Calculations for Additional lift

We already showed that the total lift coefficient of an arbitrary circulation distribution is proportional to the coefficient of $\sin\theta$ in the Fourier series describing the distribution.

Examining experimental results for many untwisted planforms and devised the approximate rule that the distribution of additional lift. that is, the lift associated with the chord distribution without twist, is nearly proportional at every point to the ordinate that lies halfway between the elliptical and actual chord distributions for the same total area and span. thus

$$L_a' = \frac{1}{2}\left[c + c_{SE}\sqrt{1 - \frac{y^2}{(2b)}}\right]\frac{L}{S}$$

C: is actual chord.

C_{SE}: chord at plane of symmetry for the elliptical planform

$$S = \int_{-b/2}^{b/2} c\, dy = \frac{\pi}{4} b \cdot c_{SE}$$

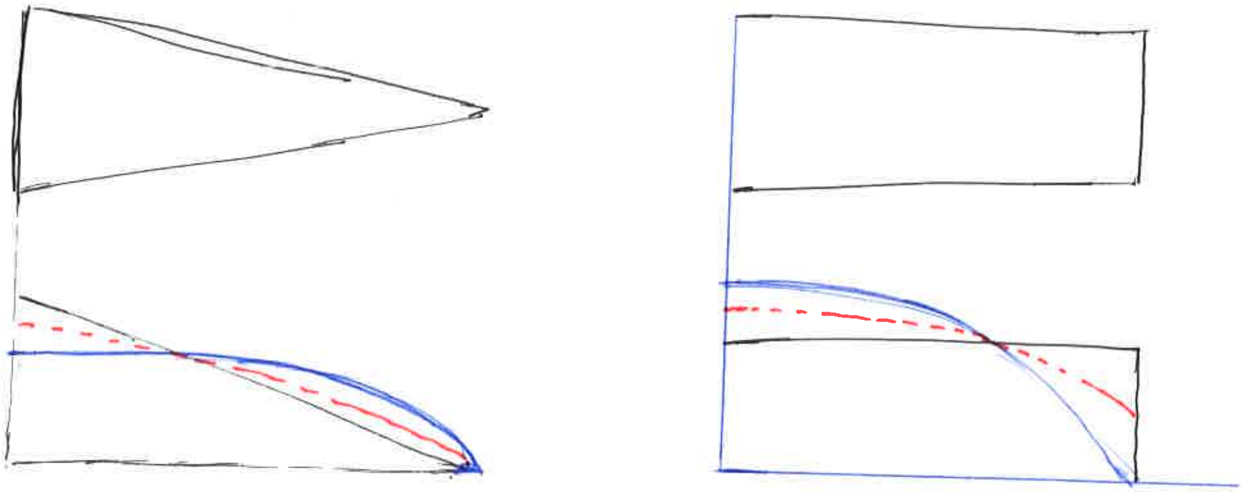
By the use of the relations

$$C'_{L_a} = \frac{L_a'}{\frac{q}{f_\infty} c . C_L} \quad \text{and} \quad \bar{C} = \frac{S}{b}$$

we find Schrenk's approximate relation

$$C'_{L_a} = \frac{1}{2}\left[1 + \frac{4}{\pi}\frac{\bar{C}}{C}\sqrt{1 - \left(\frac{y}{b/2}\right)^2}\,\right]$$

This relation shows clearly the effect of taper on the spanwise lift distribution



The red lines are drawn halfway between the actual chord and that for an ellipse of the same area and semispan. We can observe that the effect of taper is to increase the load in the outbord portion above that which occur if the additional lift were proportional to the chord.

