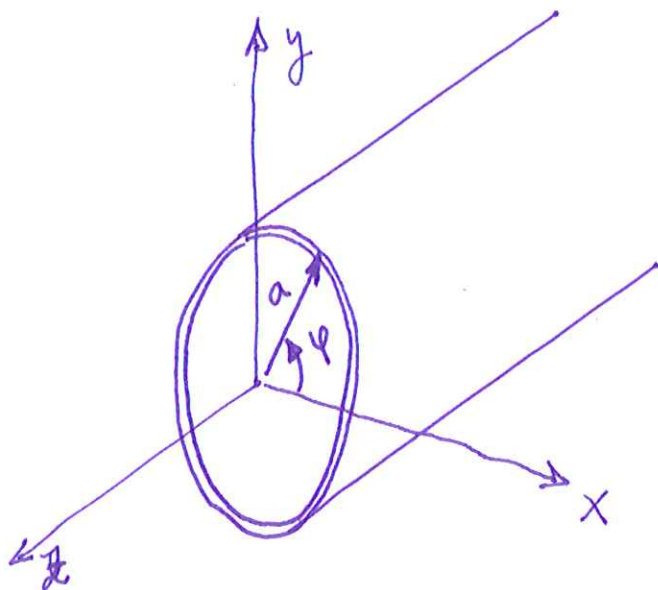


Guías Conductoras Sección Circular.

①



De nuevo, de la ec. de las Ondas EM en medios materiales:

$$\nabla^2 \vec{E} + \omega^2 \epsilon \mu \vec{E} + j\omega \sigma \mu \vec{E} = 0$$

En el interior de la guía $\sigma = 0$, quedando

$$\nabla^2 \vec{E} + \omega^2 \epsilon \mu \vec{E} = 0$$

La solución a esa ec. es una onda que se propaga en $+z$

$$\vec{E}(r, \varphi, z) = E(r, \varphi) \cdot e^{-j\beta z}$$

Sustituyendo en la ec. y teniendo en cuenta que el Laplaciano de una función en coord. cilíndricas es:

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

La ecuación queda:

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r} \frac{\partial E}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E}{\partial \varphi^2} + \beta^2 E + \omega^2 \epsilon \mu E = 0$$

(2)

Con el Laplaciano podemos considerar: $\nabla^2 \vec{E} = (\nabla_\rho^2 + \nabla_z^2) \vec{E}$

Sustituyendo en la ec llegamos a que:

$$E_\rho = \frac{-j}{\underbrace{(\omega^2 \mu \epsilon - \beta^2)}_{K_c^2}} \left[\beta \frac{\partial E_z}{\partial \rho} + \frac{\omega \mu}{\rho} \frac{\partial H_z}{\partial \varphi} \right]$$

$$E_\varphi = \frac{-j}{K_c^2} \left[-\frac{\beta}{\rho} \frac{\partial E_z}{\partial \varphi} + \omega \mu \frac{\partial H_z}{\partial \rho} \right]$$

$$H_\rho = \frac{j}{K_c^2} \left[\frac{\omega \epsilon}{\rho} \frac{\partial E_z}{\partial \varphi} - \beta \frac{\partial H_z}{\partial \rho} \right]$$

$$H_\varphi = \frac{-j}{K_c^2} \left[\omega \epsilon \frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho} \frac{\partial H_z}{\partial \varphi} \right]$$

Lo que nos lleva a que: Calculando E_z y H_z obtenemos el resto de los componentes de campo eléctrico y magnético.
Jugando con estas expresiones llegamos a:

$$\nabla^2 E_z + (\omega^2 \mu \epsilon - \beta^2) E_z = 0$$

En cuya resolución empleamos el método de separación de variables

$$E_z(\rho, \varphi) = R(\rho) \cdot \Phi(\varphi)$$

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \varphi^2} + \underbrace{(w^2 \mu \epsilon - \beta^2)}_{K_c^2} E_z = 0 \quad (3)$$

Substituyendo $E_z = R(\rho) \cdot \Phi(\varphi)$ y operando, se tiene:

$$\underbrace{\rho^2 \frac{1}{R} \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + \rho^2 K_c^2}_{\text{Solo depende de } \rho} = \underbrace{- \frac{1}{\Phi} \frac{d^2 \Phi}{d\varphi^2}}_{\text{Solo depende de } \varphi}$$

La única forma de cumplirse esto es que ambos lados sean iguales a una constante que llamaremos m^2

$$\begin{cases} \rho^2 \frac{1}{R} \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + \rho^2 K_c^2 = m^2 \\ -\frac{1}{\Phi} \frac{d^2 \Phi}{d\varphi^2} = m^2 \end{cases}$$

Las ecuaciones a resolver son entonces:

$$\begin{cases} \left[\frac{d^2 R}{d\rho^2} + \frac{1}{\rho} \frac{dR}{d\rho} + \left(K_c^2 - \frac{m^2}{\rho^2} \right) R = 0 \right] \Rightarrow \text{Soluciones F. Bessel.} \\ \left[\frac{d^2 \Phi}{d\varphi^2} + m^2 \Phi = 0 \right] \rightarrow \text{Soluciones tipo "sen" "cos"} \end{cases}$$

$$R = A J_m(K_c \cdot \rho) + B N_m(K_c \cdot \rho)$$

$$\Phi = C \cos m\varphi + D \sin m\varphi$$

J_m : funciones de Bessel (1ª especie)

N_m : funciones de Neumann (Bessel de 2ª especie)

Modos TM

$$E_z(\rho, \varphi) = R(\rho) \cdot \Phi(\varphi) = \left[A J_m(k_c \rho) + B N_m(k_c \rho) \right] \left[C \cos m\varphi + D \sin m\varphi \right]$$

Aplicamos las condiciones de contorno:

* $E_z(\rho=a) = 0 \Rightarrow J_m(k_c \rho) = 0 \Rightarrow$ Ceros de la F. Bessel.
 $X_{m,n}$

* El campo ha de ser finito en todos los puntos $\Rightarrow B=0$ pues las F. Neumann son descont. en $\rho=0$.

