TEMA 2: TRABAJO Y ENERGIA

ESTE AÑO

- Teorema de la energía (energía cinética / fuerzas vivas):

↱ multiplicamos por $\vec{v}$

$$m\frac{d\vec{v}}{dt} = \Sigma\vec{F} \Rightarrow m\frac{d\vec{v}}{dt}\cdot\vec{v} = \Sigma\vec{F}\cdot\vec{v}$$

↓ es la derivada

$$\frac{d}{dt}\Big(\underbrace{\tfrac{1}{2}mv^2}_{\text{energía cinética} \equiv T}\Big)$$

$\vec{F} = m\cdot\vec{a}$

$$\vec{F}\cdot\vec{v} = m\cdot\frac{d\vec{v}}{dt}\cdot\vec{v} = \frac{d}{dt}\Big(\underbrace{\tfrac{1}{2}m\cdot\vec{v}\,\vec{v}}_{T \equiv \text{energía cinética}}\Big)$$

$$\frac{dT}{dt} = \vec{F}\cdot\vec{v}$$

$$\int_{T_A}^{T_B} dT = \int_{t_A}^{t_B}\vec{F}\cdot\vec{v}\cdot dt \Rightarrow T_B - T_A = \underbrace{\int_{t_A}^{t_B}\vec{F}\,\vec{v}\cdot dt}_{\text{W realizado por } \vec{F}}$$

$$\int_{t_1}^{t_2}\frac{d}{dt}\Big(\tfrac{1}{2}mv^2\Big)dt = \int_{t_1}^{t_2}\vec{F}\cdot\vec{v}\cdot dt$$

me da igual que sean conservativas que no conservativas.

$$\tfrac{1}{2}mv^2\Big]_{V_1}^{V_2} = \underbrace{T_2 - T_1}_{\Delta E_c} = \underbrace{\int_{t_1}^{t_2}\vec{F}\cdot\vec{v}\cdot dt}_{W \equiv \text{trabajo}}$$

T^a de la energía: W realizado por todas las fuerzas que actuan sobre un cuerpo es igual a la variación de su energía cinética.

- Conservación de energía en movimiento rectilíneo. (en movimientos 1D.)

$$W = \int_{t_1}^{t_2}\vec{F}\cdot\vec{v}\cdot dt = \int_{t_1}^{t_2}\vec{F}(x)\frac{d\vec{x}}{\cancel{dt}}\cancel{dt} = \int_{x_1}^{x_2}\vec{F}(x)d\vec{x} = \int_{x_1}^{x_2} -\frac{dV(x)}{\cancel{dx}}\cancel{dx} = *$$

Exigimos $\vec{F} = F(x)$

$\vec{F} \equiv$ campo $\Rightarrow \vec{F}(x) = -\frac{dV(x)}{dx}$

↓

$V(x) \equiv$ energía potencial

↳ $F(x) = -\dfrac{dV(x)}{dx}$ ⤳ $V(x) \equiv$ energía potencial

$V(x) = -\int F(x)dx$

$$* = -V(x_2) + V(x_1) = T_2 - T_1$$

$$V(x_1) - V(x_2) = E_2 - E_1$$

≫ E es lo mismo que T (energía cinética)

$$\boxed{T_1 + V(x_1) = T_2 + V(x_2)}$$

(Potencial + cinética) en 1 = (Potencial + cinética) en 2.

* peso mg $\rightarrow V(y) = m\cdot g\cdot y + C$ (con $y = h$)

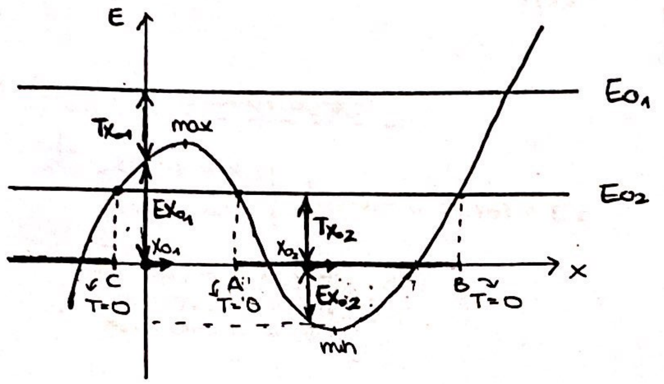
↳ se obtiene considerando un sistema de referencia.

AÑADIDO ESTE AÑO AL MOVIMIENTO 1D:

Características generales del movimiento 1D:

$$\frac{1}{2} m \cdot v_0^2 + V(x_0) = E_0 = \frac{1}{2} m \cdot v^2 + V(x)$$

* Partícula limitada a los puntos tal que $V(x) \leq E_0$:



EXPLICADO POR PAPÁ A LAPIZ EN LA SIG HOJA !

$V_{0_1}, x_{0_1} \Rightarrow E_{0_1}$

$V_{0_2}, x_{0_2} \Rightarrow E_{0_2}$

Tendremos movimientos acotados (oscilará) y movimientos no acotados

* Periodo de oscilación:

$$\frac{1}{2} m v^2 + V(x) = E_0$$

$$v^2 = \frac{(E_0 - V(x)) \cdot 2}{m} \Rightarrow v = \pm \sqrt{\frac{(E_0 - V(x)) \cdot 2}{m}} = \frac{dx}{dt}$$

velocidad (desde A hasta B) apunta a la dcha ($+$)

velocidad apunta a la izqd (desde B hasta A) ($-$)

