Tema 1 - Señales en el Dominio Temporal

Cuestiones resueltas

C2 Calcular y representar:

- Real, imag.
- Módulo, fase.

Para un número complejo: $v = v_r + j v_i = v_m e^{j v_f}$

Para una señal compleja: $v(t) = v_r(t) + j v_i(t) = v_m(t) e^{j v_f(t)}$

• $x_1(t) = \cos(\pi t) + j \operatorname{sen}(\pi t)$

$$x_r(t) = \cos(\pi t)$$

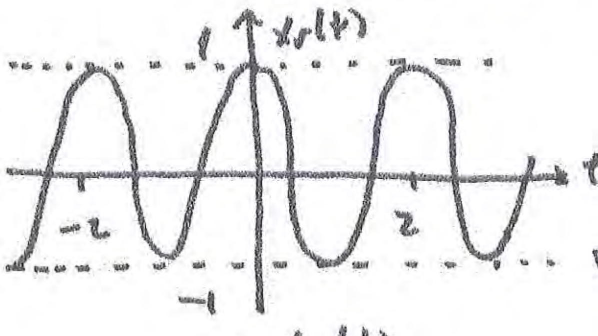
$$x_i(t) = \operatorname{sen}(\pi t)$$

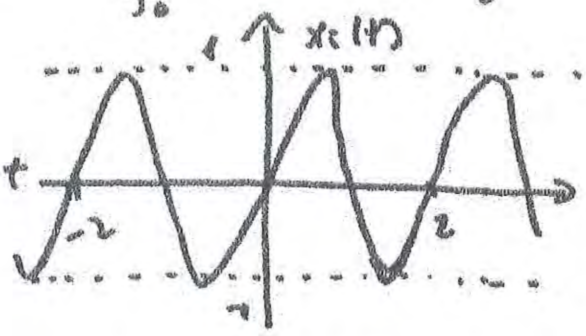
$$x_m^2(t) = x_r^2(t) + x_i^2(t) = \cos^2(\pi t) + \operatorname{sen}^2(\pi t) = 1.$$

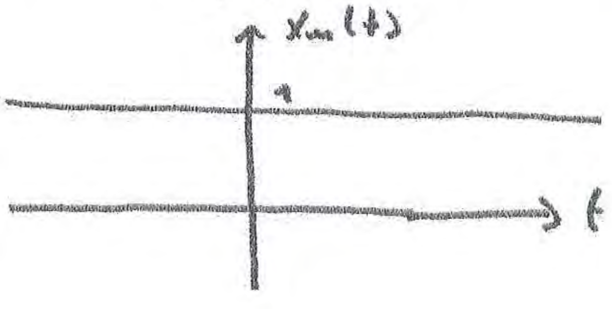
$$x_f(t) = \operatorname{arctg} \frac{x_i(t)}{x_r(t)} = \operatorname{arctg}\left(\frac{\operatorname{sen}(\pi t)}{\cos(\pi t)}\right) =$$

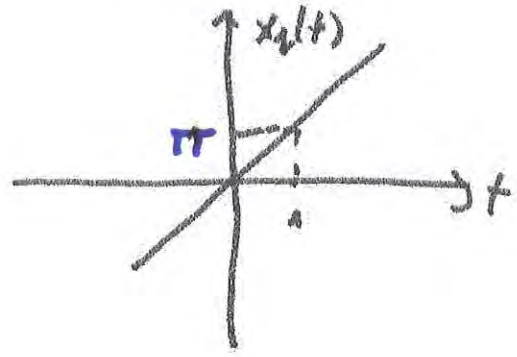
$$= \operatorname{arctg}(\operatorname{tg}(\pi t)) = \pi t.$$

Calculamos el periodo: $w_0 = \pi = \frac{2\pi}{T_0} \Rightarrow T_0 = 2$ seg.

