19/1/16

AERODYNAMICS

Fundamentos aerodinámicos _ John Anderson

- Aerodynamics for eng. students. - E.L

- Intro. to structural Aerodynamics

Content

1. Basic concepts.
2. Aerodynamic moment and forces
3. L and D and field velocity.
4. 2D wing theory and finite wing theory.
5. Aeroelasticity static and dynamic
6. CFD

Bernoulli Ecuation

$$p + \frac{1}{2}\rho U^2 = P_\infty + \frac{1}{2}\rho U_\infty^2 = P_R$$

Presión de remanso

• Bernoulli equation can be applied in streamlines

$$\boxed{\begin{gathered} P_\infty + \frac{1}{2}\rho V_\infty^2 = P + \frac{1}{2}\rho V^2 \\ V = \sqrt{\frac{2(P_\infty - P)}{\rho} + V_\infty^2} \end{gathered}}$$

$\rho = \rho$ Density.

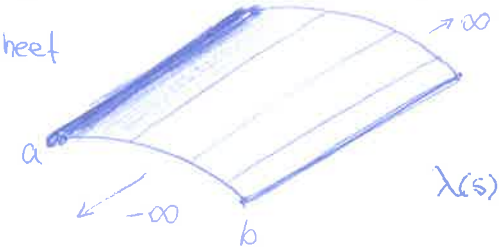
• Kutta Jou-Kowsky

$$\frac{L}{b} = \rho_\infty V_\infty \Gamma$$

• Source Panel Method

Non Lifting flows over arbitrary Bodies.

Source sheet



$$d\phi = \frac{\lambda\, ds}{2\pi} \cdot \ln r$$

$\lambda(s)$ = Volume flow rate/unit depth in z direction.

$$\phi_{(x,y)} = \int_a^b \frac{\lambda\, ds}{2\pi} \cdot \ln r$$

$$C_{p_i} = 1 - \left(\frac{V_i}{V_\infty}\right)^2$$

Exam (March)
Aerodynamics

(1)

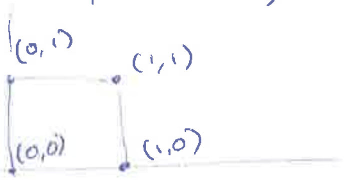
- Ideal α
- $\alpha_{L=0}$
- C_L C_m
- x_{cp}.

$$z = c\varepsilon\left[1-\left(\frac{x}{c}\right)^2\right]$$

(2) Are similar the dynamic (conditions) for equal μ and $\overbrace{a \text{ (speed of sound)}}^{?}$?

Airfoil 1	Airfoil 2
$c = x$	$c = 2x$
$\rho = 1'23$	$\rho = 1'74$
$V = 100$	$V = 200$
$T = 200K$	$T = 800K$

(3) Circulation w/ velocity vectors $u = xy^2$ $\quad v = xy$



…ación corticobasal

- A… rigidez extrapiramidal, al… …as asimétricas y disfunción se…
- S… …haplotipo MALT que PSP

Degeneración corticobasal. AP

- Pérdida neuronal con gliosis y nª balonizadas en corteza premotora, motora y parietal anterior
- Pérdida de nª pigmentadas e inclusiones argirófilas en sustancia negra y locus ceruleus
- Inmunorreactividad frente a tau en
  - Astrocitos "astrocitos en ovillo"
  - Oligodendrocitos "cuerpos en bobina"

COEFFICIENTS

Lift Coefficient

- $C_L = C_n \cos\alpha - C_a \sin\alpha$

$$C_L = \frac{L}{q_\infty \cdot S}$$

$$C_M = \frac{M}{q_\infty S l}$$

l = Reference length
S = Reference surface $\rightarrow S = c\,(l)$

Drag Coef. $C_D = C_{D_\infty} + \underbrace{C_{Di}}_{\text{induced drag from vortex.}}$

- $C_d = C_n \sin\alpha + C_a \cos\alpha$

$$C_D = \frac{D}{q_\infty S}$$

Same for N (normal) and A (xial) coefficient.

Pressure Coef.

$$C_p = \frac{p - p_\infty}{q_\infty}$$

p_∞ - Freestream pressure

Skin Friction C.

$$c_f = \frac{\tau}{q_\infty}$$

Normal and Axial Coefficients:

$C_{p_L} = C_{p,L}$

- $$C_n = \frac{1}{c}\left[\int_0^c (C_{p_L} - C_{p_u})\,dx + \int_0^c \left(C_{f_u}\frac{dy_u}{dx} + C_{f_L}\frac{dy_L}{dx}\right) dx\right]$$

- $$C_a = \frac{1}{c}\left[\int_0^c \left(C_{p,u}\frac{dy_u}{dx} - C_{p,L}\frac{dy_L}{dx}\right)dx + \int_0^c (C_{f,u} + C_{f,L})\,dx\right]$$

Moment at Leading Edge Coefficient.

Ecuation of State (P)

- $P = \rho R T$

Speed of Sound (a)

- $a = \sqrt{\gamma R T}$

γ = Specific heat ratio

$\gamma_{air} = 1'4$

R = Gas Constant =
= 8'314 J/mol·K

$R_{specific} = 287$ J/kg·K

Dynamic Pressure: (q_∞) [N/m²]

- $q_\infty = \frac{1}{2} \rho_\infty V_\infty^2$

Drag coefficient for entire a/c $(C_{D,E})$

- $C_{D,T} = C_{D_0} + \frac{C_L^2}{\pi e AR}$
$= C_{D_0} + C_L^2 \left(r + \frac{1}{\pi e AR} \right)$

Oswald Factor (e)

- $e = 1'78 (1 - 0'045 \, AR^{0'68}) - 0'64$

$C_{D,0} \rightarrow$ Zero-Lift C_D

$C_{D,0} = C_D - C_{D,i}$

$\frac{C_L}{C_D} = \frac{C_L}{C_{D_0} + \frac{C_L^2}{\pi e AR}}$

Aspect Ratio

$AR = \frac{b^2}{S}$

Fourier Coefficient

- $A_0 = \alpha - \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{dz}{dx}\, d\theta$

$\frac{x}{c}$

- $A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{dz}{dx} \cos(n)\, d\theta$

- $C_{M_{LE}} = -\frac{C_L}{4} + \frac{\pi}{4}\left(A_2 - A_1\right)$

$C_L = 2\pi\left(\alpha - \alpha_{L=0}\right)$

$C_L = a\left(\alpha - \alpha_{L=0}\right)$

a = slope in C_L vs α

$C_L = 2\pi\left(A_0 + \frac{A_1}{2}\right)$

In terms of Mach n°: $C_L = \frac{C_{L_i}}{\sqrt{1-M_\infty^2}}$

Incomp. $C_L = 2\pi\alpha$

In

In terms of Mach No:

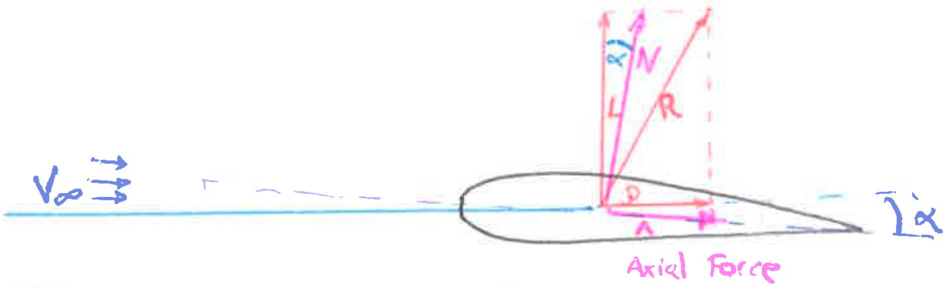
$C_{P_{crit.}} = \frac{C_{P_i}}{\sqrt{1-M_\infty^2}}$

M_∞ = Mach No.

AERODYNAMICS

$D = N \cdot \sin\alpha + A \cdot \cos\alpha$

$L = N \cos\alpha - A \sin\alpha$



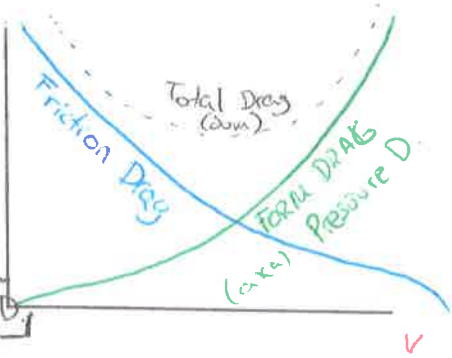
A' axial and N' normal forces PER UNIT SPAN

DRAG

Friction Drag

Form Drag

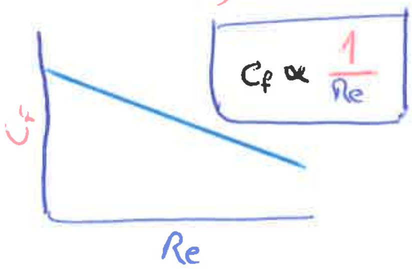
$\boxed{C_{D_\infty} = \text{Form D.} + \text{Friction D.}}$



FORM DRAG = Pressure Drag + Skin Friction

≃

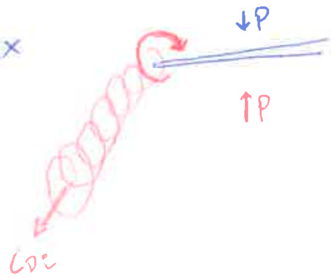
Skin Friction Coeff. vs Reynolds



C_f decreases with Re (and V), as

Induced Drag

Wingtip Vortex



$$\boxed{C_{D_{wing}} = C_{D_\infty} + C_{D_i} \cdot f(C_L^2)}$$

$C_D' = \frac{D'}{\quad} = \frac{D'}{\quad}$

$M = \frac{V_\infty}{c}$

$M \cdot c = V$

$\alpha = 0°$

$5°$ half angle wedge.