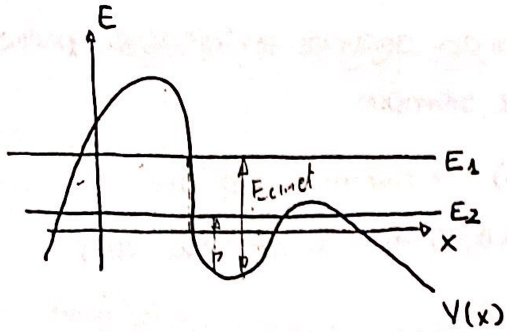
$$a \to b)\quad v = \frac{dx}{dt} = + \sqrt{\frac{2(E_0 - V(x))}{m}} \to \int_a^b \frac{dx}{\sqrt{\frac{2(E_0-V(x))}{m}}} = \int_0^{\tau_1} dt \to \boxed{\tau_1 = \int_a^b \frac{dx}{\sqrt{\frac{2(E_0-V(x))}{m}}}}$$

$$b \to a)\quad v = \frac{dx}{dt} = - \sqrt{\frac{2(E_0 - V(x))}{m}} \to -\int_b^a \frac{dx}{\sqrt{\frac{2(E_0-V(x))}{m}}} = \int_0^{\tau_2} dt \to \boxed{\tau_2 = \int_a^b \frac{dx}{\sqrt{\frac{2(E_0-V(x))}{m}}}}$$

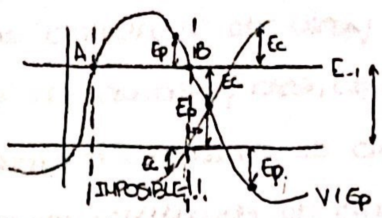
$$\tau = \text{periodo} = \tau_1 + \tau_2$$

$$\tau = 2 \int_a^b \frac{dx}{\sqrt{\frac{2(E_0-V(x))}{m}}}$$

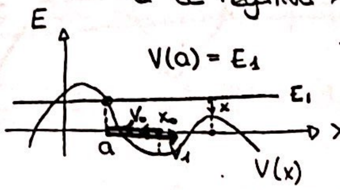
$$\boxed{\overset{Ec}{T} + \overset{Ep}{V} = \overset{Em}{E}}$$



La energía mecánica tiene que ser



La Ep(V) + Ec = Em por lo que la Ec va a ser la diferencia entre Em - Ep.
Podemos considerar la Ep negativa (ej: debajo de un pozo) pero no podemos considerar la Ec negativa xq la v no puede ser negativa, por ello hay una zona imposible y la Em va o de $(\infty, A]$ o de $[B, \infty)$



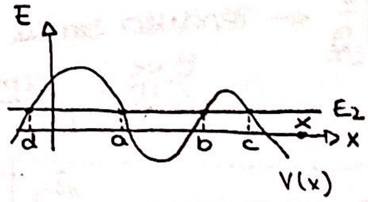
- **Región movimiento:**

① $\left.\begin{matrix} x(0) = x_0 \\ v(0) = v_0 \end{matrix}\right\} \frac{1}{2} m v_0^2 + V(x_0) = E_1$

$(-\infty, b] \; / \; [a, +\infty)$
poniendo x_0 al otro lado de E

② $\left.\begin{matrix} x(0) = x_0' \\ v(0) = v_0' \end{matrix}\right\} \frac{1}{2} m v_0'^2 + V(x_0') = E_2$

$$V(x) = E_2 \Rightarrow \begin{cases} \left.\begin{matrix} a \\ b \end{matrix}\right\} [a,b] \\ c \quad [c, +\infty) \\ d \quad (-\infty, a] \end{cases}$$



Periodo de oscilación: $\frac{1}{2} m v^2 + V(x) = E$

$$v^2 = \frac{2(E - V(x))}{m}$$

$$v = \pm \sqrt{\frac{2(E - V(x))}{m}} = \frac{dx}{dt}$$

velocidad + → tiempo que tarda de ir de a hasta b.
velocidad - → tiempo que tarda ir de b hasta a.

- $a \to b$: $\displaystyle\int_a^b \frac{dx}{\sqrt{\frac{2(E - V(x))}{m}}} = \int_0^{\tau_1} dt \longrightarrow \int_a^b \frac{dx}{\sqrt{\frac{2(E - V(x))}{m}}} = \int_0^{\tau/2} dt \Rightarrow \boxed{\tau = 2 \int_a^b \frac{dx}{\sqrt{\frac{2(E - V(x))}{m}}}}$

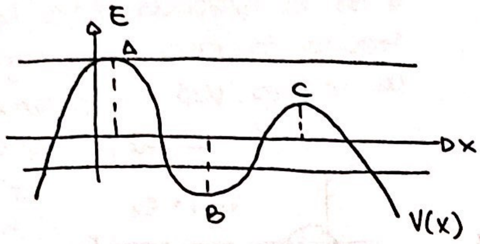
$\tau = \tau_1 + \tau_2 = 2\tau_1$
$\tau_1 = \tau/2$

- $b \to a$: $\displaystyle\int_b^a \frac{dx}{\sqrt{\frac{2(E - V(x))}{m}}} = \int_0^{\tau_2} dt$

Punto de equilibrio: max y min de la función potencial

- 'a' es punto de equilibrio si cuando dejamos en 'a' una partícula en reposo, esta permanece en 'a' por siempre.

  * Punto de equilibrio estable (B) → mínimos de $V(x)$
  * Punto de equilibrio inestable (A, C) → máximos de $V(x)$



Aproximación a pequeñas oscilaciones:

$$\left[T = 2\pi\sqrt{\frac{l}{g}} \leftarrow \text{Péndulo simple} \quad \theta \ll \quad \operatorname{sen}\theta \simeq \theta \right]$$

Recuerda!!