* Por simplificación tomaremos $D=0$ (Aunque algunos libros prefieren no hacerlo (Balmain))

Quedando:

$$E_z(\rho, \varphi) = \overset{A \cdot C}{\textcircled{A}} J_m(k_c \rho) \cdot \cos m\varphi$$

La condición que nos lleva a los ceros de la función de Bessel implica:

$$k_c \cdot a = X_{m,n}$$

siendo a el radio de la guía.

$$\sqrt{\omega^2 \mu \epsilon} \cdot a = X_{m,n}$$

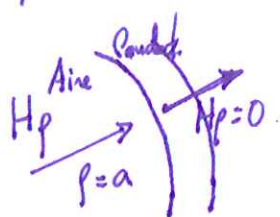
$$f_{c_{m,n}} = \frac{X_{m,n}}{2\pi a \sqrt{\mu \epsilon}} \Rightarrow$$

Frecuencia de corte del modo $TM_{m,n}$

Módos TE

$$H_z(r, \varphi) = B J_m(k_c r) \cdot \cos m \varphi$$

Dando una de las condiciones de contorno, obligaba a que la componente H_r fuera cero en $r=a$



$$H_r(r=a) = 0 \Rightarrow \left. \frac{\partial H_z}{\partial r} \right|_{r=a} = 0$$

Y a que

$$H_r = \frac{j}{k_c^2} \left[\frac{\omega \epsilon}{r} \frac{\partial E_z}{\partial \varphi} - \beta \frac{\partial H_z}{\partial r} \right]$$

Modo TE ($E_z=0$)

El hecho de que $\left. \frac{\partial H_z}{\partial r} \right|_{r=a} = 0$ lleva a considerar como solución,

los "ceros" de la "derivada de la función de Bessel"

$$J'_m(k_c r) = 0 \Rightarrow$$

$$X'_{m,n}$$

ceros de la derivada de la función de Bessel de orden m

Con lo que:

$$k_c r = X'_{m,n}$$

$$f_{c_{m,n}} = \frac{X'_{m,n}}{2\pi a \sqrt{\mu \epsilon}}$$

Modes TE ($H_z=0$) Balanis

$$E_\rho = -A_{mn} \frac{m}{\epsilon \rho} J_m(\beta_\rho \rho) [-C_2 \sin m\phi + D_2 \cos m\phi] e^{-j\beta_z z}$$

$$E_\phi = A_{mn} \frac{\beta_\rho}{\epsilon} J'_m(\beta_\rho \rho) [C_2 \cos m\phi + D_2 \sin m\phi] e^{-j\beta_z z}$$

$$E_z = 0$$

$$H_\rho = -A_{mn} \frac{\beta_\rho \beta_z}{\omega \mu \epsilon} J'_m(\beta_\rho \rho) [C_2 \cos m\phi + D_2 \sin m\phi] e^{-j\beta_z z}$$

$$H_\phi = -A_{mn} \frac{m\beta_z}{\omega \mu \epsilon} \frac{1}{\rho} J_m(\beta_\rho \rho) [-C_2 \sin m\phi + D_2 \cos m\phi] e^{-j\beta_z z}$$

$$H_z = -j A_{mn} \frac{\beta_\rho^2}{\omega \mu \epsilon} J_m(\beta_\rho \rho) [C_2 \cos m\phi + D_2 \sin m\phi] e^{-j\beta_z z}$$

Modes TM ($H_z = 0$) Balanis.

$$E_z = -j B_{mn} \frac{\beta_p^2}{\omega \mu \epsilon} J_m(\beta_p \rho) [C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$$

$$E_\rho = -j B_{mn} \frac{\beta_p \beta_z}{\omega \mu \epsilon} J'_m(\beta_p \rho) [C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$$

$$E_\phi = - B_{mn} \frac{m \beta_z}{\omega \mu \epsilon \rho} J_m(\beta_p \rho) [-C_2 \sin(m\phi) + D_2 \cos(m\phi)] e^{-j\beta_z z}$$

onde $\beta_p = \frac{x_{mn}}{a}$

$$\beta_z = \sqrt{\omega^2 \mu \epsilon - \frac{x_{mn}^2}{a^2}} = \underbrace{\beta}_{\text{end kawas } \beta} \sqrt{1 - \left(\frac{\beta_c}{\beta}\right)^2}$$