Characteristic of Finite Wing

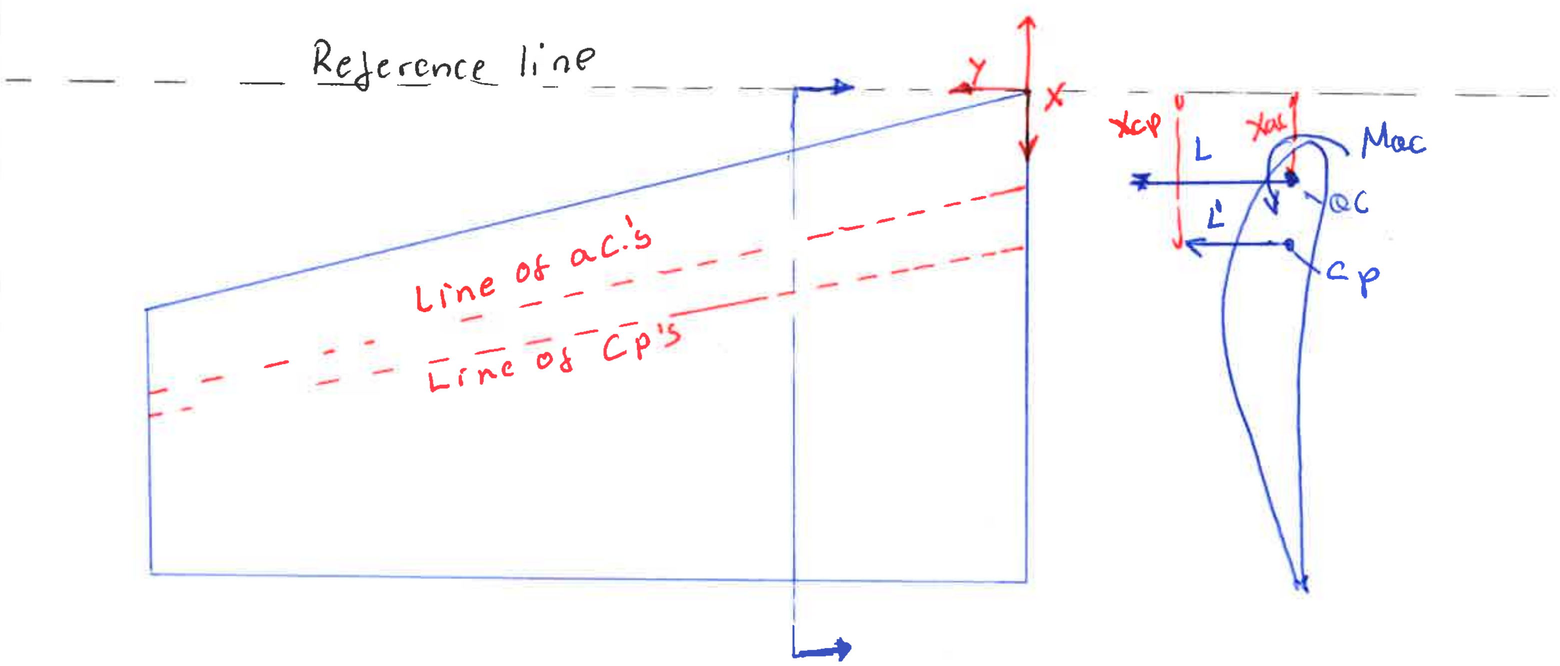
Center of pressure, aerodynemic center and moments about finite wing are calculated as the weighted mean averages of the section characteritics.

The fore and aft location of the center of pressure an important design parameter, will then be the weighted average of x_{cp}. the location for the sections. The distance from a reference line to the center of pressure line for the entire wing X_{cp} for a given angle of attack. is

$$X_{cp} = -\frac{M_{RL}}{L}$$

M_{RL} : The moment about the reference line given by

$$M_{RL} = -\int_{-b/2}^{b/2} L' x_{cp} \, dy = -\int_{-b/2}^{b/2} L' x_{ac} \, dy + \int_{-b/2}^{b/2} M'_{ac} \, dy$$



In this equation X_{ac} and M'_{ac} are function only of Y; since they are independent of the angle of attack, however, L' is a function of Y, therefore of the sectional absolute angle of attack, α_a. It follows that M_{RL} will be a function of α_{aw}, the absolute angle of attack of the wing and X_{cp} will be also be a function of α_{aw}.

$$X_{cp} = - \frac{\int_{-b/2}^{b/2} \left[-(C_{lb} + C'_{la} \cdot C_L) X_{ac} \, c + C_{mac} c^2 \right] dy}{\int_{-b/2}^{b/2} C_{la} \cdot C_L \cdot c \, dy}$$

$$X_{ac} = - \frac{\int_{-b/2}^{b/2} C_{la} \cdot C_b \, X_{ac} \cdot dy}{\int_{-b/2}^{b/2} C_{la} \cdot c \, dy}$$

From C_{lb} and C_{mac} terms must determine M_{AC}; the moment about the A.C., this follows from the definition of M_{AC} as independent of the angle of attack and, therefore of the additional lift.