$P_\infty = 1$ Pa

$M_\infty = 2$

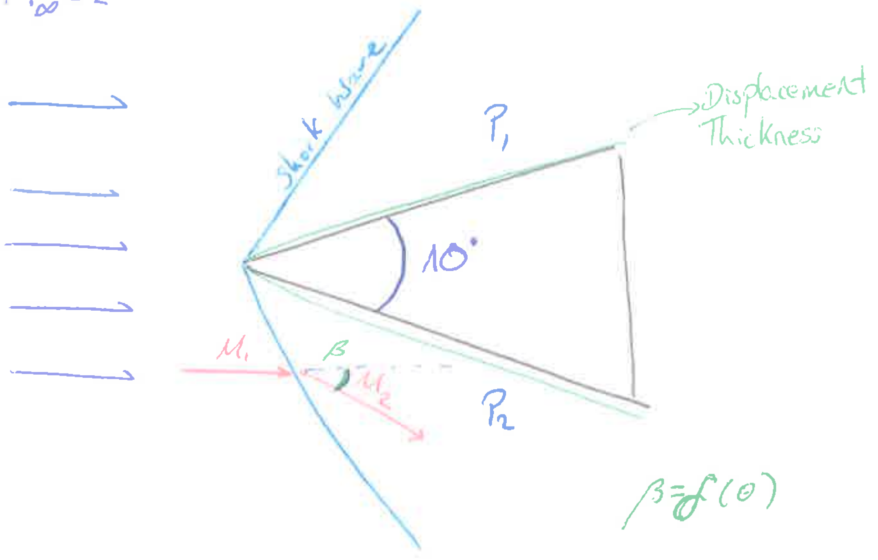
$\rho_\infty = 1'23 \; kg/m^3$

$P_1 = 1{,}31 \cdot 10^5$ Pa

$P_2 = P_\infty$

Shear $\tau_\omega = 431 \cdot s^{-2} \; N/m^2$

$L = 2$

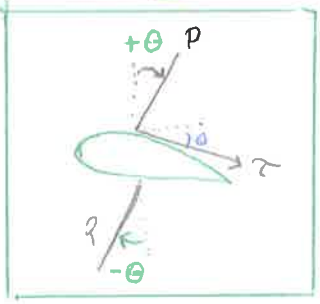


$\beta = f(\theta)$

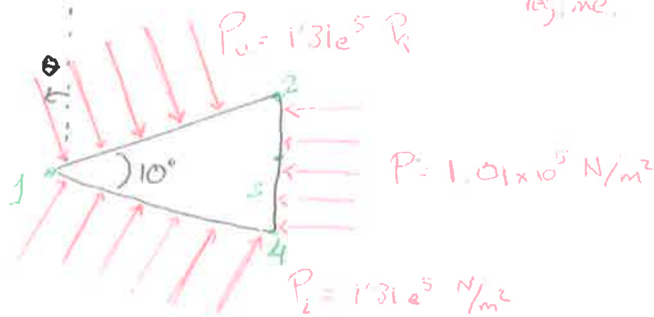
↳ weak shock wave
Flow after it, keeps supersonic regime.

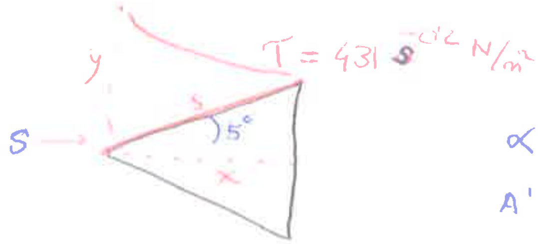
Find C_D

What is θ ? ±?



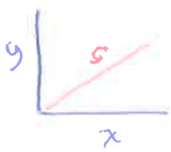
Sign convention





$\alpha = 0$

$A' = D'$



$$A' = \int_{LE}^{TE} \left(-P_u \cdot \sin\theta + \tau_u \cdot \cos\theta\right) ds_u + \int_{LE}^{TE} \left(P_l \cdot \sin\theta + \tau_u \cos\theta\right) ds_l$$

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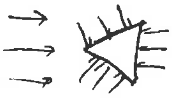


Continuation:

$$\int_{LE}^{TE} P_L \cdot \sin\theta \, ds_L = \int_{s_1}^{s_4} P_L \cdot \sin(5)\, ds_L + \int_{s_4}^{s_3} P \cdot \sin(90)\, ds_L$$

$$= \boxed{5260\, N}$$

$$\int_{LE}^{TE} T_u \cdot \cos\theta \, ds_u = \int_{s_1}^{s_2} 431 \cdot s^{-0'2} \cdot \cos(-5)\, ds_u$$

$$= 431 \cdot \left[\frac{s^{0'8}}{0'8}\right]_{s_1}^{s_2} \cdot \cos(-5)$$

$$= 429\,(s_2^{0'8} - s_1^{0'8})\,\frac{1}{0'8} = \boxed{936'5\, N}$$

$$\int_{LE}^{TE} T_L \cdot \cos\theta \, ds_L = 936'5\,N$$

$$D' = A' = \underbrace{5260\,N + 5260\,N}_{\text{Pressure Drag } \sim(85\%)} + \underbrace{936'5\,N + 936'5\,N}_{\text{Shear Force (Friction Drag)}(\propto V)\ \sim(15\%)} = \boxed{12\,393\,N}$$

As its supersonic, shock wave drag must be

$Pg\ 1 \longrightarrow C_D'$. $\quad M = \frac{V_\infty}{L}$

12 393 KN

Pitching Moment

Upper

$$dM'_u = (P_u \cdot \cos\theta + \tau_u \cdot \sin\theta)\, x\, ds_u + (-P_u \cdot \sin\theta + \underbrace{\tau_u \cdot \cos\theta}_{})\, y\, ds_u$$

Lower

Distribution of shear stress at u or l sfc

$$dM'_l = (-P_l \cos\theta + \tau_l \cdot \sin\theta)\, x\, ds_l + (P_l \sin\theta + \underbrace{\tau_l \cdot \cos\theta}_{})\, y\, ds_l$$

Moment @ LE

$$M'_{LE} = \int_{LE}^{TE} (dM'_u + dM'_l)\, ds$$

$$M'_{LE} = -x_{cp} L' = -\frac{c}{4} L' + M_{c/4}$$

$D' = \oint P\, ds$

i.e:

Consider an incompressible flow laminar BL growing along the surface of a flat plate. Chord = c. BL thickness at TE $\rightarrow$ $\boxed{\delta = \frac{5 \cdot c}{\sqrt{Re}}} \rightarrow \frac{1'328}{\sqrt{Re_c}}$

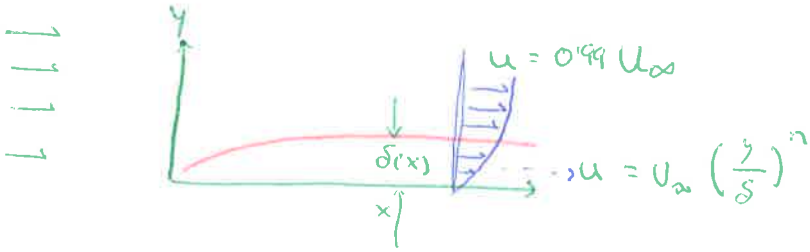
Skin friction coeff. for flat plate laminar $\rightarrow$ $\boxed{C_f = \frac{D'}{q_\infty c (1)}}$ unit span

Assuming velocity profile through the BL given by power law. $u = U_\infty \left(\frac{y}{\delta}\right)^n$

Calculate the value of n to satisfy

Assumptions

- Ideal Gas
- Steady State
- Laminar BL
- Incompressible
- No heat transfer
- Viscous flow.



Solution

$$C_f = \frac{D'}{q_\infty c} = \frac{\rho_\infty \int_0^\delta u_2 (u_1 - u_2)\, dy}{q_\infty \cdot c}$$

$$\left(q_\infty = \frac{1}{2} u^2_\infty \rho \right)$$

$$= \frac{\cancel{\rho_\infty}\, 2}{u_\infty^2 \cancel{\rho_\infty}\, c} \int_0^\delta \frac{u_2 (u_1 - u_2)\, dy}{\cancel{q}}$$

$u_1 = u_\infty$

$$= \frac{2}{c} \int_0^\delta \frac{u_2}{u_\infty} \left(\frac{u_1 - u_2}{u_\infty} \right) dy$$

$\delta = f(x)$

$\delta = \frac{5c}{}$

Change of variable

$0 \quad u_\infty \quad \left(1 \quad U_\infty \quad \delta \right) \quad \delta (\%)$

$$C_f = 2\int_0^{\delta} \frac{(y/c)^n}{(y/c)^n}\left(1 - \frac{(y/c)^n}{(\delta/c)^n}\right) d(y/c)$$

$$C_f = 2\int_0^{\delta/c} \left[\left(\frac{y/c}{\delta/c}\right)^n - \left(\frac{y/c}{\delta/c}\right)^{2n}\right] d(y/c)$$

Replace

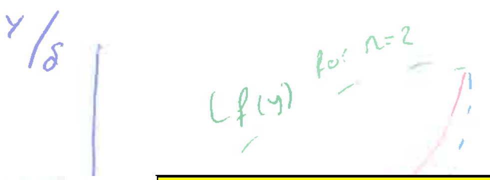
$$C_f = \frac{2}{n+1}\left(\frac{\delta}{c}\right) - \frac{2}{2n+1}\left(\frac{\delta}{c}\right) = \frac{1'328}{\sqrt{Re_c}}$$

$$= \frac{2}{n+1}\left(\frac{5\cdot c}{\sqrt{Re}}\,\frac{1}{c}\right) - \frac{2}{2n+1}\left(\frac{5\cdot c}{\sqrt{Re}\cdot c}\right) = \frac{1'328}{\cancel{\sqrt{Re_c}}}$$

$$\Rightarrow \frac{1}{n+1} - \frac{1}{2n+1} = \frac{1'328}{10}$$

$$0'2656\, n^2 - 0'6016\, n + 0'1328 = 0$$

$$\boxed{n = 2 \quad \text{or} \quad n = 0'25}$$

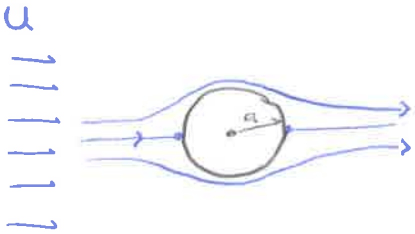


$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^n$$

Potential Flow

Potential Flow ⟨ Inviscid, Incompressible, Irrotational ⚠

Potential calculation:



$$w(z) = f(z) + \overline{f\left(\frac{a^2}{z}\right)}$$ → conjugated

$$\left[w(z) = Uz + \frac{Ua^2}{z} \right]$$ Real Part, No circulation.

With circulation $$\left[w(z) = U \cdot z + U\frac{a^2}{z} - \frac{i\Gamma}{2\pi} \log \cdot z \right]$$

Stream function ψ

$$\psi = U \cdot y \left(1 - \frac{a^2}{x^2+y^2}\right) - \frac{\Gamma}{2\pi} \cdot \log\left(x^2+y^2\right)$$

$z^2 = x^2 + y^2$ As is a circle $a^2 = x^2 + y$.