$\sqrt{\omega^2 \mu \epsilon}$

$$H_\rho = B_{mn} \frac{m}{\mu} \frac{1}{\rho} J_m(\beta_p \rho) [-C_2 \sin(m\phi) + D_2 \cos(m\phi)] e^{-j\beta_z z}$$

$$H_\phi = - B_{mn} \frac{\beta_p}{\mu} J'_m(\beta_p \rho) [C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$$

$$H_z = 0$$

REPRESENTACION GRAFICA DE LAS FUNCIONES DE BESSEL

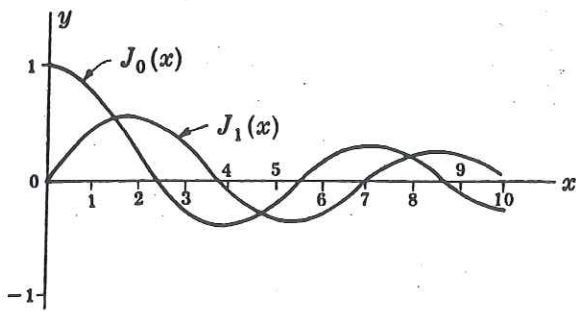


Fig. 24-1

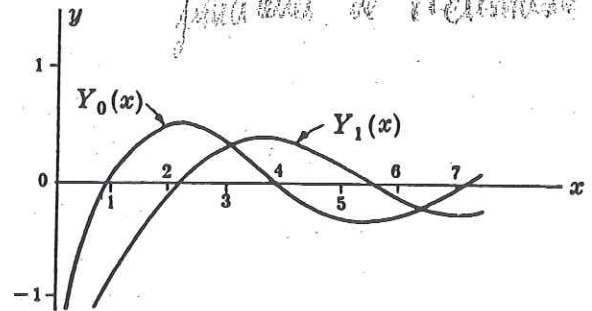


Fig. 24-2

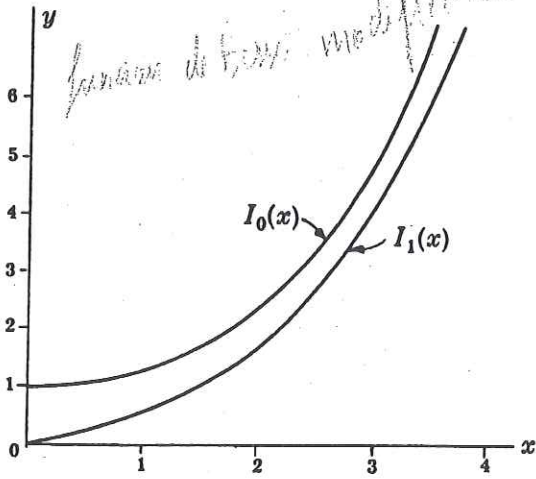


Fig. 24-3

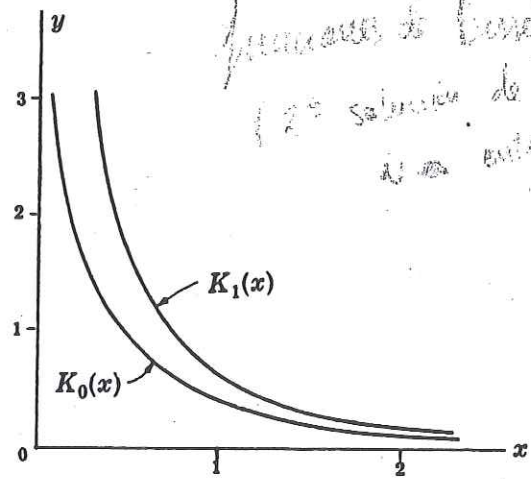


Fig. 24-4

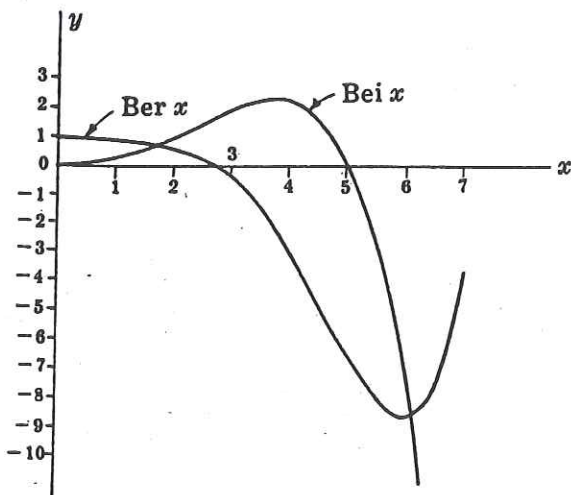


Fig. 24-5

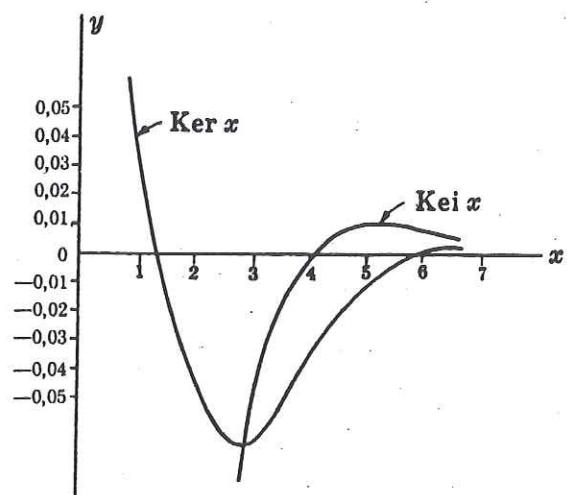


Fig. 24-6

TABLE 9-1

Zeros χ'_{mn} of derivative $J'_m(\chi'_{mn}) = 0$ ($n = 1, 2, 3, \dots$) of the Bessel function $J_m(x)$