Serie de Taylor:

sea $x = a$ un punto equilibrio estable:

$$V(x) = V(a) + (x-a)V'(a) + \frac{1}{2}(x-a)^2 V''(a)\ldots$$

$$T + V(x) = Em$$

⇓

$$\frac{1}{2}mv(t)^2 + V(a) + (x-a)V'(a) + \frac{1}{2}(x-a)^2 V''(a) = Em$$

$\frac{dx}{dt}$ $\quad \frac{d}{dt}$

$$\frac{1}{\cancel{2}}m\cancel{2}v(t)\frac{d^2x(t)}{dt^2} + / \; + \; / \; + \frac{1}{\cancel{2}}\cancel{2}(x-a)\frac{dx}{dt}V''(a) = 0$$

$$m \cdot v(t) \cdot \frac{d^2x(t)}{dt} + (x-a)v(t)V''(a) = 0$$

1) $v(t) = 0$ → sist. no oscila
2) * Todo lo demás menos $v(t)$ es 0.

* $m\dfrac{d^2x(t)}{dt^2} + (x-a)V''(a) = 0$

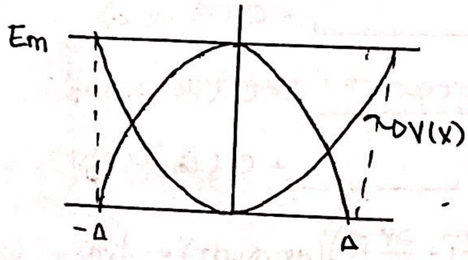
$$m\frac{d^2x(t)}{dt^2} = -(x-a)V''(a)$$

$$\Big\downarrow \quad x-a=u, \quad \frac{dx}{dt}=\frac{du}{dt}$$

$$m\frac{d^2u(t)}{dt^2} = -V''(a)\cdot u(t) = -K\,u(t) \rightarrow \text{Ley de Hoocke}$$

$\downarrow$

M.A.S (Mov. Armónico Simple)



$$\omega = \sqrt{\frac{K}{m}} \rightarrow \omega = \sqrt{\frac{V''(a)}{m}} \ (\mathrm{rad/s})$$

$$\downarrow$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{V''(a)}}$$

- <u>Conservación de la energía en campos fuerza 3D:</u>

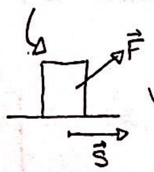
$1D \rightarrow F(x) = -\dfrac{dV(x)}{dx}$

$T+V=\text{cte}$

$$W_{A\to B} = \int_{t_A}^{t_B} \vec{F}(\vec{r})\cdot\vec{v}(t)\,dt = \int \vec{F}(\vec{r})\,\frac{d\vec{r}(t)}{dt}\,dt = \boxed{\underbrace{\int_{\vec{r}_A}^{\vec{r}_B} \vec{F}(\vec{r})\,d\vec{r}}_{\text{Integral de línea}} \Leftrightarrow \Delta T = T_B - T_A}$$

$$3D \rightarrow \vec{F}(\vec{r}) = -\operatorname{grad} V(x,y,z) = -\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)V(x,y,z)$$

$V(x,y,z) \equiv$ energía potencial



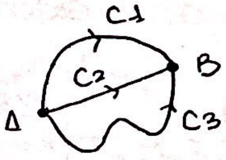
$W = \vec{F}\cdot\vec{S} = F\,s\cdot\cos\alpha$

$$W_{A\to B} = -\Delta V = V_A - V_B = T_B - T_A$$

$$\boxed{T_A + V_A = T_B + V_B}$$ Consev. Energía Mecánica

$F(x,y,z)$ es conservativo: $\longrightarrow$ $\underbrace{\exists V}_{\text{existe energía potencial}}(x,y,z) \equiv$ energía potencial $\Rightarrow W_{A\to B} = -\Delta V = V_B + V_A$

1) $\underbrace{\displaystyle\int_{A,C_1}^{B} \vec{F}\,d\vec{r}}_{\text{por el camino 1}} = \underbrace{\displaystyle\int_{A,C_2}^{B} \vec{F}\,d\vec{r}}_{\text{por el camino 2}}$



2) $\oint \vec{F}\,d\vec{r} = 0$

$$\oint \vec{F} d\vec{r} = \int_{A,C_1}^{B} \vec{F} d\vec{r} + \int_{B,C_1}^{A} \vec{F} d\vec{r} = 0$$

3) $\vec{F}$ es conservativa $\rightarrow \underbrace{\text{rot}}_{\text{rotacional}} \vec{F} = 0 = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ F_x & F_y & F_k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix} = \vec{i}\left(\frac{\partial^2 V}{\partial y \partial z} - \frac{\partial^2 V}{\partial z \partial y}\right) \ldots$ → producto escalar de las derivadas; 0

$$\vec{F}(x,y,z) = -\text{grad } V(x,y,z) = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$

- $F_x = -\frac{\partial V}{\partial x} \rightarrow V(x,y,z) = -\int F_x dx = \_\_\_\_\_\_ + C(\mathbb{R},y,z)$
- $F_y = -\frac{\partial V}{\partial y} \rightarrow V(x,y,z) = -\int F_y dy = \_\_\_\_\_\_ + C(\mathbb{R},x,z)$
- $F_z = -\frac{\partial V}{\partial z} \rightarrow V(x,y,z) = -\int F_z dz = \_\_\_\_\_\_ + C(\mathbb{R},x,y)$

* $W_{A\rightarrow B} = \int \vec{F} d\vec{r} = \int -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)(dx, dy, dz) = -\int_A^B dV = -V_B + V_A$

PARCIAL →

* EJERCICIOS:

1. $\vec{F}(x) = -\alpha x \vec{i}$

Sabemos que es conservativa porque depende solo de una constante

$$V(x) = -\int \vec{F}_x d\vec{x} = -\int -\alpha x = -\frac{1}{2}\alpha x^2 + C$$

FALTA APARTADO B)