Defining Δx_{ac} as the distance from the A.C. line to the section aerodynamic center (a.c.) and we define $C_{MAC} = \frac{M_{AC}}{q_{\infty} \bar{c} S}$ => $\bar{c} = \frac{S}{b}$ mean chord

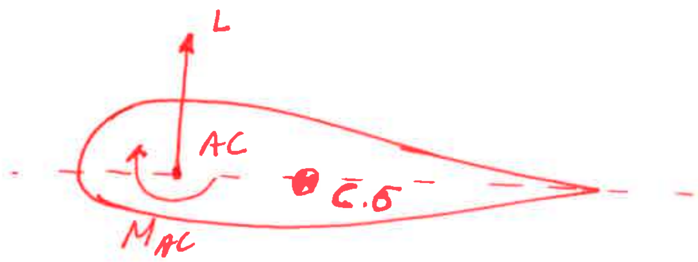
$$C_{MAC} = \int_{-b/2}^{b/2} \left[-C_{L_b} \cdot \frac{\Delta x_{ac} \cdot c}{\bar{c}^2} + C_{mac} \left(\frac{c}{\bar{c}} \right)^2 \right] d\left(\frac{y}{b}\right)$$

The integrand of this equation can also be interpreted as the result of the transfer of moments from the a.c. at a given (y) to the A.C., the centroid of the additional lift of the wing. so

$$M_{AC} = \int_{-b/2}^{b/2} \left(M'_{ac} - L'_b \, \Delta x_{ac} \right) d\left(\frac{y}{b}\right)$$

Stability and trim of wings.

A wing is termed statically stable if, as a result of a small angular disturbance from equilibrium in steady flight, an aerodynamice moment is generated tending to return the wing to equilibrium.



Wing cross section is shown with the load systems acting at aerodynamic center. Consider a wing as a rigid body, any unbalanced moments will cause it to rotate about its center of Gravity.

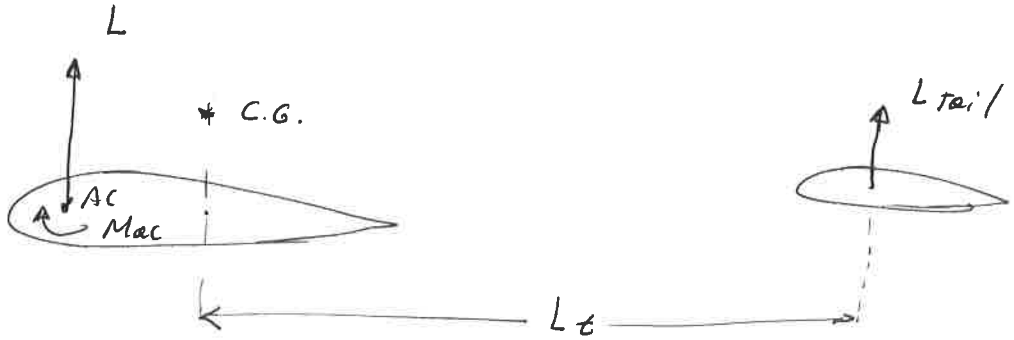
If C.G is behind the A.C. and M_{CG} is zero or balanced externally, the increment of lift $+\Delta L$, that results from an increment in the angle of attack will cause a moment $+\Delta M_{CG}$.

Conversely, if the angle of attack decreases, the resulting ΔL and M_{CG} will both be negative.

In either case $dM_{CG}/dL > 0$; so, since the moment generated is in the direction to increase the deviation from equilibrium, this inequality identifies the configuration of the figure unstable.

therefore the wing is stable if $X_{AC} > X_{CG}$ so that $dM_{CG}/dL < 0$.

