\left(e^{-t/\tau}\right) - \frac{F}{b}$$

$$\left[Vt = \left(V_0 \cdot e^{-t/\tau}\right) + \frac{F}{b}\left(e^{-t/\tau} - 1\right)\right]$$

$F = \mu \cdot N \; ; \quad F = \mu \cdot m \cdot g \; ; \quad \rho = \frac{m}{V} \; ; \quad m = \rho \cdot V$

$F = \mu \cdot \rho \cdot V \cdot g \Rightarrow F = \mu \cdot \rho \cdot \frac{4}{3}\pi R^3 \cdot g$

$v(t) = \frac{dy}{dt} \; ; \quad dy = v(t) \cdot dt \; ; \quad y_0 = 0$

$$\frac{dy}{dt} = \left(\frac{F}{b} + V_0\right)\left(e^{-t/\tau}\right) - \frac{F}{b}$$

$$dy = \left(\frac{F}{b} + V_0\right)\int e^{-t/\tau}\, dt - \int \frac{F}{b}\, dt$$

$$dy = \left(\frac{F}{b} + V_0\right)(-\tau)\left(e^{-t/\tau} - 1\right) - \frac{F}{b}\, t$$

$$dy = \left(\frac{F}{b} + V_0\right)(\tau)\left(1 - e^{-t/\tau}\right) - F/b\, t$$

$\Sigma F = m \cdot a$

$m \cdot \frac{dv}{dt} = -bv$

$\int \frac{dv}{v} = \int \frac{-b}{m} dt$

$\ln v \Big]_{v_0}^{vt} = \frac{-b}{m} t$

$\ln \frac{vt}{v_0} = -t/\tau$

$\frac{vt}{v_0} = e^{-t/\tau}$

$vt = v_0 \cdot e^{-t/\tau}$

$\frac{dx}{dt} = v_0 \cdot e^{-t/\tau}$

$dx \Big]_{x_0}^{x} = v_0 \cdot \int e^{-t/\tau} dt$

$x - x_0 = v_0(-\tau) \int e^{-t/\tau} \cdot \frac{-1}{\tau} dt$

$x = x_0 - \tau v_0 (e^{-t/\tau} - 1)$

Horizontal

$\Sigma F = m \cdot a$

$m \cdot \frac{dv}{dt} = -bv - F$

$m \cdot \frac{dv}{dt} = -b(v + \frac{F}{b})$

$\int \frac{dv}{v + \frac{F}{b}} = \int \frac{-b}{m} dt$

$du = dv$

$u = v + \frac{F}{b}$

$\ln \frac{F}{b} + v \Big]_{v_0}^{vt} = -t/\tau$

$\frac{F/b + vt}{F/b + v_0} = e^{-t/\tau}$

$vt = (F/b + v_0)(e^{-t/\tau}) - \frac{F}{b}$

$\frac{dx}{dt} = (F/b + v_0)(e^{-t/\tau}) - F/b$

$dx \Big]_{x_0}^{x} = F/b + v_0 \int e^{-t/\tau} dt - \int F/b \, dt$

$x - x_0 = -\tau (F/b + v_0) \int e^{-t/\tau} \cdot \frac{-1}{\tau} dt - F/b \, t$

$x = x_0 - \tau (F/b + v_0)(e^{-t/\tau} - 1) - F/b \, t$

Horizontal + Froz

$\Sigma F = m \cdot a$

$m \cdot \frac{dv}{dt} = -mg - bv$

$m \cdot \frac{dv}{dt} = -b(v + \frac{mg}{b})$

$\int \frac{dv}{v + Vlm} = \int \frac{-b}{m} dt$

$\ln v + Vlm \Big]_{v_0}^{vt} = \frac{-b}{m} t$

$\frac{vt + Vlm}{v_0 + Vlm} = e^{-t/\tau}$

$vt = (v_0 + Vlm)(e^{-t/\tau}) - Vlm$

$\frac{dy}{dt} = (v_0 + Vlm)(e^{-t/\tau}) - Vlm$

$y = v_0 + Vlm \int e^{-t/\tau} dt - \int Vlm \, dt$

$y = (-\tau)(v_0 + Vlm)(e^{-t/\tau} - 1) - Vlm \, t$

Vertical

$\Sigma F = m \cdot a$

$m \cdot \frac{dv}{dt} = -cv^2$

$\frac{dv}{v^2} = \frac{-c}{m} dt$

$\int v^{-2} dv = \int \frac{-c}{m} dt$

$-\frac{1}{v} \Big]_{v_0}^{vt} = \frac{-c}{m} t$

$\frac{-1}{vt} + \frac{1}{v_0} = \frac{-c}{m} t$

$\frac{1}{vt} = \frac{1}{v_0} + \frac{c}{m} t$

$\frac{1}{vt} = \frac{1}{v_0}(1 + \frac{cv_0}{m} t)$

$\frac{1}{vt} = \frac{1}{v_0}(1 + t/\tau)$

$vt = \frac{v_0}{1 + t/\tau}$

$\frac{dx}{dt} = \frac{v_0}{1 + t/\tau}$

$x - x_0 = \tau v_0 \int \frac{\frac{1}{\tau}}{1 + t/\tau} dt$

$x = x_0 - \tau v_0 \ln(1 + t/\tau)$

Horizontal cuadratica

$\Sigma F = m \cdot a$

$m \cdot \frac{dv}{dt} = mg - cv^2$

$\frac{dv}{dt} = g - \frac{cv^2}{m}$

$dv = a(1 - \frac{cv^2}{mg})$

$\frac{v}{Vlm} = u$

$dv = Vlm \cdot du$

$\int \frac{Vlm \cdot du}{(1 - u^2)} = \int g dt$

$\frac{vt}{Vlm} = \tanh\left(\frac{gt}{Vlm}\right)$

$vt = Vlm \cdot \tanh\left(\frac{gt}{Vlm}\right)$

$dy = Vlm \left[\tanh\left(\frac{gt}{Vlm}\right) dt\right.$

Vertical cuadratica