Bioengineering and Aerospace Engineering Dept.
Advanced Flight Mechanics

Navigation

1   Session 1

The aim of the second assignment is to understand the role of the navigation function in the GNC loop. The navigation block is composed by: 1) sensors in charge of obtaining the measurements from the state of the system and 2) statistic filters to filter out to a certain extent the noise of the measurements. The aerospace vehicle under investigation is the same as in the first assignment, the RPAS Fulmar, and flying in the same conditions. So far, perfect navigation has been considered, that is, the evolution of the state of the aircraft is available at any instant of time (according to the dynamical model). In real world, this is never the case, and in this assignment, we will see the effect of including the navigation function. There are plenty of different sensors in an aircraft to provide state-related data. However, in this assignment we will only deal with gyroscopes. In addition, the statistic treatment of the measurements can be done using different methods, but we will focus on the Kalman filter. To begin with, built-in functions of Simulink will help us in designing the navigation block.

1.1   Linear Model in Simulink

In this assignment, we will make use of built-in block of Simulink. Therefore, the first step should be designing a Simulink model corresponding to the linear dynamical model of the RPAS developed in the first assignment, in case you have not done it before. Only system matrix A for both longitudinal and lateral motion will be needed because the excitation of the dynamical modes will be done modifying the initial conditions, as in the first assignment (instead of using the more realistic approach of using the action of the actuators).

1.2   Gyroscope

The only sensor to be considered in this assignment is a 3-axis gyroscope. It is, then, able to measure the angular velocity of the aircraft. A built-in model of a gyroscope in the Aerospace Blockset of Simulink is to be used. The block implements a model of the sensor noise, incorporated to the actual value of the angular velocity.
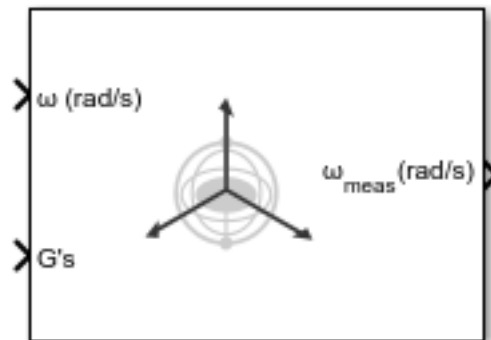
The detailed model of the gyroscope noise can be found in [1]. The detailed information on the block can be found in the Mathworks website [1].

1.3   Kalman Filter

The measurement provided by the gyroscope is filtered by a Control System Toolbox Simulink block that implements a Kalman Filter algorithm. The fundamentals of the Kalman Filter can be found in the slides of the course and in (for instance) [1, 2]. For a detailed description of the different inputs and outputs of the block,

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[1] https://www.mathworks.com/help/aeroblks/threeaxisgyroscope.html#mw_9410d74b-253c-4696-81bc-b0e10044cd50

you can check the mathworks documentation [2]. For an example on how the Kalman filter enters into a GNC complete loop to estimate the continuous state of a simple vehicle, you can find it out here [3]

1.4   Task

The study case for this assignment should be the excitation of the phugoid in the longitudinal dynamics and the dutch roll in lateral dynamics. A comparison should be made among the actual evolution of the state, the measurement produced by the gyroscope and the filtered signal after the Kalman filter block. A parametric assessment of the variation on the process and sensor noise covariances (Q and R) is also to be carried out.

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[2]https://mathworks.com/help/control/ref/kalmanfilter.html
[3]https://www.mathworks.com/help/control/ug/state-estimation-using-time-varying-kalman-filter.html

2   Session 2

The second session is devoted to code a Kalman filter and compare its performance to the built-in Simulink block. The code should return the estimate of the state vector $\hat{\mathbf{x}}$ given 5 inputs:

- Initial conditions $\hat{\mathbf{x}}_0, P_0^-$. Components related to angular rates (p, q, r) should be initialized with the first measure. Some random noise can be added to the rest.

- Gyroscope outputs $\mathbf{y}$.

- Process noise covariance Q.

- Sensor noise covariance R.

Additionally, the dynamics should be discretized as follows:

$$\dot{\mathbf{x}} = A\mathbf{x} \rightarrow \mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k$$

The required pseudocode is presented down below.

Algorithm 1 $\hat{\mathbf{x}} \leftarrow \text{KF}(\hat{\mathbf{x}}_0, P_0^-, \mathbf{y}, Q, R)$

1: **while** $k \leq N_{steps} - 1$ **do**
2:     $dt_k = t_{k+1} - t_k$;
3:     $\mathbf{x}_{k+1}^- = \Phi_k \hat{\mathbf{x}}_k^+$;
4:     $P_{k+1}^- = \Phi_k P_k^+ \Phi_k^T + Q$;
5:     $K = P_{k+1}^- C^T (R + C P_{k+1}^- C^T)^{-1}$;
6:     $\hat{\mathbf{x}}_{k+1} = \mathbf{x}_{k+1}^- + K(\mathbf{y}_{k+1} - C\mathbf{x}_{k+1}^-)$;
7:     $P_{k+1}^+ = (I - KC) P_{k+1}^-$;
8: **end while**
9: **return** $\hat{\mathbf{x}}$

2.1   Task

The study case for this session should be the application of the Kalmar filter to the measures provided by the gyroscope for the Phugoid and Dutch Roll. A comparison should be made among the measurement, the Simulink Kalman filter and the filter from this session. A parametric assessment of the variation of the process and sensor noise covariance (Q and R) should also be carried out.

References

[1] Robert M Rogers. *Applied mathematics in integrated navigation systems.* American Institute of Aeronautics and Astronautics, 2007.

[2] William E Wiesel. *Modern orbit determination*, volume 2. Aphelion Press Beavercreek, 2010.