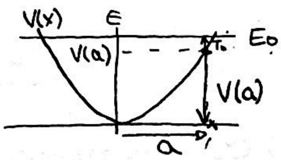
2. m

$x_0 = a$

$v_0 = \mu$

$\vec{F} = -m\omega^2 x \vec{i}$

[Aplico la conservación de la E potencial porque se trata de un campo conservativo]



$$V(x) = -\int F(x) dx = -\int -m\omega^2 x dx = \frac{m\omega^2 x^2}{2} + C$$

$$E_0 = T_0 + V_0 = \frac{1}{2}m\mu^2 + V(a) = \frac{1}{2}m\mu^2 + \frac{m\omega^2 a^2}{2} + C$$

$x_{max} \Rightarrow v = 0 \Rightarrow E_0 = V(x)$

despejamos x

$$\frac{1}{2}m\mu^2 + \frac{m\omega^2 a^2}{2} + \cancel{C} = \frac{m\omega^2 x^2}{2} + \cancel{C} \Rightarrow \boxed{x =}$$

$V_{max} \Rightarrow T_{max} \Rightarrow V_{min} \Rightarrow V(x) = 0$

$E_0 = T_{max} + V_{min} = \frac{1}{2} m v^2_{max} + \cancel{c} = \frac{1}{2} m \mu^2 + \frac{m\omega^2 a^2}{2} + \cancel{c}$

energía inicial

$$V_{max} = \sqrt{\mu^2 + \omega^2 a^2}$$

3. HACERLO

4. $\vec{F} = A(\operatorname{sen}\omega t\,\vec{i} + \cos\omega t\,\vec{j})$

$x_0 = (0,0)$

$v_0 = (0,0)$

$W = \frac{A^2}{m\omega^2}(1 - \cos\omega t)$ ← Supongamos que es cierta (luego lo comprobamos)

$\downarrow \frac{d}{dt}$

$Q = \frac{A^2}{m\omega} \operatorname{sen}\omega t$

1°) Aplico el teorema de las fuerzas vivas ($W = \Delta E_c$)

$\vec{F} = A(\operatorname{sen}\omega t\,\vec{i} + \cos\omega t\,\vec{j}) \Rightarrow \vec{a} = \frac{\vec{F}}{m} = \frac{A}{m}(\operatorname{sen}\omega t\,\vec{i} + \cos\omega t\,\vec{j})$

$v(t) = \int a(t) = \frac{A}{m}\left[\left(\frac{-\cos(\omega t)}{\omega} + c_x\right)\vec{i} + \left(\frac{\operatorname{sen}(\omega t)}{\omega} + c_y\right)\vec{j}\right]$ → se calculan con las condiciones iniciales

$v(0) = (0,0) = \frac{A}{m\omega}[(-1 + c_x), (0 + c_y)]$ ↗ $c_y = 0$ ↘ $c_x = \frac{A}{m\omega}$ $\quad c_x = 1$

$W = \Delta E_c = E_{c_f} - \cancel{E_{c_i}} = \frac{1}{2} m|\vec{v}|^2$

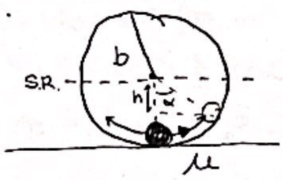
$\phantom{W = \Delta E_c = E_{c_f} -} 0$

Para calcular $\vec{v}$ sustituimos c_x y c_y en la ecuación de $v(t)$

$W = \cancel{m}\frac{A^2}{m^{\cancel{2}}\omega^2}(1 - \cos\omega t)$

FALTAN EJERCICIOS (1 día)

15.



* Principio conservación de la energía

$E_c = \frac{1}{2} m v^2$

$E_{p_0} = V_0 = -mg\cos\theta$

} $E_m = cte$

$$E_m = \frac{1}{2}m\mu^2 - mgb = \frac{1}{2}mv^2 - mgb\cos\theta$$

$$\frac{1}{2}mv^2 = \frac{1}{2}m\mu^2 - mgb + mgb\cos\theta$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{1}{2}\cancel{m}\mu^2 - \cancel{m}gb(1-\cos\theta)$$

$$v^2 = \mu^2 - 2gb(1-\cos\theta)$$

Dirección normal $\rightarrow \Sigma F_n = m \cdot a_n$

$$N - mg\cos\theta = m \cdot \frac{v^2}{b} = \frac{m}{b}[\mu^2 - 2gb(1-\cos\theta)]$$

$$N = \frac{m\mu^2}{b} - 2mg(1-\cos\theta) + mg\cos\theta$$

$$N = \frac{m\mu^2}{b} - mg(2-3\cos\theta)$$

$$N = \frac{m\mu^2}{b} + mg(3\cos\theta - 2)$$

$\mu = \sqrt{3gb} \rightarrow$ Para este caso $\begin{cases} v=0 & ① \\ N=0 & ② \end{cases}$ Para ver cual de los ángulos cumple las condiciones.

① $v=0$

$$v^2 = \mu^2 - 2gb(1-\cos\theta) = \overbrace{3gb}^{\mu^2} - 2gb + 2gb\cos\theta = gb(1+2\cos\theta) = 0$$

$$\theta = 120°$$

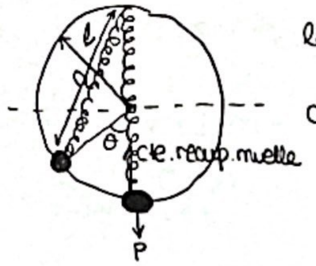
② $N=0$

$$N = \frac{m\overbrace{3g\cancel{b}}^{\mu^2}}{\cancel{b}} + mg(3\cos\theta - 2) = 3mg + 3mg\cos\theta - 2mg = mg + 3mg\cos\theta =$$

$$= mg(\underbrace{1+3\cos\theta})$$

$\theta = 109° \rightarrow$ La normal se hace 0 antes que la velocidad por ello la pelota solo llega a $\underline{\underline{109°}}$.