	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$
$n = 1$	3.8318	1.8412	3.0542	4.2012	5.3175	6.4155	7.5013	8.5777	9.6474	10.7114	11.7708	12.8256
$n = 2$	7.0156	5.3315	6.7062	8.0153	9.2824	10.5199	11.7349	12.9324	14.1155	15.2867	16.4479	17.6007
$n = 3$	10.1735	8.5363	9.9695	11.3459	12.6819	13.9872	15.2682	16.5294	17.7740	19.0046	20.2230	21.4307
$n = 4$	13.3237	11.7060	13.1704	14.5859	15.9641	17.3129	18.6375	19.9419	21.2291	22.5014	23.7607	25.0087
$n = 5$	16.4706	14.8636	16.3475	17.7888	19.1960	20.5755	21.9317	23.2681	24.5872	25.8913	27.1820	28.4607

TABLE 9-2

Zeros χ_{mn} of $J_m(\chi_{mn}) = 0$ ($n = 1, 2, 3, \dots$) of Bessel function $J_m(x)$

	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$
$n = 1$	2.4049	3.8318	5.1357	6.3802	7.5884	8.7715	9.9361	11.0864	12.2251	13.3543	14.4755	15.5898
$n = 2$	5.5201	7.0156	8.4173	9.7610	11.0647	12.3386	13.5893	14.8213	16.0378	17.2412	18.4335	19.6160
$n = 3$	8.6537	10.1735	11.6199	13.0152	14.3726	15.7002	17.0038	18.2876	19.5545	20.8071	22.0470	23.2759
$n = 4$	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801	20.3208	21.6415	22.9452	24.2339	25.5095	26.7733
$n = 5$	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178	23.5861	24.9349	26.2668	27.5838	28.8874	30.1791

Atenuación (por pérdidas en conductores)

Modos TE

$$\alpha_{c_{TE_{m,n}}} = \frac{R_s}{a \cdot \eta \cdot \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left[\left(\frac{f_c}{f}\right)^2 + \frac{m^2}{\left(\chi'_{m,n}\right)^2 - m^2} \right] \frac{N_p}{m}$$

Modos TM

$$\alpha_{c_{TM_{m,n}}} = \frac{R_s}{a \cdot \eta \cdot \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \frac{N_p}{m}$$

Modos TE_{0,m}

Ventajas:

Observar cómo su atenuación disminuye con las altas frecuencias.
Se emplean en transporte de señal de alta frecuencia.

Atenuación guía rectangular $\approx 120 \text{ dB/Km.}$ (WR90 de lu)

Atenuación fibra óptica $\approx 0.5 \text{ dB/Km}$ (1.3 μm)

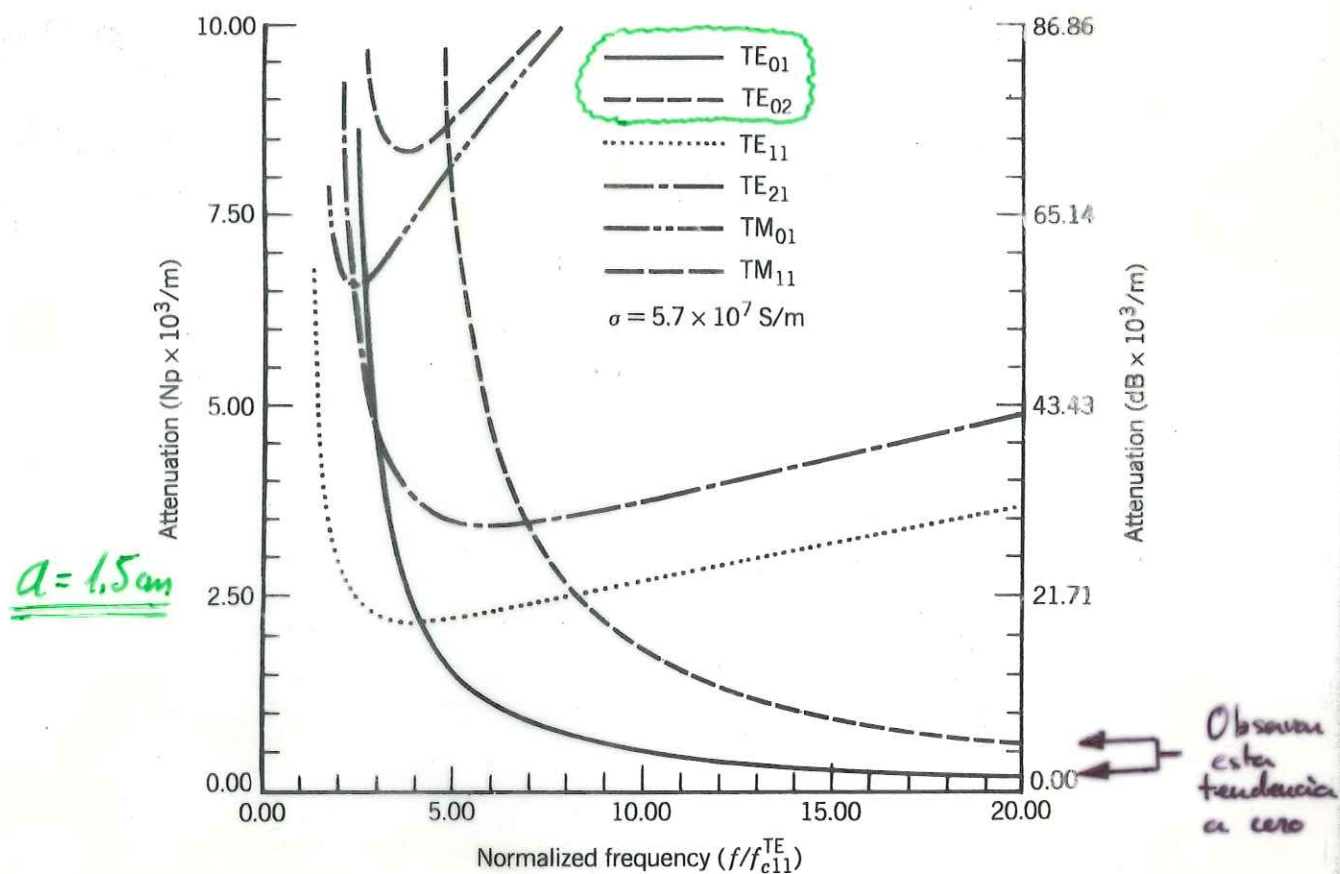
Atenuación guía circular $\approx 1.5 \text{ dB/Km.}$