Streamline → $\psi = -\frac{\Gamma}{2\pi} \log(a)$

Any stagnation points in the flow must satisfy (2D-Flow)

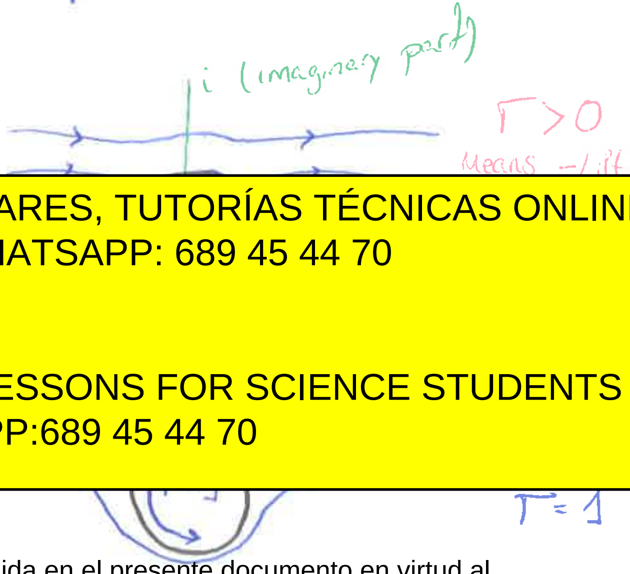
$$\left[u - iv = 0 = \frac{dw}{dz} = U - \frac{Ua^2}{z^2} - \frac{i\Gamma}{2\pi z} \right]$$

$$0 = z^2 U - \frac{i\Gamma z}{2\pi} - Ua^2$$

$$\left(\frac{z}{a}\right) = \frac{i\Gamma}{2\pi a} \pm \sqrt{1 - \Gamma^2 / 8\pi^2 U^2 a^2}$$

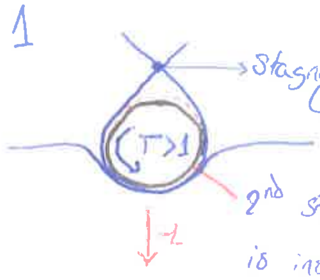
(1) No circulation → $\Gamma = 0$

$\frac{z}{a} = \pm 1$ → $z = \pm a$



$\Gamma > 1$

(2.2) when $\frac{\Gamma}{4\pi U a} > 1$



Stagnation Point (out of cylinder)

2^{nd} stagnation Point is inside the cylinder.

$$(F_x - iF_y) = i\frac{\rho}{2}\oint\left(\frac{\partial w}{\partial t}\right)^2 dz$$

Solution of the integral:

$$\oint_{|z|=a}\left(u - \frac{Ua^2}{z^2} - \frac{i\Gamma}{2\pi z}\right)^2 dz$$

radius

by Cauchy Residue:

$$\oint_z f(z)\,dz = 2\pi i \sum_{K=1}^{n} \text{Residue}(f(z)\cdot z)$$

Pole $\rightarrow z_0 = 0$
aka Singularity

Laurent Series $(-\infty, \infty)$

$$f(z) = \dots + \frac{a_{-2}}{(z-z_0)} + \frac{a_{-1}}{(z-z_0)} + a_0 + a_1(z-z_0)^1 \dots n$$

Cauchy Residue

a coefficient should be calculated.

$$\text{Res}(f(z);z_0) = a_j$$

For single pole $\frac{f_0(z)}{z-z_0} =$

$$= \frac{a_{-1}}{(z-z_0)} + a_0 + a_1(z-z_0) + a_2(z-z_0)^2$$

$$(z-z_0)\cdot f_0(z) = a_1 + a_0(z-z_0) + a_1(z-z_0)^2$$

Drop out

$$\oint f(z)\,dz = 2\pi i\,\text{Res}(f(z);z_0)$$

$$a_{-1} = \frac{1}{2\pi i}\oint f(z)\,dz = \text{Res}(f(z);z_0)$$

Example

$$f(z) = \frac{z+3}{z+2}$$

Singularity $z_0 = -2$

$$\oint_C \frac{z+3}{z+2}\,dz = 2\pi i \,\mathrm{Res}\,(f(z); -2)$$

$$\mathrm{Res}\,(f(z); -2) = \lim_{z \to -2} \frac{z+3}{\cancel{z+2}} \cancel{(z-z_0)} = 1$$

Solution:

$$\boxed{\oint \frac{z+3}{z+2}\,dz = 2\pi i\,(1) = 2\pi i}$$

For $z_0 = 0$

$$\mathrm{Res}\,(f(z); 0) = -\frac{i\Gamma U}{\pi}$$

$$\oint_{|z|=a} \left(\frac{dw}{dz}\right)^2 dz = 2\cancel{\pi} i\left(-i\frac{\Gamma U}{\cancel{\pi}}\right) = 2\Gamma U$$

$$= \frac{i\rho}{2} \int \left(\frac{dw}{dz}\right)^2 dz = \frac{i\rho}{2}\, 2\Gamma U = \rho U \Gamma i$$

$F_x = 0 \longrightarrow$ Drag $= 0$

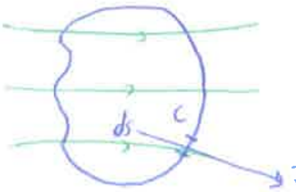
$\boxed{F_y = -\rho U \Gamma = \boxed{\text{LIFT}}}$ ↑ $\Gamma < 0 \longrightarrow +(\text{Positive})$ LIFT ↑

CIRCULATION [Γ]

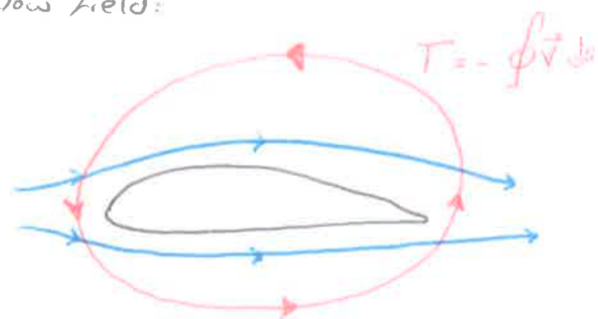
$$\Gamma = -\oint \vec{V} \cdot ds$$

(is a finite value)
If $\Gamma = \infty \rightarrow \Gamma = 0$

Consider a close curve (c) in a flow field:



$\vec{V}$ is the velocity of ds (a segment) of the total shape



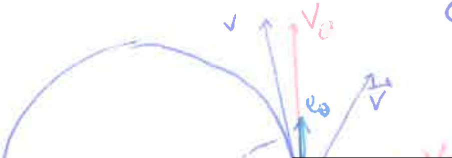
Kutta - Joukoski Theorem

$$\boxed{F_y = -\rho U \Gamma \rightarrow \text{Lift}}$$

$$\Gamma = -\oint \vec{v}\, ds = \iint_S \underbrace{(\vec{\nabla} \times \vec{V})}_{\text{vorticity}} ds$$

$r \alpha \frac{1}{\nabla V}$

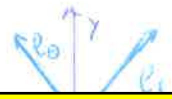
eg: $V \left(\underbrace{\frac{y}{x^2+y^2}}_{u\,(m/s)} ; \underbrace{\frac{-x}{x^2+y^2}}_{v\,(m/s)} \right)$ Calculate the circulation around a circular path of radius 5m.



Cylindrical coords!

$X = r \cos \theta$

e_0 & e_r is a transformation of axis



$$u = \frac{y}{x^2+y^2} = \frac{r \cdot \sin\theta}{r^2} = \frac{\sin\theta}{r}$$

$$v = \frac{-x}{x^2+y^2} = -\frac{r \cdot \cos\theta}{r^2} = -\frac{\cos\theta}{r}$$

Replacing into V_r and V_θ:

$$V_r = \frac{\sin\theta}{r} \cdot \cos\theta + \left(-\frac{\cos\theta}{r}\right) \cdot \sin\theta = 0$$

$$V_\theta = -\frac{\sin\theta}{r} \cdot \sin\theta + \left(-\frac{\cos\theta}{r}\right) \cdot \cos\theta = -\frac{1}{r}$$

$$\vec{V}\,ds = (V_r + V_\theta)\,(\vec{e}_\theta + \vec{e}_r)\,(dr\,\vec{e}_r + r\,d\theta\,\vec{e}_\theta)$$

$$= (V_r e_r + V_\theta e_\theta)\,(dr \cdot \vec{e}_r + r\,d\theta\,\vec{e}_\theta)$$

$$= \vec{V}_r\,dr + r\,\vec{V}_\theta\,d\theta$$

$$= 0\,dr + r\left(\frac{-1}{r}\right)d\theta \longrightarrow \vec{V}\,ds = -d\theta$$

$$\boxed{\Gamma = -\oint \vec{V}\,ds = -\int_0^{2\pi} -d\theta = +2\pi\left(\frac{m^2}{s}\right)}$$

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$V = 10\,m/s$

$\Gamma = 20\,m^2/s$

C_p Vs θ

$$C_p = 1 - 4\sin^2\theta\left(1 + \frac{\Gamma}{4\pi a V_\infty \sin\theta}\right)^2$$

$$= \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2}$$

Statements of Kutta condition.

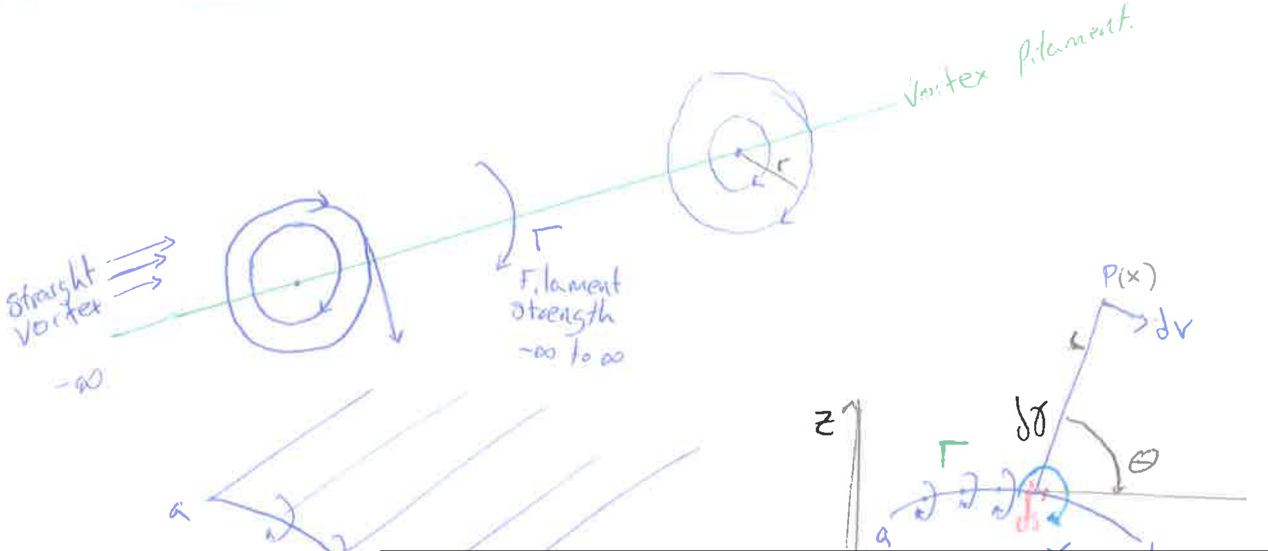
1°/ For any given Γ is such that the flow leaves the T.E. smoothly.

2°/ In the T.E. must exist an stagnation point.

3°/ Cusped t.E. and exit velocities must be equal in magnitude and direction.

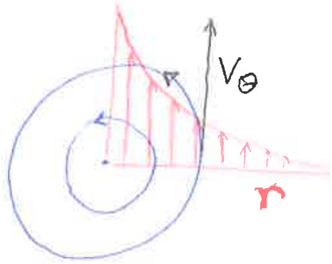
4.4 Theorical solutions for low speed flow over airfoils: is the vortex sheet.

Pg 285
3ED



Pg 232
3ed

Vortex Flow



$V_\theta = \frac{c}{r}$. (Determine constant c)

$$\Gamma = -\oint \vec{V}\, ds$$

$$= \iint_s (\vec{\nabla} \times \vec{V})\, ds \longleftarrow [\text{Escalar}]$$

$\vec{\nabla} \times \vec{V} =$ Vorticity.

$$\Gamma = -\int_0^{2\pi} \vec{V}_\Theta \cdot r\, d\Theta$$

$$= -V_\Theta\, 2\pi r$$

$$V_\Theta = \frac{-\Gamma}{2\pi r} = \frac{c}{r} \longrightarrow c = \frac{-\Gamma}{2\pi} \longrightarrow \boxed{\Gamma = -c\, 2\pi}$$

what is ϕ?