Most configuration, the C.G. lies behind the A.C. and stability is generally achieved by placing horizontal stabilizer behind wing.



It is seen that the tail contributes a stabilizing moment when the wing-tail configuration is disturbed from equilibrium, and by a proper adjusment of the tail area and tail length ~~Lt~~, this stabilizing moment can be made easily to outweigh the destabilizing effect of wing.

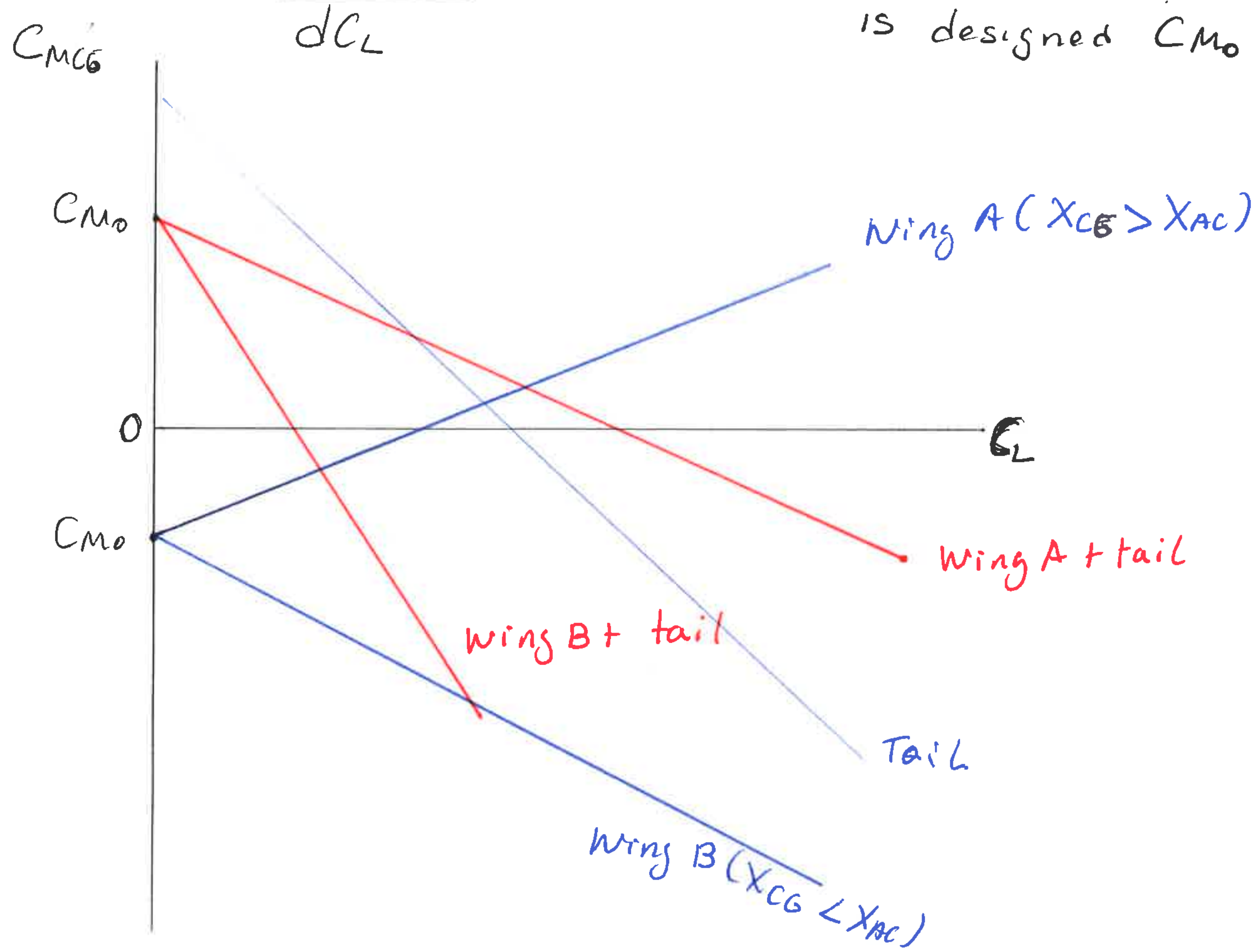
The airplane must be trimmed, meaning that the net moment acting must vanish. the airplane is stable if $dM_{CG}/dL < 0$; if trim is also to be achieved in steady flight ($L>0$), it is necessary that $M_{CG}=0$ at $L=L_{Trim}$. Thus, to satisfy both conditions, it is required that $M_{CG}>0$ at $L<L_{Trim}$. In terms of coefficients the two conditions trim and stability, for steady equilibrium flight are