Inconvenientes:

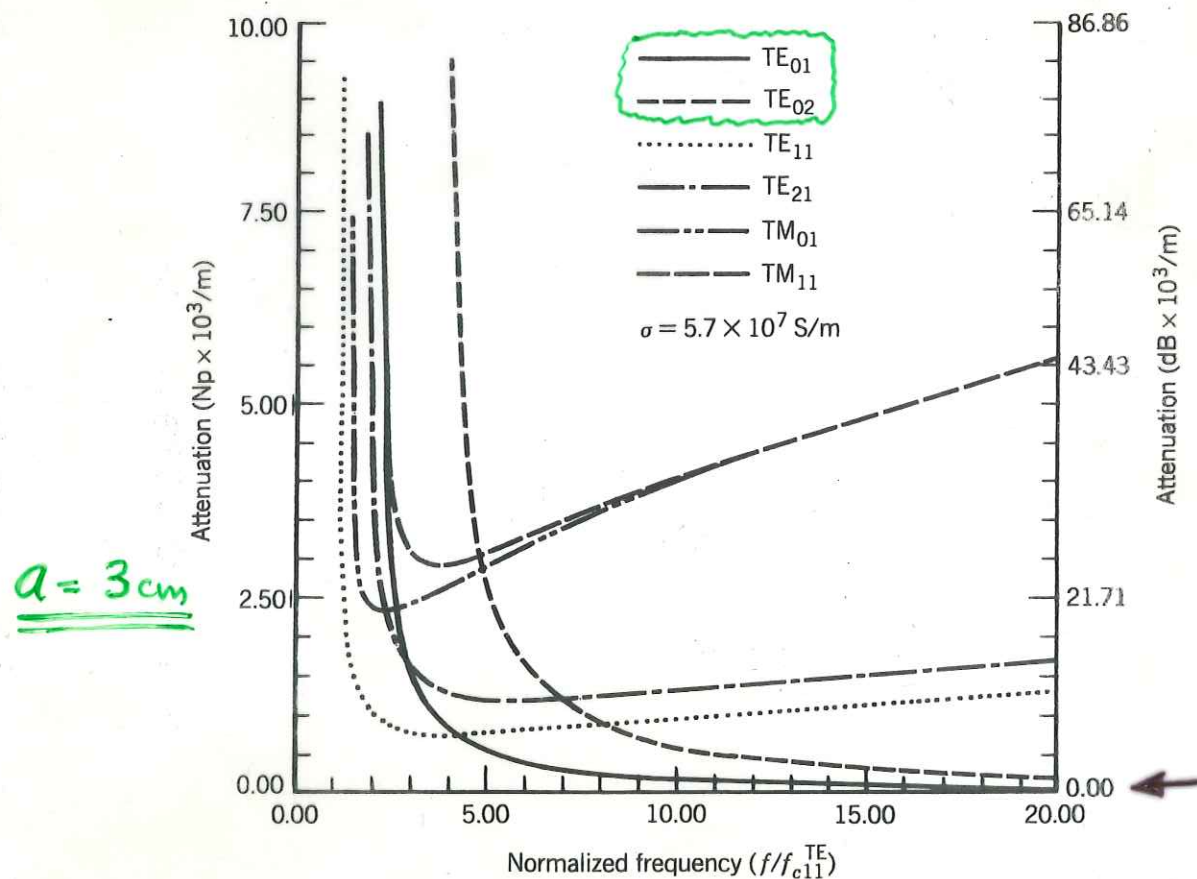
No es el modo dominante.

Atenuación (por pérdidas en dieléctricos)

$$\alpha_d = \frac{\omega \cdot \sqrt{\epsilon'} \cdot \frac{\epsilon''}{\epsilon'}}{2 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$



(a)



(b)

FIGURE 9-4 Attenuation for TE_{mn}^z and TM_{mn}^z modes in a circular waveguide. (a) $a = 1.5 \text{ cm}$. (b) $a = 3 \text{ cm}$.