19.



longitud natural $= \frac{3a}{2}$

cte. recuperadora $= \alpha$
del muelle

y la N que no sabemos
para donde va pero
sabemos que $\Sigma F = 0$
pero como no realizan
trabajo nos olvidamos de ella

$$\underset{\text{elástica}}{V_e} = E_{pe} = \frac{1}{2}\overset{\alpha}{\overset{\shortparallel}{K}}(\overset{\Delta l}{\overset{\shortparallel}{\Delta x}})^2$$

$\Delta l =$ longitud en determinado instante (l) - longitud natural ($\frac{3a}{2}$)

$$l^2 = (a\,\text{sen}\theta)^2 + [a(1+\cos\theta)]^2 = a^2\text{sen}^2\theta + a^2\cos^2\theta + a^2 + 2a^2\cos\theta = 2a^2(1+\cos\theta)$$

$$l = a\sqrt{2(1+\cos\theta)} = a\sqrt{\frac{2(1+\cos\theta)2}{2}} = 2a\sqrt{\frac{1+\cos\theta}{2}} \overset{*}{=} 2a\cos(\theta/2)$$

↳ NOS LO DARÍA ÉL.

$$V_g = -mga\cos\theta$$

$$V_e = \frac{1}{2}\alpha\left[2a\cos(\theta/2) - \frac{3a}{2}\right]^2$$

$$V_T = \frac{\alpha a^2}{8}(4\cos(\theta/2) - 3)^2 - mga\cos\theta$$

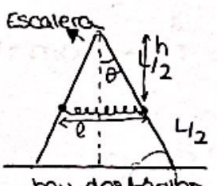
$$V' = \frac{3}{2}\alpha a^2\,\text{sen}(\theta/2) + (mga - \alpha a^2)\,\text{sen}\,\theta$$

$$V'' = \frac{3}{4}\alpha a^2\cos(\theta/2) + (mga - \alpha a^2)\cos\theta$$

$$V''(0) = \frac{3}{4}\alpha a^2 + (mga - \alpha a^2)$$

- si $\alpha = \frac{2mg}{a} \Rightarrow V'' > 0 \Rightarrow$ ESTABLE
- si $\alpha = \frac{5mg}{a} \Rightarrow V'' < 0 \Rightarrow$ INESTABLE

26. Escalera



hay dos barritas

longitud natural $= 0$

$$V_g = 2\left(mg\frac{L}{2}\cos\theta\right) = mgL\cos\theta$$

$$V_e = \frac{1}{2}K(\Delta x)^2 = \frac{1}{2}K\left(\cancel{2}\frac{L}{\cancel{2}}\text{sen}\theta\right)^2 = \frac{1}{2}KL^2\text{sen}^2\theta$$

$\Delta x = \Delta l = l - 0 = l$

$$V_T = mgL\cos\theta + \frac{1}{2}KL^2\text{sen}^2\theta$$

equilibrio

$$V'_T = -mgL\sin\theta + \frac{1}{\cancel{2}} KL^2 \cancel{2} \sin\theta\cos\theta = (-mgL + KL^2\cos\theta)\sin\theta) = 0$$

$$\sin\theta = 0 \qquad -mgL + KL^2\cos\theta = 0$$

$$\cos\theta = \frac{Mg}{KL}$$

Escaneado con CamScanner

* EJERCICIOS FISICA TEMA 2. EJ. 15 y 10 y 12 y 1

① dV?

a) $\vec{F} = -\alpha x \vec{i}$

$$V(x) = -\int \vec{F}_x\, dx = -\int \vec{F}_y\, dy = -\int \vec{F}_z\, dz$$

$$V(x) = -\int \vec{F}_x\, dx = -\int -\alpha x\, dx = +\frac{1}{2}\alpha x^2 + C$$

$$\int x\, dx = \frac{x^2}{2}$$

② m

$x = a$

$v = \mu$

$\vec{F} = -m\omega^2 x \vec{i}$

$$E = T + V$$

$$V(x) = -\int F(x)dx = -\int -m\omega^2 x\, dx = \frac{m\omega^2 x^2}{2} + C$$

$$E = T + V = \frac{1}{2}m\mu^2 + \frac{m\omega^2 x^2}{2} + C = \frac{1}{2}m\mu^2 + \frac{m\omega^2 a^2}{2} + C$$

- $x = max \Rightarrow \overset{\text{velocidad}}{v = 0} \Rightarrow E = V$

$$\frac{1}{2}m\mu^2 + \frac{m\omega^2 a^2}{\cancel{2}} + \cancel{C} = \frac{m\omega^2 x^2}{\cancel{2}} + \cancel{C}$$

$$m(\mu^2 + \omega^2 a^2) = m\omega^2 x^2$$

$$x^2 = \frac{\cancel{m}(\mu^2 + \omega^2 a^2)}{\cancel{m}\omega^2} = \frac{\mu^2 + \omega^2 a^2}{\omega^2}$$

$$x = \sqrt{\frac{\mu^2 + \omega^2 a^2}{\omega^2}}$$

$v = max \Rightarrow \overset{Ec}{T} = max \Rightarrow \overset{Ep}{V} = min \Rightarrow V(x) = 0 \Rightarrow E = T$

$$\frac{1}{\cancel{2}}m\mu^2 + \frac{m\omega^2 a^2}{\cancel{2}} + \cancel{C} = \frac{1}{\cancel{2}}mv^2 + \cancel{C}$$

$$v^2 = \mu^2 + \omega^2 a^2$$

$$v = \sqrt{\mu^2 + \omega^2 a^2}$$

(4) masa = m

$\vec{F} = A(\text{sen}\,\omega t\,\vec{i} + \cos\omega t\,\vec{j})$ Teorema de las Fuerzas Vivas $\Rightarrow W = \Delta Ec$

