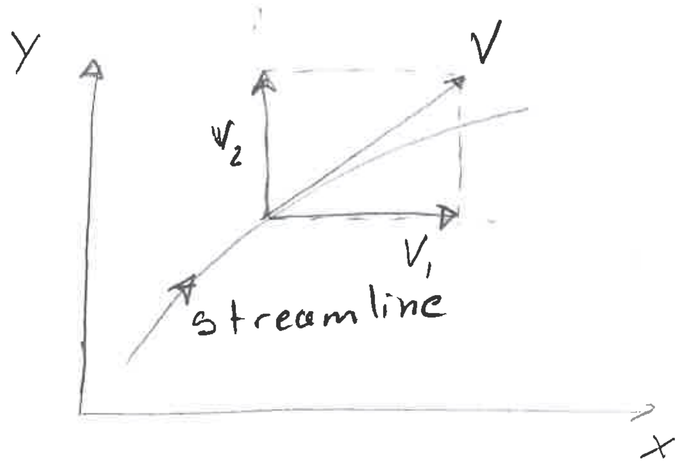
Fluid Flow (Potential theor

Laplace's equation also plays a basic
hydrodynamics, steady nonviscous flu
under physical conditions. Assuming
dimensional analysis, so that the velo
Vector (V) by which the motion of th
can be given depends only on two
X and Y (space variables)



Then we ... e
for V a
function

$V = V_1$

Giving a magnitude $|V|$ and direction
of velocity at each point $z = x + i$
V_1 and V_2 are components of the velo
in the X and Y directions. V is tan
to the path of the moving particles
streamline of the motion.

For a given flow exits an analyti[...]on

$F(z) = \phi(x,y) + i\psi(x,y)$, this is [...]

complex potential of the flow, such [...]

the streamline are given by $\psi(x,y)$ [...] t

and the velocity is given by

$$V = V_1 + iV_2 = \overline{F'(z)};$$

The bar denotes the complex conjug[...]

ψ is the stream function. The function [...]

called the velocity potential. The curve[...]stan[...]

are called equipotential lines. V is [...]ien[...]

of ϕ; by definition,

$$V_1 = \frac{\partial\phi}{\partial x} \qquad V_2 = \frac{\partial\phi}{\partial y}$$ This [...]

for the Cauchy-Riemann equation, [...]

analytic in a domain D if only if the [...]

derivative satisfy the two

$$\frac{\partial\psi}{\partial x} = -\frac{\partial\phi}{\partial y} \quad \text{and} \quad \frac{\partial\psi}{\partial y} = \frac{\partial\phi}{\partial x}$$

$$F'(z) = \frac{\partial\phi}{\partial x} + i\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial y} - i\frac{\partial\phi}{\partial y}$$

where $\frac{\partial\psi}{\partial x} = -\frac{\partial\phi}{\partial y}$

Example

The complex potential

$F(z) = z^2 = x^2 - y^2 + 2ixy$ describe

with Equipotential $\phi = x^2 - y^2 =$ const

Streamlines $\psi = 2xy =$ consta

the velocity vector is obtain as.

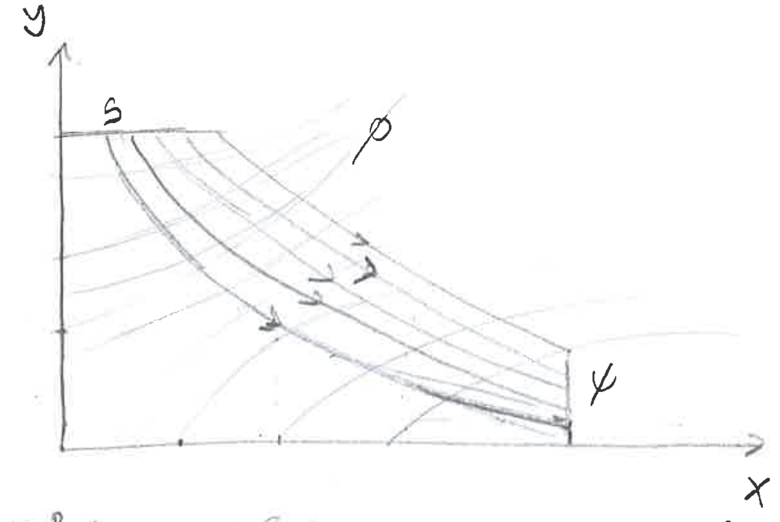
$\overline{F'(z)} = V$

$F'(z) = 2x + i2y$

$\overline{F'(z)} = 2x - i2y = V$

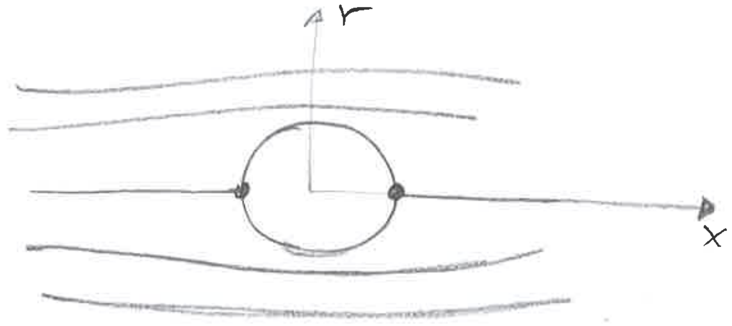
$V_1 = 2x$ and $V_2 = -2y$

The magnitude is $|V| = \sqrt{4x^2 + 4y^2} =$



$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0$ Continuity

$2 - 2 = 0$ satisfyes the cont equation



Special ...

$\psi(x,y) =$ ...

(1) $\left(r - \frac{1}{r}\right) =$ ...

(2) $\sin\theta = 0$

$r = \pm 1$ we fa... positive, the ...ine is the circle ...s $r = 1$ and x-ax... ...nd

Stagnation points

$V = 0$ for stagnation point

$$\left[F'(z) = 1 - \frac{1}{z^2} = 0 \qquad z = \pm 1\right]$$