Guías de ondas circulares (radio: a)

A19 bis.

(Punto)
 $\beta_c = \beta_p$

TE_{m,n}

$\frac{X'_{m,n}}{a}$

Ceros de las funciones
Derivadas de Bessel.

TM_{m,n}

$\frac{X_{m,n}}{a}$

Ceros de las funciones
de Bessel.

f_c

$\frac{X'_{m,n}}{2\pi a \sqrt{\mu \epsilon}}$

$\frac{X_{m,n}}{2\pi a \sqrt{\mu \epsilon}}$

λ_c

$\frac{2\pi a}{X'_{m,n}}$

$\frac{2\pi a}{X_{m,n}}$

$\beta_z (f \geq f_c)$
constante de fase

$\beta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

igual

$\lambda_g (f \geq f_c)$
long. de onda de la guía.

$\frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$

igual

$u_p (f \geq f_c)$

$\frac{u}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$

igual.

$Z (f \geq f_c)$
Impedancia de la onda

$\frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$

$\eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

TABLE 9-3
Summary of TE_{mn}^z and TM_{mn}^z mode characteristics of circular waveguide

	$TE_{mn}^z \left(\begin{matrix} m = 0, 1, 2, \dots, \\ n = 1, 2, 3, \dots \end{matrix} \right)$	$TM_{mn}^z \left(\begin{matrix} m = 0, 1, 2, 3, \dots, \\ n = 1, 2, 3, 4, \dots \end{matrix} \right)$
E_ρ^+	$-A_{mn} \frac{m}{\epsilon \rho} J_m(\beta_\rho \rho) [-C_2 \sin(m\phi) + D_2 \cos(m\phi)] e^{-j\beta_z z}$	$-B_{mn} \frac{\beta_\rho \beta_z}{\omega \mu \epsilon} J_m'(\beta_\rho \rho) [C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$
E_ϕ^+	$A_{mn} \frac{\beta_\rho}{\epsilon} J_m'(\beta_\rho \rho) [C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$	$-B_{mn} \frac{m \beta_z}{\omega \mu \epsilon \rho} J_m(\beta_\rho \rho) [-C_2 \sin(m\phi) + D_2 \cos(m\phi)] e^{-j\beta_z z}$
E_z^+	0	$-j B_{mn} \frac{\beta_\rho^2}{\omega \mu \epsilon} J_m(\beta_\rho \rho) [C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$
H_ρ^+	$-A_{mn} \frac{\beta_\rho \beta_z}{\omega \mu \epsilon} J_m'(\beta_\rho \rho) [C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$	$B_{mn} \frac{m}{\mu \rho} J_m(\beta_\rho \rho) [-C_2 \sin(m\phi) + D_2 \cos(m\phi)] e^{-j\beta_z z}$
H_ϕ^+	$-A_{mn} \frac{m \beta_z}{\omega \mu \epsilon \rho} J_m(\beta_\rho \rho) [-C_2 \sin(m\phi) + D_2 \cos(m\phi)] e^{-j\beta_z z}$	$-B_{mn} \frac{\beta_\rho}{\mu} J_m'(\beta_\rho \rho) [-C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$
H_z^+	$-j A_{mn} \frac{\beta_\rho^2}{\omega \mu \epsilon} J_m(\beta_\rho \rho) [C_2 \cos(m\phi) + D_2 \sin(m\phi)] e^{-j\beta_z z}$	0
	$\frac{\partial}{\partial(\beta_\rho \rho)}$	
$\beta_c = \beta_\rho$	$\frac{\chi'_{mn}}{a}$	$\frac{\chi_{mn}}{a}$
f_c	$\frac{\chi'_{mn}}{2\pi a \sqrt{\mu \epsilon}}$	$\frac{\chi_{mn}}{2\pi a \sqrt{\mu \epsilon}}$
λ_c	$\frac{2\pi a}{\chi'_{mn}}$	$\frac{2\pi a}{\chi_{mn}}$

TABLE 9-3 (Continued).

	$TE_{mn}^z \left(\begin{matrix} m = 0, 1, 2, \dots, \\ n = 1, 2, 3, \dots \end{matrix} \right)$	$TM_{mn}^z \left(\begin{matrix} m = 0, 1, 2, 3, \dots, \\ n = 1, 2, 3, 4, \dots \end{matrix} \right)$
$\beta_z (f \geq f_c)$	$\beta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \beta \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}$	
$\lambda_g (f \geq f_c)$	$\frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$	
$v_p (f \geq f_c)$	$\frac{v}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{v}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$	
$Z_w (f \geq f_c)$	$\frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\eta}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$	$\eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \eta \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}$
$Z_w (f \leq f_c)$	$j \frac{\eta}{\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}} = j \frac{\eta}{\sqrt{\left(\frac{\lambda}{\lambda_c}\right)^2 - 1}}$	$-j \eta \sqrt{\left(\frac{f_c}{f}\right)^2 - 1} = -j \eta \sqrt{\left(\frac{\lambda}{\lambda_c}\right)^2 - 1}$
α_c	$\frac{R_s}{a \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left[\left(\frac{f_c}{f}\right)^2 + \frac{m^2}{(\chi'_{mn})^2 - m^2} \right]$	$\frac{R_s}{a \eta} \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$