$x_0 = 0,0$

$v_0 = 0,0$

$$W = \Delta Ec = \Delta \frac{1}{2} m v^2$$

$\downarrow$

$?? \rightarrow v = \int a\, dt$

$$F = m \cdot a \Rightarrow a = \frac{F}{m} = \frac{A}{m}(\text{sen}\,\omega t\,\vec{i} + \cos\omega t\,\vec{j})$$

$$v(t) = \int a(t)\,dt = \int \frac{A}{m}(\text{sen}\,\omega t\,\vec{i} + \cos\omega t\,\vec{j})\,dt$$

$$v(t) = \frac{A}{m}\left[\left(-\frac{\cos\omega t}{\omega} + C_x\right)\vec{i} + \left(\frac{\text{sen}\,\omega t}{\omega} + C_y\right)\vec{j}\right]$$

$$\cancel{v(t) = \frac{A}{m\omega}\left[(-\cos\omega t + C_x\omega)\vec{i} + (\text{sen}\,\omega t + C_y\omega)\vec{j}\right]}$$

$$\cancel{v(0) = \frac{A}{m\omega}\left[(-1 + C_x\omega), (0, C_y\omega)\right]}$$

$$v(t) = \left(\frac{-A\cos\omega t}{m\omega} + C_x\right)\vec{i} + \left(\frac{A\,\text{sen}\,\omega t}{m\omega} + C_y\right)\vec{j}$$

$$v(0,0) = \left[\left(\frac{-A}{m\omega} + C_x\right), (0, C_y)\right]$$

$\rightarrow C_y = 0$

$\rightarrow C_x = +\dfrac{A}{m\omega}$ Condiciones inicio

$$W = \Delta Ec = Ec_f - \cancel{Ec_i}^{\,0} = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{2A^2}{m^2\omega^2}\right)(1 - \cos\omega t) = \frac{A^2}{m\omega^2}(1 - \cos\omega t) = W$$

$$V_x = \frac{-A\cos\omega t}{m\omega} + \frac{A}{m\omega} = \frac{A}{m\omega}(1 - \cos\omega t)$$

$$V_y = \frac{A\,\text{sen}\,\omega t}{m\omega} + 0 = \frac{A}{m\omega}(\text{sen}\,\omega t)$$

sen + cos = 1

$$V^2 = \frac{A^2}{m^2\omega^2}(1 + \cos^2\omega t - 2\cos\omega t + \text{sen}^2\omega t)$$

$$V^2 = \frac{A^2}{m^2\omega^2}(2 - 2\cos\omega t)$$

$$V^2 = \frac{2A^2}{m^2\omega^2}(1 - \cos\omega t)$$

$$Q = \frac{dW}{dt} = \frac{A^2}{m\omega}\,\text{sen}\,\omega t = Q$$ Potencia

③ $\vec{F} = 20\vec{i} - 30\vec{j} + 15\vec{k}$

A(2,7,-3) hasta B(5,-3,-6)

$$W_{F(A\to B)} = \int_A^B \vec{F} d\vec{r} = \int_A^B 20dx - \int_A^B 30dy + \int_A^B 15dz = 20x\Big]_2^5 - 30y\Big]_7^{-3} + 15z\Big]_{-3}^{-6} =$$

$+210-(-90)$

$$= -60 - 300 + 45 = \underline{-315 J}$$

⑤ $m = 2$

$$\left.\begin{array}{l} \vec{F} = (4/x^2 - 1) \\ x_0 = 4 \\ v_0 = 0 \end{array}\right\} \text{DATOS}$$

En los extremos del período $\underset{\text{velocidad}}{v=0} \Rightarrow \underset{Ec}{T} = 0 \Rightarrow \underset{Em}{E} = \underset{Ep}{V}$

$\int \frac{1}{x} = \ln x \quad \int \frac{1}{x^2} = \frac{1}{x} \quad \int \frac{1}{x^3} = \frac{1}{2x^2} \quad \int \frac{1}{x^4} = \frac{1}{3x^3}$

$$V(x) = -\int F dx = -\int \frac{4}{x^2} - 1 dx = -\int \frac{4}{x^2} dx + \int 1 dx = -\left(-\frac{4}{x}\right) + x + C$$

$$V(x) = \frac{4}{x} + x + C$$

$$\overbrace{\frac{1}{2} m \cancel{v^2}^{0} + \frac{4}{\underset{4}{x}} + \overset{4}{x} + \cancel{C}}^{E} = \overbrace{\frac{4}{x} + x + \cancel{C}}^{V}$$

$$5 = \frac{4}{x} + x \Rightarrow 5 = \frac{4 + x^2}{x} \Rightarrow 5x = 4 + x^2$$

$$x^2 - 5x + 4 = 0$$

$$x = \frac{5 \pm \sqrt{25-16}}{2} = \begin{cases} 4 \to \text{esta es la inicial} \\ 1 \end{cases}$$

Los puntos extremos son $\underline{4 \text{ y } 1}$.

$$T + V = E \to \frac{1}{2} mv^2 + V(x) = \frac{1}{\cancel{2}} \cancel{m v^2}^{0} + V(x)$$

$$\frac{1}{2} mv^2 + \frac{4}{x} + x + \cancel{C} = \frac{4}{x} + \overset{4}{x} + \cancel{C}$$

$$v^2 = \frac{(5 - 4/x - x)\cancel{2}}{m=2} = 5 - \frac{4}{x} - x \Rightarrow v = \frac{dx}{dt} = \sqrt{5 - \frac{4}{x} - x}$$

$$\frac{dx}{\sqrt{5-\frac{4}{x}-x}} = dt$$

$$\int_1^4 \frac{dx}{\sqrt{5-\frac{4}{x}-x}} = \int_0^{\tau/2} dt \Rightarrow \tau = 2\int_1^4 \frac{1}{\sqrt{5-\frac{4}{x}-x}}\,dx \ldots = 9'64seg = \tau$$