Velocity potential of the vortex.

$$V_r = \frac{d\phi}{dr} = 0 \qquad V_\Theta = \frac{1}{\cancel{r}} \frac{d\phi}{\delta\Theta} = -\Gamma / 2\pi \cancel{r}$$

$$\cancel{d\Theta}\ d\phi = \frac{-\Gamma}{2\pi}\, d\Theta$$

ϕ Velocity potential $\rightarrow$ $\boxed{\phi = \frac{-\Gamma}{2\pi}}$ $\qquad \Gamma = \gamma\, ds$

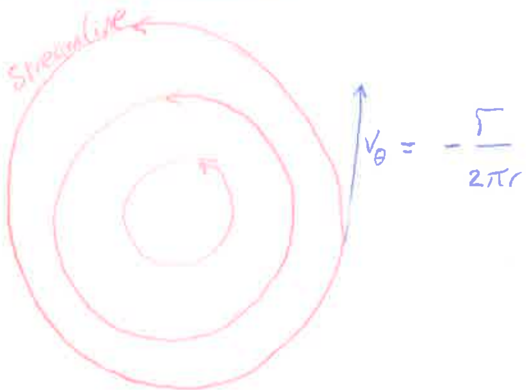
$$d\phi = (-\gamma\, ds / 2\pi)\ \Theta$$

$$\phi(x,z) = \left(\int_a^b \frac{-\gamma\, \Theta\, ds}{2\pi} \right)$$

This is important for numerical vortex panel method. Pg 247 / 3ed.

$\Gamma \quad \int^b$

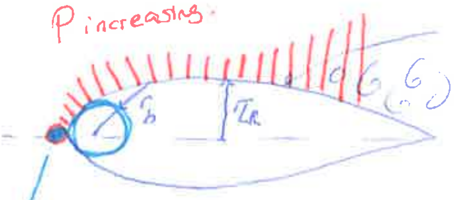
3.14 Vortex Flow



Airfoil Defined geometrically by:

• THICKNESS τ_R • CURVATURE ε • α

= Espesor Relativo ε =
= Espesor max / c

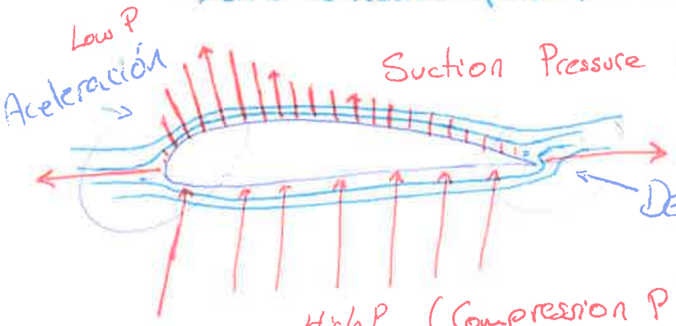


r_b = Radio de curvatura

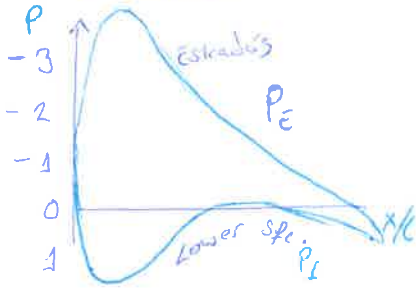
P=∞ → Pico de succión de borde de ataque (Normal a la superficie)
(Al haber presión baja)
Inverso al radio de curvatura.
← En el LE las presiones generan tracción
→ En el TE resistencia (Mayor que la tracción LE)

Diseño de planos:
$\boxed{\text{* Borde de salida (TE) * Borde de ataque (LE)}}$

Kutta Th. → Refleja la importancia del TE para la generación de sustentación



$L = P_E - P_I$ (Ambas P van en la misma dirección)



3.16
pg 244
3ed.

Kutta theorem.

La circulación al rededor de un perfil ha de ser la apropiada para que el punto de remanso posterior no esté en el estradós ni

Kutta ideal circulation.

Como la P_R (P de remanso) es uniforme, por la ec. de Bernoulli se deduce que igualdad de P requiere igualdad de V.

Bernoulli

$$p + \frac{1}{2} \rho U^2 = p_\infty + \frac{1}{2} \rho U_\infty^2 = P_R$$

Circulation

$$\boxed{\Gamma = \oint_C V \cdot ds}$$

Lift/unit span

$$L' = \rho_\infty V_\infty \Gamma$$

* $\oint_C$ → Line integral $\int_C$
Normaly is anticlockwise.
In aerodyn. better clockwise.

Se habla de circulación para referirse a que la line integral es finita.

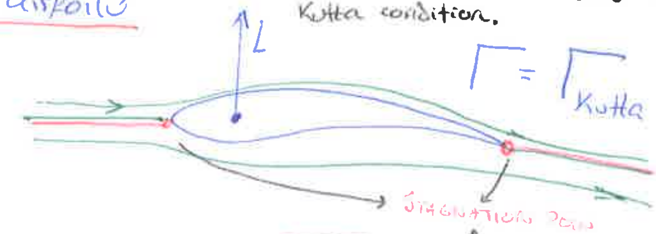
ds → Means we are moving along a closed curve "C" (instead of x, y axis)

«Circulation is an ABSTRACT, mathemathical concept»

19/1/16

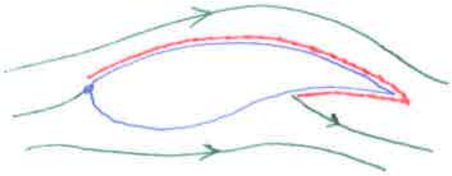
Incompressible flow over airfoils

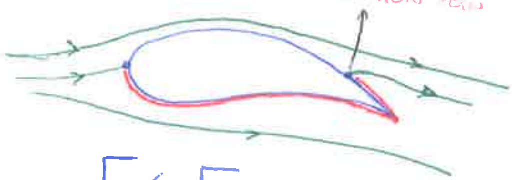
(1) circulation that satisfies the Kutta condition.



Kutta - Condition.

Flow around the trailing edge.





Circulation $\Gamma > \Gamma_{Kutta}$ No Lift

$\Gamma < \Gamma_{Kutta}$ No Lift

Theory of Cambered Airfoil

Symmetric Airfoil

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\,d\xi}{x-\xi} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

Transformation

$$\xi = \frac{c}{2}(1-\cos\theta)$$

$$d\xi = \frac{c}{2}\sin d\theta$$

$$x = \frac{c}{2}(1-\cos\theta_0)$$

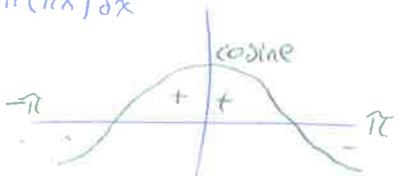
$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\cdot\sin\theta\,d\theta}{\cos\theta-\cos\theta_0} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx + B_n \sin(nx)$$

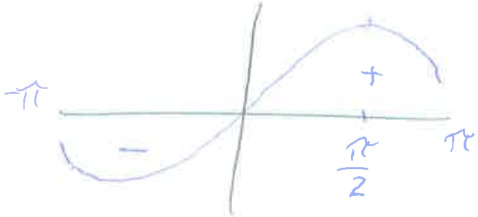
Fourier Series
- Obtain the coefficient.

$P = 2\pi$

$$\int_{-\pi}^{\pi} f(x)\,dx = \underbrace{\int_{-\pi}^{\pi} A_0\,dx}_{X+\pi-(-\pi)} + \underbrace{\int_{-\pi}^{\pi} A_n \cos nx + B_n \sin(nx)\,dx}_{=0}$$



$$\int_{-\pi}^{\pi} f(x)\,dx = 2\pi A_0$$



$$A_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

To obtain A_n multiply by $\cos mx$

$$\int_{-\pi}^{\pi} f(x)\cos mx\,dx = \overbrace{\int_{-\pi}^{\pi} A_0 \cos mx\,dx}^{=0} + \sum_{n=1}^{\infty}\left[\int_{-\pi}^{\pi} A_n \cos nx \cdot \cos mx + \int_{-\pi}^{\pi} B_n \sin(nx)\cos mx\,dx\right]$$

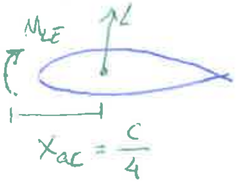
$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(mx) = A_n$$

$B_n \sin(mx)$

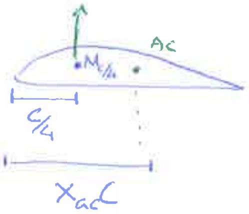
$$\int_{-\pi}^{\pi} dx \sin(mx)\,dx = \int^{\pi} \ldots \sum^{\infty} \int^{\pi} \ldots = 0$$

Aerodynamic center

Symmetric a/f.



Cambered A/f.



$C_{M_{ac}}$ = Moment Coef. at A.C.

$X_{ac} = X\% C$

$$C_{m_{ac}} = C_L (X_{ac} - 0'25) + C_{m_{c/4}}$$

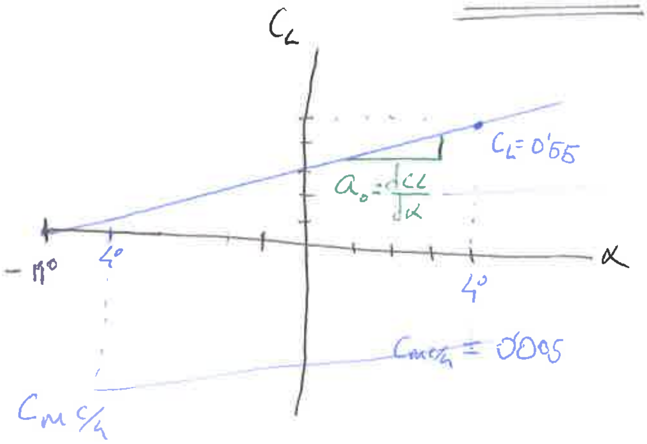
i.e: NACA 23012

For $\alpha = 4° \longrightarrow C_L = 0'55 \longrightarrow C_{m_{c/4}} = -0'005$

For $\alpha_0 =$ Zero-Lift $= 11°(-1) \longrightarrow \alpha_0 = -11°$

For $\alpha = -4° \longrightarrow C_{m_{c/4}} = -0'0125$

Obtain AC. (X_{ac})



a_0 = Slope of $\frac{dC_L}{d\alpha}$

$$\frac{dC_L}{\alpha} = \frac{0'55 - 0}{4 - 11} \qquad \frac{dC_L}{\alpha} = 0'1078 \text{ per Degree.}$$

$$\frac{dC_{m/4}}{d\alpha} = m_0 = \frac{-0'005 - (-0'0125)}{4 - (-4)}$$

$$m_0 = 9'375 \times 10^{-5} \text{ per degree.}$$

Pregunta de Examen

Example Mean Cambered.