Fijase en la
 dirección de las líneas
 de E.
 Similar al TE₁₀
 en guías rectangulares
 BAJA ATENUACIÓN

Table 8.9
Summary of Wave Types for Circular Guides^a

Wave Type	TM ₀₁	TM ₀₂	TM ₁₁	TE ₀₁	TE ₁₁
Field distributions in cross-sectional plane, at plane of maximum transverse fields					
Field distributions along guide					
Field components present	E_z, E_r, H_ϕ	E_z, E_r, H_ϕ	$E_z, E_r, E_\phi, H_r, H_\phi$	H_z, H_r, E_ϕ	$H_z, H_r, H_\phi, E_r, E_\phi$
p_{nl} or $-p'_{nl}$	2.405	5.52	3.83	3.83	1.84
$(\zeta_c)_{nl}$	$\frac{2.405}{a}$	$\frac{5.52}{a}$	$\frac{3.83}{a}$	$\frac{3.83}{a}$	$\frac{1.84}{a}$
$(\zeta_c)_{nl}$	2.61a	1.14a	1.64a	1.64a	3.41a
$(\zeta_c)_{nl}$	$\frac{0.393}{a\sqrt{\mu\epsilon}}$	$\frac{0.877}{a\sqrt{\mu\epsilon}}$	$\frac{0.609}{a\sqrt{\mu\epsilon}}$	$\frac{0.609}{a\sqrt{\mu\epsilon}}$	$\frac{0.293}{a\sqrt{\mu\epsilon}}$
Attenuation due to imperfect conductors	$\frac{R_s}{a\eta} \frac{1}{\sqrt{1 - (\zeta_c/f)^2}}$	$\frac{R_s}{a\eta} \frac{1}{\sqrt{1 - (\zeta_c/f)^2}}$	$\frac{R_s}{a\eta} \frac{1}{\sqrt{1 - (\zeta_c/f)^2}}$	$\frac{R_s}{a\eta} \frac{(\zeta_c/f)^2}{\sqrt{1 - (\zeta_c/f)^2}}$	$\frac{R_s}{a\eta} \frac{1}{\sqrt{1 - (\zeta_c/f)^2}} \left[\left(\frac{\zeta_c}{f} \right)^2 + 0.420 \right]$

^a Electric field lines are shown solid and magnetic field lines are dashed.

together with
uation from
velocity, gro
are of the sa
plane and re
is substituted
earlier.

TE Waves
expressed in

The solution

Here we have
the same just
to (4) with E_z

where

In this case
requires that l

where $J'_n(p'_{nl})$
along the guid

The field di
in Table 8.9. I
the TE₁₀ mode
the guide, so th
guide were pro
cutoff frequen
the TE₀₁ mode