⑥ $$\vec{F}(x,y,z) = \overbrace{(y^2-2xyz^3)\vec{i}}^{F_x} + \overbrace{(3+2xy-x^2z^3)\vec{j}}^{F_y} + \overbrace{(6z^3-3x^2yz^2)\vec{k}}^{F_z}$$

$\vec{F}$ conservativo $\Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix} = 0$ F conservativo $\Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix} = 0$

$$\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\vec{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\vec{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\vec{k} = 0$$

$\vec{i}$: $-3x^2z^2 - (3x^2z^2) = 0$ ✓
$\vec{j}$: $-6xyz^2 - (-6xyz^2) = 0$ ✓ El campo es conservativo
$\vec{k}$: $2y - 2xz^3 - (2y - 2xz^3) = 0$ ✓

Si $\vec{F} = 0 \Rightarrow \vec{F}(x,y,z) = -\nabla V = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$ $\vec{F}(x,y,z) = -\nabla V$

$F_x = -\frac{\partial V}{\partial x} \Rightarrow V(x,y,z) = -\int y^2 - 2xyz^3\,dx = -y^2x + x^2yz^3 + C(\mathbb{R},y,z)$

??

$F_y = -\frac{\partial V}{\partial y} \Rightarrow V(x,y,z) = -\int 3+2xy-x^2z^3\,dy = -3y - xy^2 + x^2z^3y + C(\mathbb{R},x,z)$

$F_z = -\frac{\partial V}{\partial z} \Rightarrow V(x,y,z) = -\int 6z^3 - 3x^2yz^2\,dy = -6z^3y + \frac{3}{2}x^2y^2z^2 + C(\mathbb{R},x,y,$

$V(x,y,z) =$

??

Escaneado con CamScanner

⑦ $V = -xy^2 + x^2yz^3 - 3y - \frac{3}{2}z^4$

$A(2,-1,2)$ hasta $B(-1,3,-2)$ $\qquad W_F = -\Delta V$

$W_{F(A \to B)} = -(V_B - V_A) = V_A - V_B = -55 + 48 = \underline{-7J}$

⑧ $\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k} = 0$

¿a,b,c?

¿$V(x,y,z)$?

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix} = 0 \Rightarrow \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\vec{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\vec{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\vec{k} = 0$$

$\vec{i}: c + 1 = 0 \Rightarrow \underline{c = -1}$

$\vec{j}: a - 4 = 0 \Rightarrow \underline{a = 4}$

$\vec{k}: b - 2 = 0 \Rightarrow \underline{b = 2}$

$F = -\nabla V$

$F_x = -\frac{\partial V}{\partial x} \Rightarrow V(x,y,z) = -\int x + 2y + 4z \, dx = -\frac{x^2}{2} - 2yx - 4zx + C(\mathbb{R}, y, z)$

$F_y = -\frac{\partial V}{\partial y} \Rightarrow V(x,y,z) = -\int 2x - 3y - z \, dy = -2xy + \frac{3}{2}y^2 + zy + C(\mathbb{R}, x, z)$

$F_z = -\frac{\partial V}{\partial z} \Rightarrow V(x,y,z) = -\int 4x - y + 2z \, dz = -4xz + yz - z^2 + C(\mathbb{R}, x, y)$

$$\underline{V(x,y,z) = -\frac{x^2}{2} - 2xy - 4zx + \frac{3}{2}y^2 + zy - z^2}$$

9. $V = x^3 - y^3 + 2xy - y^2 + 4x$

$A(1,-1,2)$ hasta $B(2,3,-1)$

$W = -\Delta V = V_A - V_B = (1^3 - (-1)^3 + 2\cdot 1(-1) - (-1)^2 + 4) - (2^3 - 3^3 + 2\cdot 2\cdot 3 - 3^2 + 4\cdot 2) =$

$= 3 - (-8) = 11\,J$

10. $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$

dW?

a) $A(0,0,0)$ hasta $B(2,1,3)$

Parametrizamos $\rightarrow$ $x = \overset{\text{Punto A}}{0} + \overset{\text{Punto B}}{2}\lambda$; $y = 0 + \lambda$; $z = 0 + 3\lambda$

$\vec{r} = (2\lambda, \lambda, 3\lambda)$

$d\vec{r} = (2d\lambda, d\lambda, 3d\lambda)$

$\vec{F} = 3(2\lambda)^2 ,\ 2(2\lambda)(3\lambda) - \lambda ,\ 3\lambda = 12\lambda^2, 12\lambda^2 - \lambda, 3\lambda$

$V = -\int \vec{F} d\vec{r} = -\int 24\lambda^2 + 12\lambda^2 - \lambda + 9\lambda \, d\lambda = -\int 36\lambda^2 + 8\lambda \, d\lambda =$

$= -\frac{36\lambda^3}{3} - \frac{8\lambda^2}{2} \Big]_0^1 = -12 - 4 = -16J$

sustituimos A y B en x,y,z

$W = -\Delta V = -(-16) = 16J$

b) curva $\begin{cases} x = 2t \\ y = t \\ z = 4t^2 - t \end{cases}$ en $\begin{cases} t = 0 \\ t = 1 \end{cases}$ ← ya está parametrizado

$\vec{r} = (2t, t, 4t^2 - t)$

$d\vec{r} = (2dt, dt, (8t-1)dt)$

$\vec{F} = \overbrace{12t^2}^{3x^2}, \overbrace{16t^3 - 4t^2 - t}^{2xz - y}, \overbrace{4t^2 - t}^{z}$

$V = -\int \vec{F} d\vec{r} = -\int 24t^2 + 16t^3 - 4t^2 - t + \overset{32}{\cancel{28}}t^3 - 8t^2 - 4t^2 + t \, dt =$

$= -\int 44t^3 + 8t^2 \, dt = -\frac{44t^4}{4} - \frac{8t^3}{3} \Big]_0^1 = -11 - 2'6 = -13'6$