Mean Camber:

$$\frac{z}{c} = 3'212\left[\frac{x^3}{c^3} - 0'83\frac{x^2}{c^2} + 0'187\frac{x}{c}\right]$$

for $0 \leq \frac{x}{c} \leq 0'21$

$\alpha_0 = \alpha_{L=0}$
α ideal
C_L
C_M
C_{Mac}

?

$$\frac{z}{c} = 0'043\left(1 - \frac{x}{c}\right) \qquad x = \frac{c}{2}(1-\cos\theta)$$

for $0'21 \leq \frac{x}{c} \leq 1$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\,d\theta$$

Transformar $\frac{z}{c}$ en función de θ con:
$x = \frac{c}{2}(1-\cos\theta)$

$dx = \frac{c}{2}\sin\theta\, d\theta$; $d\theta = \frac{2\,dx}{\sin\theta}$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos n\theta\, d\theta$$

$$\frac{dz}{dx} \Rightarrow \underbrace{3'212\left[\frac{3x^2}{c^2} - 1'66\frac{2x}{c} + 0'187\right]}$$

$$\frac{dz}{dx} = -0'043$$

$$A_0 = \alpha - \frac{1}{\pi}\left[\int_0^{\beta} 3'212\left(\frac{3x^2}{c^2} - 1'66\frac{2x}{c} + 0'187\right)\right]d\theta +$$

$$+\int_{\beta}^{\pi} -0'043\,d\theta = \alpha - \frac{1}{\pi}\left[\int_0^{\beta}\frac{3}{4}(1-\cos\theta)^2 - 1'66(1-\cos\theta) + 0'187\,d\theta + \right.$$

Cont. Example Mean Camber.

$$\int \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$A_0 = \alpha - \frac{1}{\pi}\left[-0'723\,\theta\right]_0^{\beta} + \frac{3}{4}\left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right]_0^{\beta} + \left[0'16\sin\theta\right]_0^{\beta} -$$

$$- 0'043\,\theta\Big]_{\beta}^{\pi} =$$

$$\int \cos 2\theta\, d\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$= \alpha - \frac{1}{\pi}\Big[-0'723 \cdot 2'55$$

$\cancel{\beta = 73'14 \cdot \frac{2\pi}{180}}$ $\quad x = \frac{c}{2}(1-\cos\theta)$

$$\frac{0'21}{c} = \frac{c}{2}(1-\cos\theta)$$

$$0'21 - \frac{1}{2} = -\cos\theta\,\frac{1}{2}$$

$\cos\theta = 0'5 \cdot 0'21 .$

$\beta = 54$

Biot-Savarts

$$dw = \frac{\Gamma}{4\pi}\,\frac{\cos\beta\, ds}{r^2}$$

Relationship between the velocity induced by a vortex tube and the strengh (circulation)

$$C_L = C_n \cos\alpha - C_a \sin\alpha$$

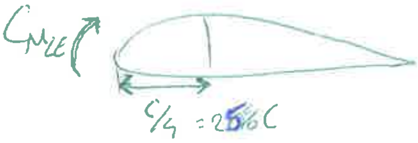
$$\boxed{C_{M_{LE}} = -\frac{C_L}{4}}$$

Moment c. in Leading Edge

$$C_n = \frac{1}{c}\int_0^c \left(\left(C_{p_L} - C_{p_u}\right) dx + \int_0^c \left(C_{f_u}\frac{dy_u}{dx} + C_{f,L}\frac{dy_L}{dx} \right) dx \right.$$

$$C_a = \frac{1}{c}\int_0^c \left(C_{p_u}\frac{dy_u}{dx} - C_{p_L}\frac{dy_L}{dx} \right) dx + \int_0^c \left(C_{f_u} + C_{f_L} \right) dx$$

$$C_{M_{c/4}} = \frac{\pi}{4}\left(A_2 - A_1\right)$$



$$C_{M_{ac}} = \frac{\pi}{4}\left(\frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos 2\theta\, d\theta - \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos\theta\, d\theta \right)$$

Moment at Aerodynamic cent.

At Aerod. Center the momen remains constant regardless α

$$C_{M_{ac}} = \frac{1}{2}\int_0^\pi \frac{dz}{dx}\left(\cos 2\theta - \cos\theta\right) d\theta \longrightarrow C_{M_{ac}} = -\frac{Z m \pi}{c}$$

Net Skin friction: (D_f) Laminar B.L.

$$C_f = \frac{D_f}{q_\infty S}$$

Lasius solution.

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = V_\infty \frac{\partial f'}{\partial y}$$

$$C_f = \frac{2\mu V_\infty}{\rho_\infty V_\infty^2}\sqrt{\frac{V_\infty}{\nu x}}\, f''(0)$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = V_\infty \sqrt{\frac{V_\infty}{2\nu x}}\, \delta''(0)$$

$$C_f = 2\sqrt{\frac{\mu}{V_\infty \rho x}}\, f''(0)$$

$$D_f = C_f \cdot q_\infty \cdot c$$

Ex: NACA 2412

$Re = 3'1 \times 10^6$

a) BL. Thickness at TE for $c = 1'5m$

b) Net Laminar skin-friction coeff.

Solution

a) $$\delta = \frac{5 \cdot c}{\sqrt{3'1 \times 10^6}} = 0'00426\,m$$

b) $$C_\delta = \frac{1'328}{\sqrt{Re}} = 7'54 \times 10^{-4}$$

Net $C_f = 2 \times 7'5 \times 10^{-4} = 0'0015$

$D_f = C_f \cdot q_\infty \cdot c$ $3'1 \times 10^6 = V_\infty \cdot 1'5m$

$$D_f = q_\infty \, x_{cr} (C_{f,s})_L + \frac{q_\infty \, C}{q_\infty \, C}$$

$$C_f = \frac{x_{cr}}{c} (C_{f,})_L + (C_{f,c})_T - \frac{x_{cr}}{c} (C_{f,1})_T$$

Skin Friction in the Laminar B.L. region

$$(C_f)_* = \frac{1'328}{\sqrt{Re_{cr}}}$$

C_{f_x}

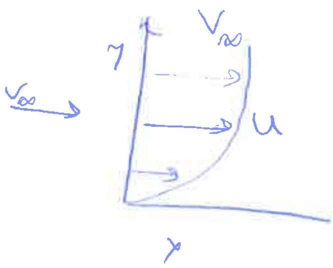
Turbulent

$$(C_f)_T = \frac{0'074}{Re_c^{1/5}}$$

$Re_{cr} = Re_x$

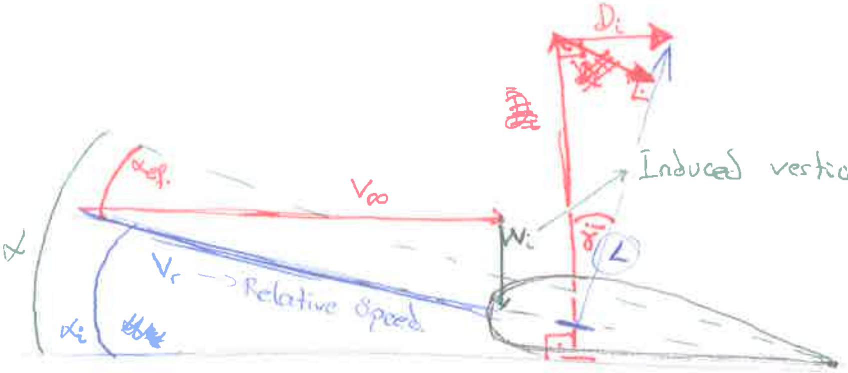
Gradient of P. different than ∅.

(→ Solve Boundary layer cndts. eqs.)

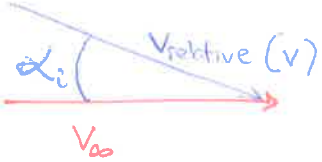


$$\frac{\partial \tau}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} = -\rho U \frac{du}{dx} = \frac{dP}{dx}$$

11/2/16



Lift → Perpendicular to relative V.

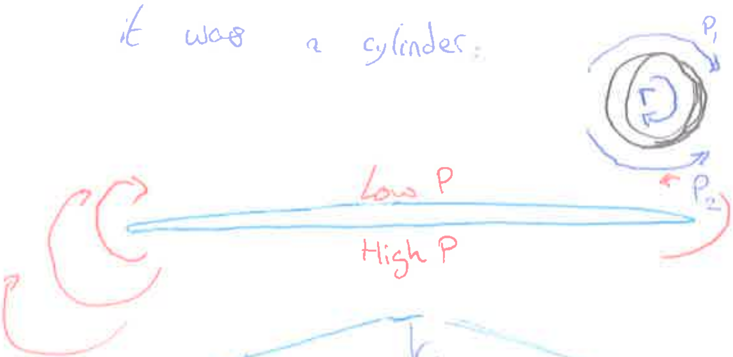


$\alpha = \alpha_i + \alpha_{eff}$

α_{eff} : Effective α

$L = \rho_\infty V_\infty \Gamma$

Assuming (just theoretically) that there is a circulation around an airfoil as it was a cylinder:

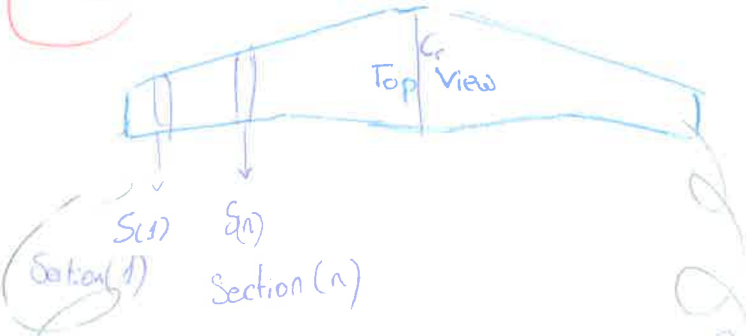


$D_i = L \cdot \sin \alpha_i$

Almost negligible

$\sin \alpha_i = \frac{W_i}{V}$

$D_i = L \cdot \alpha_i$ (Rad)



Airfoil Theory

$C_D = C_d + C_{D_i}$

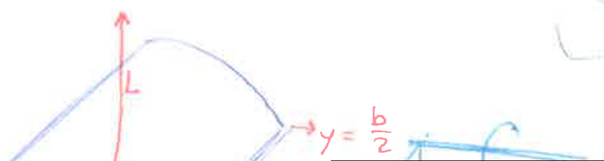
Defined by: Airfoil Parasitic or profile Drag

- Finite wing.

C_{D_i} = Induced Drag

PRANDTL'S Classical Lifting-Line Theory.

C_{D_i} → Induced Drag is less for longer wings.