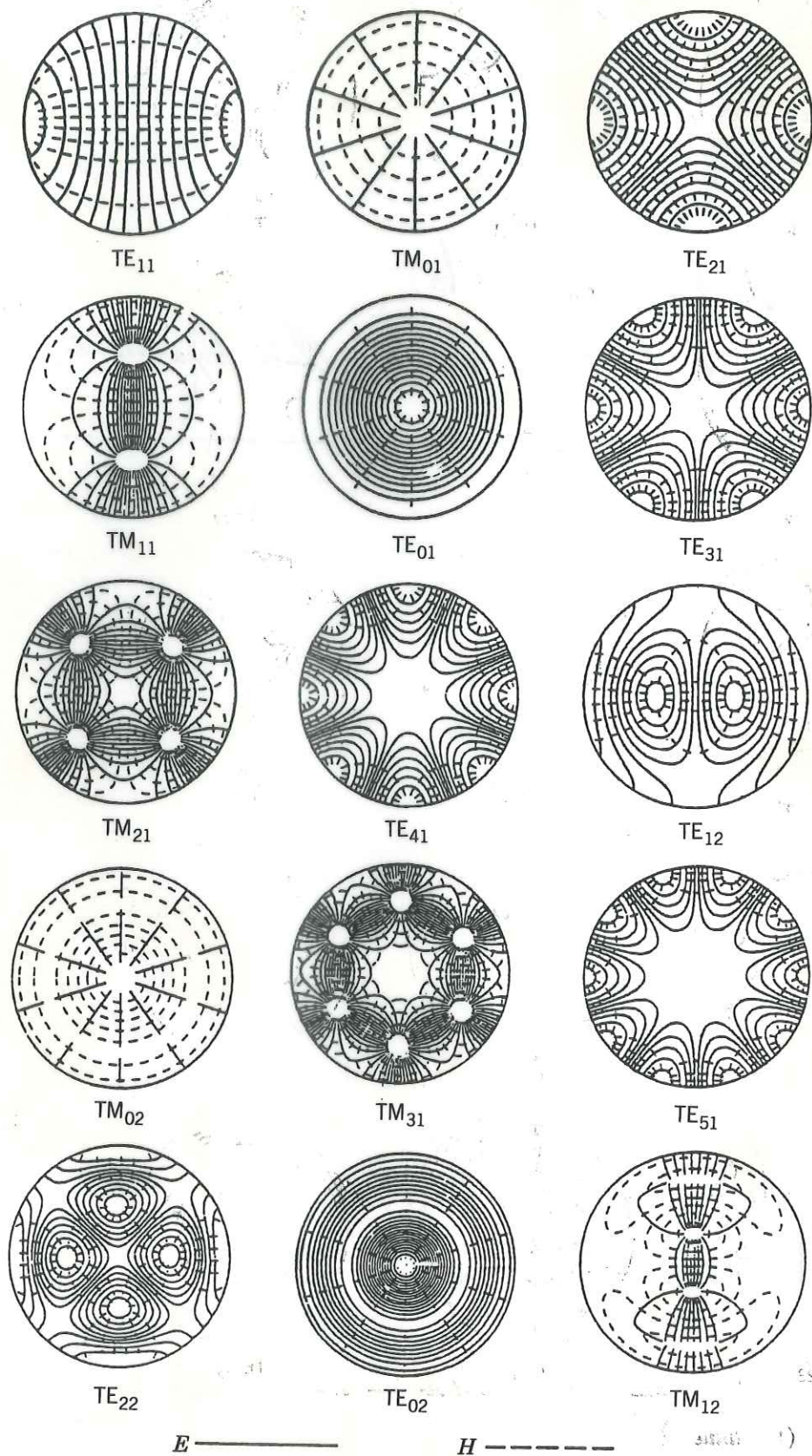


FIGURE 9-2 Field configurations of first 30 TE^z and/or TM^z modes in a circular waveguide. (Source: C. S. Lee, S. W. Lee, and S. L. Chuang, "Plot of modal field distribution in rectangular and circular waveguides," *IEEE Trans. Microwave Theory Tech.*, © 1966, IEEE.)

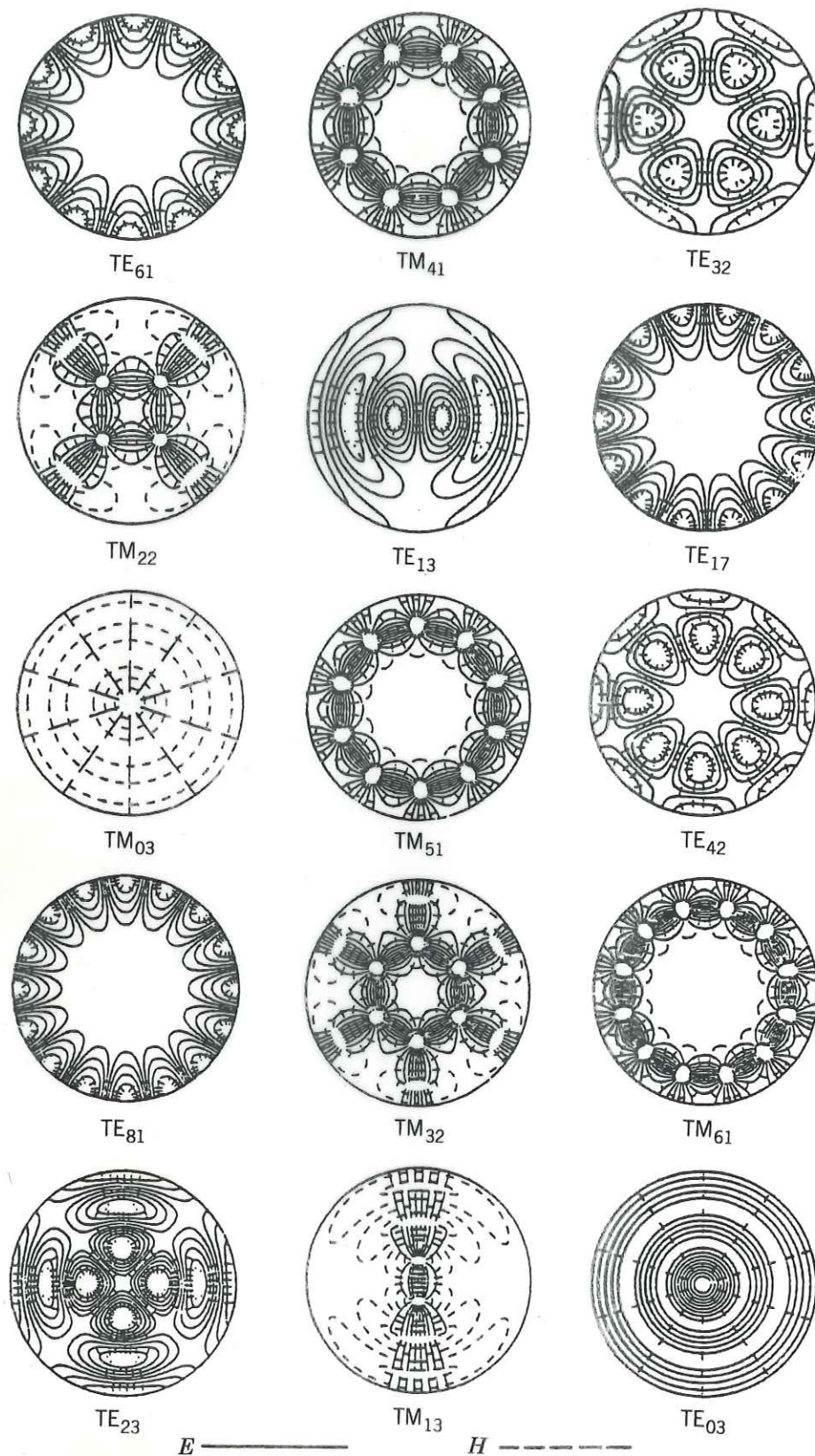


FIGURE 9-2 (Continued).