$W = -\Delta V = -(-13'6) = 13'6\,J$

(11) $\vec{F} = (x - 3y)\vec{i} + (y - 2x)\vec{j} = (2\cos t - 9\,\text{sen}\,t)\vec{i} + (3\,\text{sen}\,t - 4\cos t)\vec{j}$

$$\left.\begin{array}{l} x = 2\cos t \\ y = 3\,\text{sen}\,t \end{array}\right\} r = (2\cos t, 3\,\text{sen}\,t) \Rightarrow dr = (-2\,\text{sen}\,t\,dt, 3\cos t\,dt)$$

$$\oint \vec{F} dr = \oint -4\,\text{sen}\,t\cos t + 18\,\text{sen}^2 t + 9\,\text{sen}\,t\cos t - 12\cos^2 t\ dt =$$

$$= \oint_0^1 5\,\text{sen}\,t\cos t + 18\,\text{sen}^2 t - 12\cos^2 t\ dt =$$

(12) $V = \dfrac{x(x-3)^2}{3} = \dfrac{x(x^2 + 9 - 6x)}{3} = \dfrac{x^3 - 6x^2 + 9x}{3}$

$m = 8$

$V' = \dfrac{3x^2 - 12x + 9}{3} = x^2 - 4x + 3 \Rightarrow V' = 0 \rightarrow x^2 - 4x + 3$

$x = \begin{cases} 1 \\ 3 \end{cases}$ } 2 puntos de equilibrio

$V'' = 2x - 4$ → $V''(1) = -2 \mapsto$ máximo $\longrightarrow$ inestable

→ $V''(3) = 2 > 0 \rightarrow$ mínimo $\Rightarrow$ Estable!! $\underbrace{x = 3}$

$$m\frac{d^2x(t)}{dt^2} = -(x-a)V''(a)$$

$\downarrow$ $x - a = u(t)$, $\frac{dx}{dt} = \frac{du}{dt}$

$$m\frac{d^2u(t)}{dt^2} = -u(t)V''(a)$$

$$8\frac{d^2u(t)}{dt^2} = -\underbrace{2}_{K}u(t)$$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{V''(a)}{m}} = \sqrt{\frac{2}{8}} \rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{8}{2}} = \underbrace{4\pi}$$

(13) $F_1 = -2x\vec{i} - 2y\vec{j} - 2z\vec{k}$

$F_2 = y\vec{i} - x\vec{j}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix} = 0 \Rightarrow \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\vec{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\vec{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\vec{k}$$

a) $\vec{i} = 0 - 0 = 0$
$\vec{j} = 0$
$\vec{k} = 0$
} ES CONSERVATIVO

b) $\vec{i} = 0$
$\vec{j} = 0$
$\vec{k} = -1 + 1 = 0$
} ES CONSERVATIVO

(14) $m = 1$

$V = 6x(x-2)$

$V'(x) = 6(x-2) + 6x = 12x - 12$

$V'(x) = 0 \Rightarrow 12x - 12 = 0$

$x = 1 \rightarrow$ Punto de equilibrio

$V''(x) = 12 > 0 \rightarrow$ Mínimo = Estable

(16) $m = 4kg$ $\qquad W = \Delta E_c$

$\vec{F} = 4\vec{i} + 12t^2\vec{j}$

$V_0 = 2\vec{i} + \vec{j} + 2\vec{k}\ m/s^2$

$\vec{F}\cdot\vec{v} = (2\cdot 4) + (1\cdot 12t^2) + (0\cdot 2)$

Te lo pide el enunciado

$$\vec{W} = \int \vec{F} dr = \int_0^1 \vec{F}\vec{v}\, dt = \int_0^1 8 + 12t^2\, dt = 8t + \frac{12t^3}{3}\Big]_0^1 = 8t + 4t^3\Big]_0^1 = 12J$$

¿Normal? $\Rightarrow \Sigma F = N - P = m \cdot a_n \quad \Rightarrow a_N = \frac{v^2}{R}$

$$N - mg\,b\cos\alpha = m\,\frac{v^2}{R}$$

$$N - mg\cos\alpha = \frac{m}{b}\left(\mu^2 - 2gb(1-\cos\alpha)\right)$$

$$N = \frac{m\mu^2}{b} - 2gm(1-\cos\alpha) + mg\cos\alpha$$

$$N = \frac{m\mu^2}{b} - 2gm + 2gm\cos\alpha + mg\cos\alpha$$

$$N = \frac{m\mu^2}{b} + gm(-2 + 3\cos\alpha)$$

$$N = \frac{m\mu^2}{b} + gm(3\cos\alpha - 2)$$

Si $\mu = (3gb)^{1/2}$ ¿αmáx? El ángulo será maximo cuando $v=0$ o cuando $N=0$ (el enunciado te lo pide)

· Si $v=0$

$$\mu^2 - 2gb(1-\cos\alpha) = 0$$

$$3g\cancel{b} - 2g\cancel{b}(1-\cos\alpha) = 0 \Rightarrow 3 - 2 + 2\cos\alpha = 0$$

$$2\cos\alpha = -1$$

$$\cos\alpha = \frac{-1}{2} \Rightarrow \alpha = 2^\circ$$

· Si $N=0$

$$\frac{m\mu^2}{b} + gm(3\cos\alpha - 2) = 0$$

$$\frac{\cancel{m}3\cancel{g}\cancel{b}}{\cancel{b}} + \cancel{g}\cancel{m}(3\cos\alpha - 2) = 0 \Rightarrow 3 + 3\cos\alpha - 2 = 0$$

$$3\cos\alpha = -1$$

$$\cos\alpha = -\frac{1}{3} \Rightarrow \alpha = 1^\circ$$

La N se hace 0 antes que la v por lo que el ángulo es 1°.