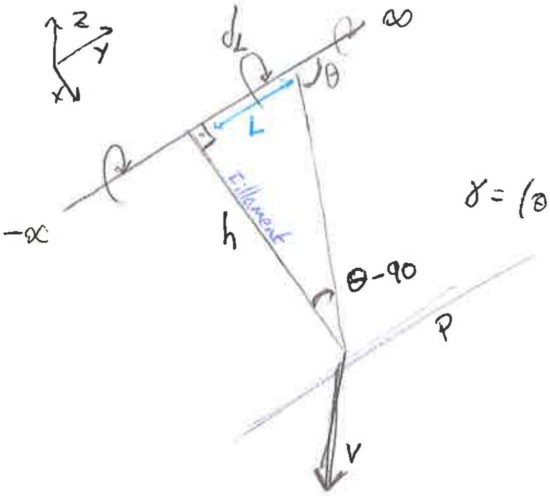


11/2/16

$$dv = \frac{\Gamma \; dL \times r}{4\pi |r|^3}$$

$$\rightarrow V = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \cdot \frac{\sin\theta \, d_L}{r^2} \qquad \underbrace{r\cos(\theta-90)}_{\sin(\theta)} = h$$

$$r = \frac{h}{\sin\theta}$$



$$L = \frac{h}{\tan\theta} \qquad dL = -\frac{h \, d\theta}{\sin^2\theta}$$

$$\gamma = (\theta - 90)$$

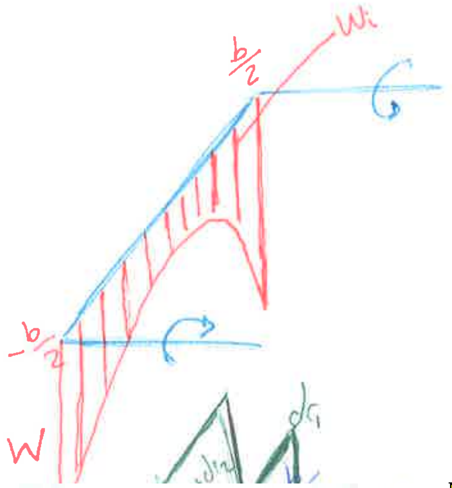
$$V = \frac{-\Gamma}{4\pi} \int_{-\infty}^{\infty} \frac{\sin\theta \, dL}{r^2} = \frac{\Gamma}{4\pi} \int_{-\infty}^{\infty} \frac{\sin\theta \, \sin^2\theta}{\cancel{h^2}} \frac{h \, d\theta}{\sin^2\theta}$$

$$V = \frac{+\Gamma}{4\pi} \int_{\pi}^{0} \sin\theta \, d\theta \qquad y = \int_a^b dx \qquad y = -\int_b^a dx$$

$$V = \frac{\Gamma}{2\pi h}$$ Infinite filament.

Semi infinite filament.

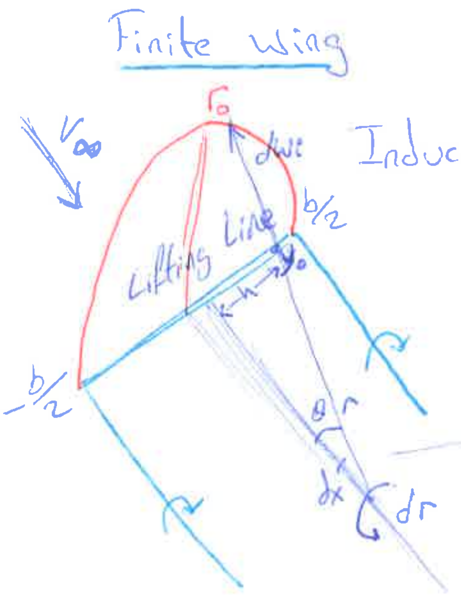
Biot-Savart $V = \frac{\Gamma}{4\pi h}$ $\quad V = W$



$$-w = \left(\frac{\Gamma}{4\pi\left(\frac{b}{2}+y\right)} + \frac{\Gamma}{4\pi\left(\frac{b}{2}-y\right)} \right)$$

16/2/16

Finite wing



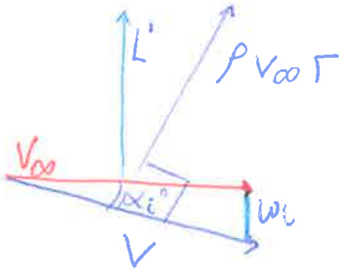
Induced velocity $(w(y)) = \dfrac{-\Gamma b}{4\pi\left[\left(\frac{b}{2}\right)^2 - y^2\right]}$

→ Biot-Savart's law

$$dw = \frac{1}{4\pi}\frac{d\Gamma}{(y_0-y)}$$

$$\left[dw = -\frac{1}{4\pi}\frac{(d\Gamma/dy)\,dy}{(y_0-y)}\right]$$

$$w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{(d\Gamma/dy)\cdot dy}{(y_0-y)}$$



$$\boxed{\begin{aligned}\alpha_i &= \tan^{-1}\left(-\frac{w}{V_\infty}\right) = -\frac{w}{V_\infty}\\ \alpha_i(y_0) &= -\frac{w(y_0)}{V_\infty}\end{aligned}}$$

$$\underline{\alpha_i(y_0)} = -\left(-\frac{1}{4\pi}\cdot\frac{1}{V_\infty}\right)\int_{-b/2}^{b/2}\frac{(d\Gamma/dy)dy}{(y_0-y)}$$

$$L = \rho_\infty V_\infty \Gamma(y_0)$$

$$= \frac{1}{2}\rho_\infty V_\infty^2 \, \overset{\text{chord}}{c(y_0)} \cdot c_L$$

$$c_L = \frac{2\Gamma(y_0)}{V_\infty c(y_0)} \qquad c_L = 2\pi(\alpha_{eff}(y_0) - \alpha_{L=0})$$

$$\frac{2\Gamma(y_0)}{V_\infty c(y_0)} = 2\pi(\alpha_{eff}(y_0) - \alpha_{L=0})$$

$$\alpha_{eff}(y_0) = \frac{\Gamma(y_0)}{V_\infty \pi c(y_0)} + \alpha_{L=0}$$

$$\alpha(y_0) = \frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0}(y_0) + \frac{1}{4\pi V_\infty}\int_{-b/2}^{b/2} \frac{(d\Gamma/dy)\,dy}{(y_0 - y)}$$

Solution $\Gamma = \Gamma(y_0)$ gives the 3 main aerodynamic characteristics of the finite wing.

1) Lift distribution is obtained from Kutta-Joukowski theorem.

- $L = \int_{-b/2}^{b/2} L'(y)\,dy = \rho_\infty V_\infty \int_{-b/2}^{b/2} \Gamma(y_0)\,dy$

- $C_L = \dfrac{L}{q_\infty S}$ $\qquad S = (b \cdot c)$

3) Induced Drag is obtained by

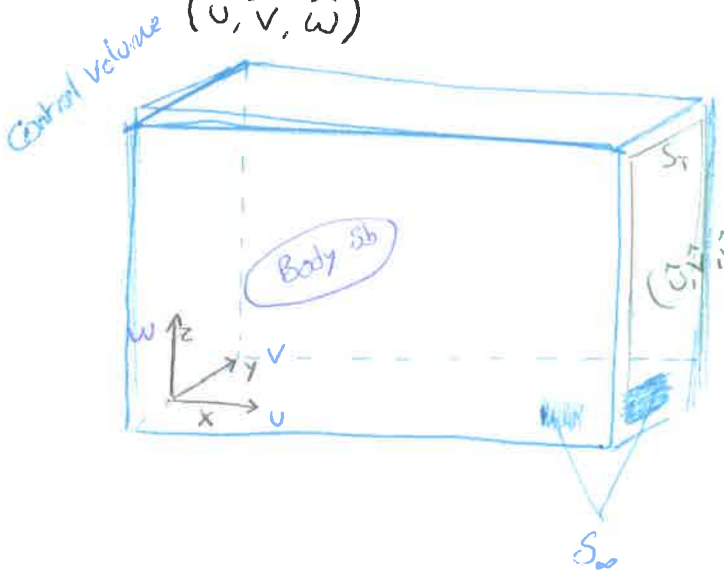
$$D'_i = L'_i\,\alpha_i$$

- $D_i = \int_{-b/2}^{b/2} \Gamma(y)\,\alpha_i(y)\,dy \cdot (\rho_\infty V_\infty)\,dy$

- $C_{D_i} = \dfrac{D_i}{q_\infty S} = \dfrac{2}{V_\infty S} \int_{-b/2}^{b/2} \Gamma(y) \cdot \alpha_i(y)\,dy$

Ex. Cosider a flow over a body of V_∞. Calculate the induce drag as a function of the velocity perturbation:

$(\hat{u}, \hat{v}, \hat{w})$



perturbation non-zero $(\hat{u}, \hat{v}, \hat{w})$ at S_T Trefftz Plane

Assumptions
- Inviscid → Bernoulli's equation
- Incompressible
- Steady state
- Potential flow

Applying the momentum integral in the x-direction:

$$\frac{d(mV)}{dt} = \underbrace{\frac{d}{dt}\iiint_V \rho\vec{U}\cdot dV}_{\text{Momentum of Volum.}} + \underbrace{\iint_{S_\infty + S_b} \rho\vec{U}(\vec{U}\cdot\vec{n})\,ds}_{\text{Surface Momentum}} + \iint_{S_\infty + S_b} P\cdot\vec{n}\,ds$$

$$\iint_{S_\infty + S_b} \rho\vec{U}\vec{U}\cdot\vec{n}\,ds = -\iint_{S_\infty + S_b} P\vec{n}\cdot ds^{*} \longrightarrow \iint_{S_\infty} \rho\vec{U}\vec{U}\,ds = \iint_{S_\infty + S_b} P\vec{n}\cdot ds$$

On body surface $\vec{U}\cdot\vec{n} = 0$

$$\iint_{S_b} P\vec{n}\,ds = \text{Force of body acting on fluid.}$$

Example continuation

* Replacing in * $-\iint p\vec{n}\,ds$

$$-\iint_{S_b} p\vec{n}\,ds = -D\hat{i} - Y\hat{j} - L\hat{k}$$

$$-\iint_{S_\infty} \rho\vec{u}\vec{u}\vec{n}\,ds - \iint p\vec{n}\,ds = D\hat{i} + Y\hat{j} + L\hat{k}$$

Looking only for the drag.

$$D = -\iint \rho\, u\cdot\vec{u}\,\vec{n}\,ds - \iint p\vec{n}\hat{i}\,ds$$

Bernoulli equation (as its inviscid flow)

$$P + \frac{1}{2}V^2\rho = P_\infty + \frac{1}{2}V_\infty\rho \qquad \underbrace{(u^2+v^2+w^2)}=V^2$$

$$p = \underbrace{P_\infty + \frac{1}{2}V_\infty\rho - \frac{1}{2}\rho V^2}$$

$$D = -\iint_{S_\infty} \rho\, u\,\vec{u}\,\vec{n}\,ds - \left(\iint_{S_\infty} p\,\vec{n}\,\hat{i}\,ds\right) \rightarrow P_\infty + \frac{1}{2}\rho V_\infty^2 \iint_{S_\infty} \vec{n}\,\hat{i}\,ds = 0$$

$$D = -\iint_{S_\infty} \rho u\cdot\vec{u}\,\vec{n}\,ds + \iint_{S_\infty} \frac{\rho}{2}(u^2+v^2+w^2)\,n\,\hat{i}\,ds$$