$$C_{MCG}>0 \quad C_L=0$$

$$\frac{dC_{MCG}}{dC_L}<0$$

These conditions are shown in below figure. $C_L=0$, the C_{MCG} ($=C_{MAC}$) is designed C_{Mo}

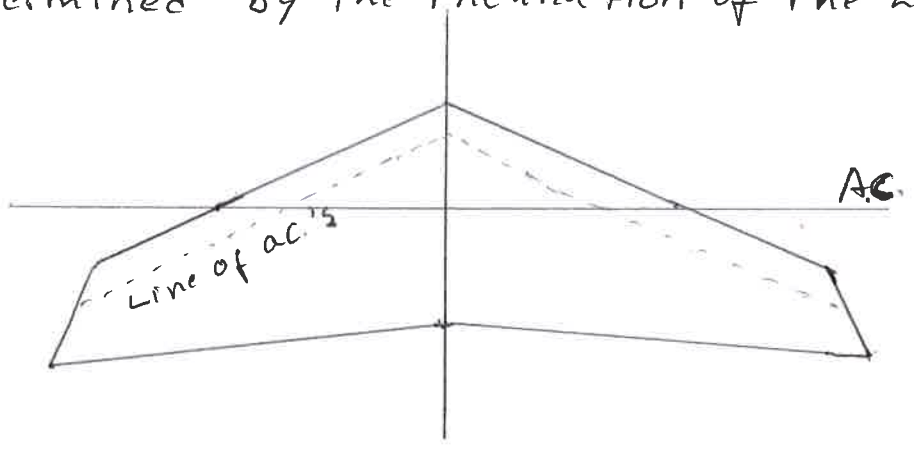
C_{Mo} ($C_{MCG}=C_{MAC}$)



The above figure shows schematically the dependence of $C_{MC\bar{O}}$ on C_L for the wings with the C.G. ahead of and behind the A.C, for tail (behind A.C) and for the wing-tail combination the wing A alone trims but is unstable if the C.G. is behind the A.C. and the wing B alone is stable but does not trim if the C.G. is ahead of A.C.

For both of these wings, stability and trim are achieved by adding the tail. For a conventional wing (Wing B) with positive camber, $C_{M_0} < 0$ so that the tail is required for steady equilibrium flight.

The contribution of the basic lift to C_{M_0} may be either positive or negative, depending on the twist and sweep of the wing. Consider the swepback wing. The direction of sweep of a wing is determined by the inclination of the Line of a.c.



Presume that the wing is set at $C_L = 0$ so that the lift acting at any section is basic lift, then If the wing is washed out at the tips, the lift acting on the outboard sections will be down, whereas the lift acting on the inboard section will be up. This consequence of the fact that the integral of the basic lift is zero. the basic lift thus distributed will cause a positive moment about the aerodynamic center.

By the same argument, it can be shown that a combination of sweepforward and washin at tips will also result in a positive contibution to the C_{MAC} by the basic lift.

In conclusions, the C_{M_0} of a wing may be made positive by reflexing the Trading edges of the wing sections or providing the proper amount of twist and sweep.

the Combination of sweepback and washout is useful in flying wing design. the sweepback moves the aerodynamic center of the wing rearward. thereby facilitating the stability condition that requires the C.G. lie infront of the aerodynamic center. the sweepback and washout obtains a positive C_{M_0} which is a necessary condition for Trim.