The perturbation flow ($\hat{u}\,\hat{v}\,\hat{w}$)

$$D=\iint_{S_\infty}\frac{1}{2}\rho\left[(V_\infty+\hat{u})^2+\hat{v}^2+\hat{w}^2\right]n_Y\,ds-\rho\iint_{S_\infty}(V_\infty+\hat{u})\,\vec{u}\,\vec{n}\,ds$$

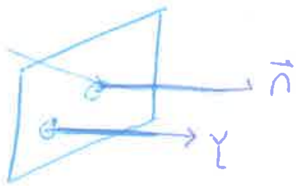
$$D=\iint_{S_\infty}\frac{1}{2}\rho\left(\cancel{V_\infty^2}^{\,0}+2V_\infty\hat{u}+\hat{u}^2+\hat{v}^2+\hat{w}^2\right)n_Y\,ds\;-$$

Conservation of mass

Incompressible flow

$$-\rho\left(\underbrace{\iint_{S_\infty}(V_\infty\,\vec{u}\cdot\vec{n}\,ds)}_{=0}+\iint_{S_\infty}\hat{u}\,\vec{u}\,\vec{n}\,ds\right)$$

$$D=\iint_{S_\infty}\cancel{\frac{1}{2}}\rho\,(\cancel{2}V_\infty\,\hat{u}\,\underbrace{\overset{\vec{n}_Y}{ds}}_{\vec{n}\,ds\,=\,\cos(0)=1\,\leftrightarrow\,\cos\theta})+\frac{1}{2}\rho\iint_{S_\infty}(\hat{u}^2+\hat{v}^2+\hat{w}^2)\,\hat{n}_Y\,ds-\rho\iint_{S_\infty}\hat{u}\,\vec{u}\,\vec{n}\,ds$$



$$D=\cancel{\rho\iint_{S_T}\frac{1}{2}\,\rho V_\infty\,\hat{u}\,ds}+$$

$$+\frac{1}{2}\rho\iint_{S_T}(\hat{u}^2+\hat{v}^2+\hat{w}^2)\,ds\;\cancel{-\rho\iint_{S_T}\rho V_\infty\,\hat{u}\,ds}$$

$$D=\frac{1}{2}\rho\iint_{S_T}(\hat{v}^2+\hat{w}^2-\cancel{\hat{u}^2})\,ds$$ → No velocity in z direction

Gauss - Theorem

$$\iint_S \vec{\nabla} F \cdot ds = \oint_C F \cdot \vec{n} \, dL \longleftarrow \text{Potential Flow}$$

$$D_i = \frac{1}{2} \rho \int_{wake} \Gamma \cdot \vec{V}_n \, dL$$

$V_n \rightarrow$ Velocity en la dirección del vector normal al plano. 3,

Special Case (Elliptical Lift Distribution)

Circulation

Circulation at origin

$$\Gamma = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2} \longrightarrow \text{Ellipse}$$

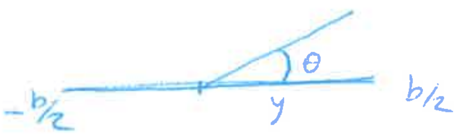
$$L'(y) = \rho_0 V_\infty \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

Downwash v.

$$w = \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{(d\Gamma/dy)}{(y_0 - y)}$$

$$\frac{d\Gamma}{dy} = \Gamma_0 \frac{1}{2} \left[\left(1 - \frac{2y}{b}\right)^2 \right]^{-1/2} \cdot \frac{4 \cdot 2y}{b^2}$$

$$w = -\frac{\Gamma_0}{4\pi b^2} \int_{-b/2}^{b/2} \frac{4 \cdot y \, dy}{\sqrt{1 - \left(\frac{2y}{b}\right)^2} (y_0 - y)}$$



$$y = \frac{b}{2} \cdot \cos\theta$$

$$w(y_0) = \frac{\Gamma_0}{\ldots} \int^{0} \frac{-b/2 \cos\theta \cdot b/2 \, \cancel{\sin\theta} \, d\theta}{\ldots}$$

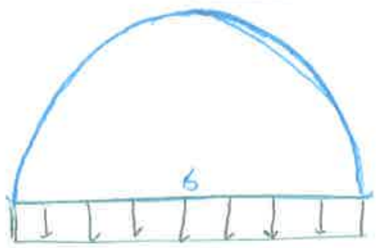
$$\omega(y_0) = \frac{+\Gamma_0}{2\pi b} \int_{\pi}^{0} \frac{\cos\theta \, d\theta}{\cos\theta_0 - \cos\theta}$$

$$\boxed{\int_0^{\pi} \frac{\cos n\theta \, d\theta}{\cos\theta - \cos\theta_0} = \pi \frac{\sin(n\theta_0)}{\sin\theta}}$$

$$\omega(y_0) = \frac{-\Gamma_0}{2\pi b} \int_0^{\pi} \frac{\cos\theta \, d\theta}{\cos\theta - \cos\theta_0}$$

$$\boxed{\omega(y_0) = \frac{-\Gamma_0}{2b}}$$

Downwash is K along b.



$$AR = \frac{b^2}{S} = \frac{b^2}{b\bar{c}} = \frac{b}{\bar{c}}$$

$\bar{c}$ → Avg Chord.

$$\alpha_i = -\frac{\omega(y_0)}{V_\infty} = \frac{S \cdot C_L}{\pi \cdot b^2} = \frac{C_L}{\pi AR}$$

$$\boxed{\alpha_i = \frac{\Gamma_0}{2bV_\infty}}$$

→ The induced drag is K along the span

$$L_{(y)} = \frac{\rho_\infty V_\infty \Gamma_0 \, b\pi}{4}$$

$$\boxed{C_{Di} = \frac{\alpha_i \Gamma_0 \, b \pi}{V_\infty S \, 2}} = \frac{C_L^2}{\pi AR} = \frac{C_L^2 \cdot c}{\pi \cdot b}$$

19/2/16

Elliptical wing

$$\boxed{C_{D_i} = \frac{C_L^2}{\pi AR e}}$$

Span efficiency factor

$$\boxed{e = \frac{1}{(1+\delta)}}$$

↳ Longer wing = Bigger AR → Reduces C_{D_i}

With tappered wings, Stall occurs first in the w/ tip. While in the rest of the wing, there is still lift.

Ex. 1

Tip: ① Calculate coefficient of Fourier Series in function of α.

$$\boxed{\alpha(\theta) = \sum_{n=1}^{N} A_n \sin(\theta n)\left(1 + \frac{n\pi}{2AR\sin\theta}\right)}$$

$$\alpha = A_1 \sin\theta\left(1 + \frac{\pi}{2\cdot AR\cdot\sin\theta}\right) + \cancel{A_2 \sin(2\theta)\left(1 + \frac{2\pi}{2\cdot AR\cdot\sin(\theta)}\right)} +$$

0 (Even no.)

$$+ A_3 \sin(\theta 3)\left(1 + \frac{3\pi}{2AR\sin(\theta)}\right) + A_5 \ldots + A_7 \ldots$$

(The more terms, the more accurate)

Sustituir en la ecuacion α_θ cada valor

Solo cambia el n para α_{θ_1}

$\alpha_{\theta} \Rightarrow \theta_1 = \frac{\pi}{8} \dashrightarrow \alpha_{\theta_1} = 0'644\, A_1 + 2'82\, A_3 + 4.0841 A_5 + 2'21 A_7$

$\alpha_{\theta_2} \Rightarrow \theta_2 = \frac{\pi}{4} \rightarrow \alpha_{\theta_2} = 0'9689\, A_1 + 1'4925\, A_3 + 2'0161\, A_5 - \ldots$

$\alpha_{\theta_3} \rightarrow \theta_3 = \frac{3\pi}{8} \rightarrow$

$\alpha_{\theta_4} \rightarrow \theta_4 = \frac{\pi}{2}$

4 Unknowns for 4 equations

$A_1 = 0'9174\alpha$ $\qquad A_5 = 0'0218\alpha$

$A_3 = 0'1104\alpha$ $\qquad A_7 = 0'0038\,\alpha$

We are calculating half of the wing.

$\boxed{C_L = A_1\, \pi \cdot AR}$ $\quad /2$

$\rightarrow C_{L_1} = 17'29\alpha\, /2 = 8'64\alpha$

$C_{L_1} = \frac{A_1 \pi \cdot AR}{2}$

For elliptical wing:

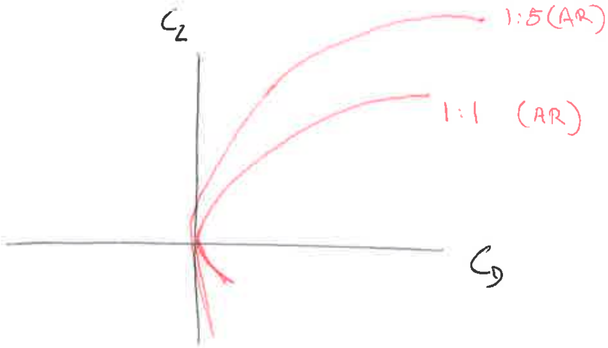
$$C_{D_i} = \frac{C_L^2 (1+\delta)}{\pi AR} = \frac{(8'64\,\alpha)^2}{\pi \cdot \frac{AR}{2}} = 7'93\alpha^2 (1+\delta)$$

EFFECT OF THE AR IN THE WING

$$AR = \frac{b}{c}$$

For general wing:

$$C_D = C_d + \frac{C_L^2}{\pi AR e}$$

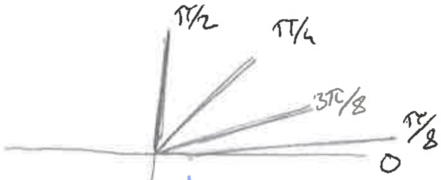


Correction for the fourier coeff

$$\frac{dC_L}{d\alpha} = a_e = \frac{a_0}{1 + a_0/\pi AR} \quad \text{(Elliptical w)}$$

$$a_{GW} = \frac{a_0}{1 + \left(\frac{a_0}{\pi AR}\right)(1+\tau)} \quad 0'05 < \tau < 0'25 \quad \leftarrow \text{General Wing.}$$

$$\boxed{\alpha_i = \frac{w_0}{V_\infty}}$$



Para la mitad del ala dividir 90° en 4 ángulos (el total serían 180°)

Induced Velocity

$$w(\theta_0) = V_\infty \sum_{1}^{N} n \cdot A_n \cdot \frac{\sin(n\theta_0)}{\sin(\theta_0)}$$

$\alpha = 5° = 0'087 \text{ rad}$

$V_\infty = 41'67 \text{ m/s}$

$\theta = \pi/8$

$$w\left(\frac{\pi}{8}\right) = 41'67 \left[0'9174\alpha \cdot \frac{\sin(\theta_0)}{\sin(\theta_0)} + \right.$$

$$\left. + \ldots + A_n \right] \rightarrow \alpha_i$$

$w\left(\frac{\pi}{8}\right) = 41'67 \times \overset{\alpha_i}{0'11?}$ (→ Migh be wrong.)

$$\boxed{w\left(\frac{\pi}{8}\right) = 4'6 \text{ m/s}}$$

$w(\pi/4) = ?$

$w(3\pi/8) = ?$

$w\left(\frac{\pi}{2}\right) = ?$

$$\boxed{\alpha_{eff} = \alpha - \alpha_{i(\theta)}}$$

For elliptical wing ~~to~~ α_i is K.

Homework 7

(2)

NACA 23012

$C_L = a(\alpha - \alpha_{L=0})$

$C_L = 0'8964$

$a =$ slope $0'108$/degree $\longrightarrow$ Slope in C_L vs α

$\alpha_{L=0} = -1'3^\circ$

L?

C_{D_i}?

$\alpha = 7^\circ$

$AR = 8$

T. ratio $0'8$

$\delta = \tau = 0'054$

(3)

$$a = \frac{a_0}{1 + \frac{a_0}{\pi AR}(1+\tau)} = 4'39 \text{ rad}$$

C_L

$C_L = 0'3415$

$$\boxed{D_i = \frac{1}{2} \cdot \rho_\infty V_\infty^2 \cdot S \cdot C_{D_i}}$$

$$\boxed{C_{D_i} = \frac{C_L^2}{\pi AR}}$$

$V = 2\pi \ m^2$

(4)

$e = 0'64$

D_i?

(6)

$AR = 5$

$c = 1'2\,m$

$b = 6\,m$

$\alpha = 4^\circ$

(Parabólico) $C_{d_0} = 0'002$

L?

C_{Di}?

$V_\infty = 200\,m/s$

$\rho_\infty = 0'98\,Kg/m^3$

D_i?

V_i?

α_i?

D?

$AR = 9$

$$\alpha(\theta) = \sum_{n=1}^{N} A_n \sin(\theta_n)\left(1 + \frac{n\pi}{2AR\sin\theta}\right)$$

3'1488

$$\cancel{A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{dz}{dx}\cos(n)\,d\theta}$$

$$\theta = \frac{\pi}{8},\ \frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{\pi}{2}$$

$\alpha_{\theta_1} = 0'6968 A_1 + 3'1992 A_3 + 4'716 A_5$

$\alpha_{\theta_2} =$

$\alpha_{\theta_3} = 1'02 A_1 + 1'6498 A_3 + (-1'033) A_5$

Sistema de ecuaciones

$$\boxed{\begin{array}{ll} A_1 = 0'05475 & A_3 = 0'0085 \\ A_5 = 0'0025 & A_7 = -0'002187 \end{array}}$$

$(2\cdot) C_L = A_n \cdot \pi \cdot AR \qquad C_L = 0'43$

$$\delta = \sum_{2}^{N} n\left(\frac{A_n}{A_1}\right)^2 \qquad \delta = 3\left(\frac{0'0085}{A_1}\right)^2 + \ldots = 0'0723 + 0'010425 + 0'011169$$

$= 0'0938$

6) For AR = 9

Cambia el C_D en función del nuevo AR.

$$C_{D_2} = C_{D_1} - \frac{C_L^2}{\pi e}\left(\frac{1}{AR_1} - \frac{1}{AR_2}\right)$$

5''

Induced velocity.

$$\boxed{w(\theta_0) = V_\infty \sum_{1}^{N} n A_n \frac{\sin(n\theta_0)}{\sin(\theta_0)}}$$

Subsonic - Airfoil - incompressible flow

3 Mar 16

Ex 2.

At a given point on a surface of an a/f, the pressure coefficient is 0'3. At very low speed. If the $M = 0'6$ calculate C_p at this point.

$$C_{p_c} = \frac{C_{p_i}}{\sqrt{1 - M_\infty^2}}$$

Air at $M = 0'6$ is compressible.
(Above $M = 0'3$ is compressible)

$C_{p_i} = 0'3$

$M = 0'6$

$C_{p_c} = -0'5$

Suppose that the a/f is thin and symmetric. Determine C_L for $M = 0'6$, $0'7$, $0'8$ and $0'9$

$$C_L = \frac{C_{L_i}}{\sqrt{1 - M_\infty^2}}$$

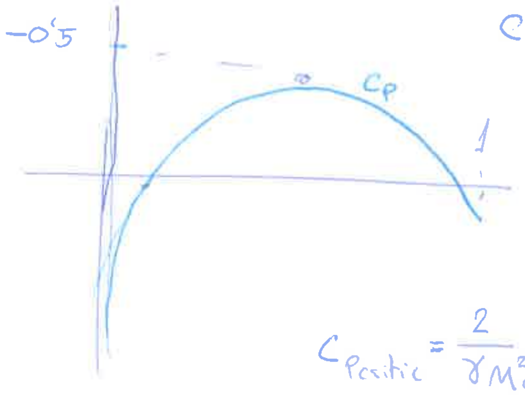
$$C_L = \frac{2\pi\alpha}{\sqrt{1 - M_\infty^2}}$$

M	C_L
.6	~~0'547~~ $7'85\alpha$
.7	$8'8\alpha$
.8	$10'46\alpha$
.9	$14'40\alpha$

If it wasn't symmetric $\rightarrow$ calculate fourier series coeff. (A_n.)

Ex. 3

Determine M_c (critical) for NACA 0012 at $\alpha = 0°$



$C_{p\,min} = -0'43$

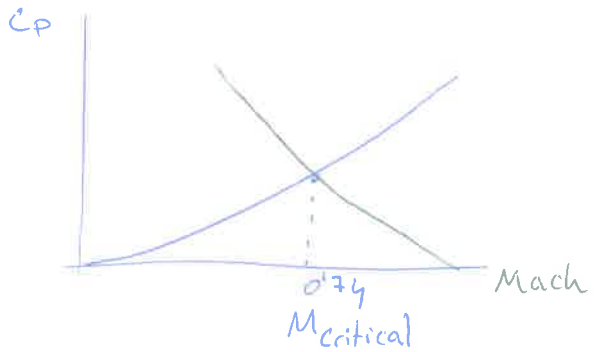
$Re = 365 \times 10^6$

$$C_{p\,critic} = \frac{2}{\gamma M_{critic}^2}\left[\left(\frac{\gamma-1}{\gamma+1} M_c^2 + \frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

$\gamma = 1'4$

M	0'4	.5	.6	.7	.8	.9	1
$C_{p\,crit.}$	-3'66	-2'13	-1'29	-0'779	-0'435	-0'188	0

$$C_{p\,critic} = \frac{(C_{p_i})}{\sqrt{1-M_\infty^2}} = \frac{-0'43}{\sqrt{1-M_\infty^2}}$$



4 Mar 16

Aerodynamics for a/c Lift and Drag

Lift

Produced by wing-Body Combination

HTP ; Canards

DRAG

$$C_D = C_d + \frac{C_L}{\pi A R e}$$

Total D.

Parasitic D. Coeff. (Not only p and skin drag also friction an P Drag from the Tail, fuselage etc...)

$$\boxed{C_D = C_{D_e} + \frac{C_L}{\pi A R e}}$$

$$\boxed{C_{De} = C_{D_0} + r\,C_L^2}$$

Parasite C_D at $L=0$ drag coeff.

Empirical constant

$L = f(\alpha)$ The C_{D_e} is also function of C_L

Drag coeff. for the entire a/c (Considering HTP, engine, etc...)

$$C_D = C_{D_0} + r\,C_L^2 + \frac{C_L}{\pi A R e}$$

$$\Rightarrow C_D = C_{D_0} + C_L^2\left(r + \frac{1}{\pi e AR}\right)$$

e includes the effect of variations of parasite Drag with Lift.

~~e = 1'78 (1 - 0'04~~

Drag Polar

$$\boxed{C_D = C_{D_0} + \frac{C_L^2}{\pi e AR}}$$

↳ Represents C_L vs C_D

$C_L \rightarrow$ of the full a/c

$AR \rightarrow$ of the wing.

Oswald Factor

4 Mar 16

Seversky P-35

w. Area = 20'4 m^2

w. Span (b) = (10'8 m)

At 18° $\longrightarrow C_{D,T} = 0'0275$

At $\alpha = ?$

$\hookrightarrow C_L = 0'15$

$C_{D_0} = ?$

$e = 0'878$

$$C_{D,T} = C_{D_0} + \frac{C_L^2}{\pi e AR}$$

$$AR = \frac{b^2}{S}$$

$AR = 5'7176$

$$C_{D,T} = 0'0275 + \frac{0'15^2}{0'878 \cdot AR}$$

$C_{D_0} = 0'026$

$$\left(\frac{C_L}{C_D}\right)_{max} = 11'8$$

⑤ Obtain (and explain how to) the wing parameters of an a/c flying at sea level condt. $V = 500 \, km/h$? and a wing load $W/S = 7 \ldots \, N/m^2$

Wing Data

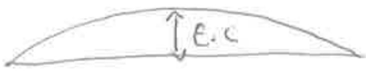
- TR =
- NACA
- $\alpha_{L=0}$
-

1)

$$z = c\,\varepsilon\left[1 + \left(\frac{x}{c}\right)^2\right]$$

Calculate:

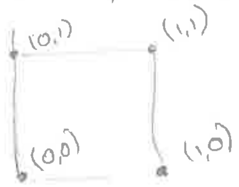
- Ideal α (Explain)
- $\alpha_{L=0}$
- C_L ; C_m
- x_{cp}



2) Are simillar flows for equal μ and c (s.o. sound)? Both proportional to $\sqrt{T}$.

Airfoil 1	Airfoil 2
$c = x$	$c = 2x$
$\rho = 1'23$	$\rho = 1'74$
$v = 100$	$v = 200$
$T = 200K$?	$T = 800k$

3) Circulation w/ velocity vectors $u = xy^2$ $v = xy$



$$\boxed{\theta^\circ = \frac{Rad\, 180^\circ}{\pi ?}\ ? \qquad Rad = \frac{\theta^\circ \cdot \pi}{180^\circ}}$$

$$P_a + \frac{1}{2}\rho V^2$$

Simmetric.

$\alpha = 1'5^\circ$

$\alpha = 1'5^\circ = 0'0261\,Rad.$

$C_l = 0'164$

$q_\infty = \frac{1}{2}\rho V^2\ (P_a)$

$C_{M,LE} = -\frac{C_L}{4} = -0'041$

Γ = Circulation $\quad \Gamma = V_\infty c\, \alpha \pi$

Elemental vortex ~~of~~ strength $\gamma(\xi)\, d\xi$

ξ Distance from LE.

$\underbrace{dL = \rho_\infty V_\infty d\Gamma}$ $\quad (L' = \text{Lift/unit span}) = \pi \alpha c \rho_\infty V_\infty^2$

Increment of L creates M about the LE $\longrightarrow dM = -\xi(dL)$

Moment about LE

$$M'_{LE} = -\int_0^c \xi\,(dL) = -\rho_\infty V_\infty \int_0^c \xi \gamma(\xi)\, d\xi$$

$$\boxed{M'_{LE} = -q_\infty c^2 \frac{\pi \alpha}{2}}$$

Lift C_l

$\boxed{C_l = 2\pi\alpha}$ in Rad.

$C_l = \frac{L'}{q_\infty S}$

Coefficient $\Rightarrow$ $\boxed{C_m)_{LE} = \frac{M'_{LE}}{q_\infty S_c}}$

$S = c(1)$

$C_{M,LE} = \frac{M'_{LE}}{q_\infty c^2} = \frac{-\pi \cdot \alpha}{2} \longrightarrow \pi\alpha = \frac{C_L}{2}$