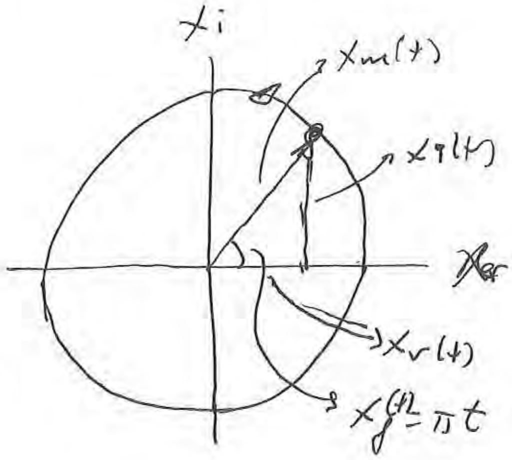




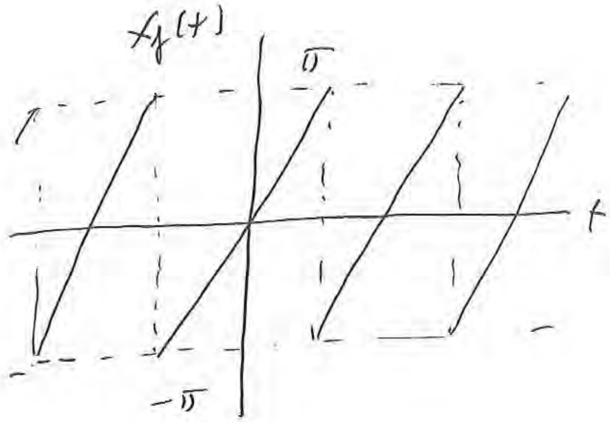




<u>Nota 1</u>: podemos ver esta señal como las coordenadas de un punto que sigue una trayectoria circular en un plano, a velocidad angular constante.



<u>Nota 2</u>: según lo anterior, la fase basta con representarla entre $[0, 2\pi)$ o entre $(-\pi, \pi]$, por ser redundante.



(2) $x(t) = \sqrt{t}$

¿Es una señal compleja? Para $t < 0 \rightarrow \sqrt{t} = \sqrt{-(-t)} = j\sqrt{(-t)}$

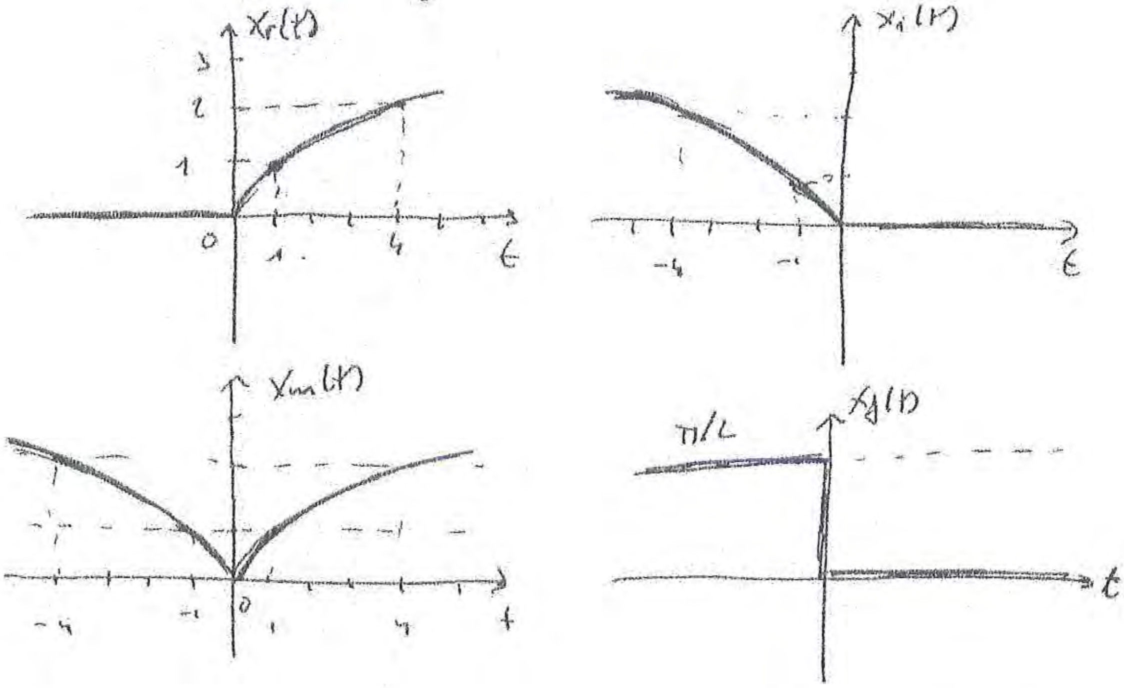
$$x(t) = \begin{cases} \sqrt{t}, & t \geq 0 \\ j\sqrt{(-t)}, & t < 0 \end{cases}$$

$$x_r(t) = \begin{cases} \sqrt{t}, & t \geq 0 \\ 0, & t < 0 \end{cases} \qquad x_i(t) = \begin{cases} 0, & t \geq 0 \\ \sqrt{(-t)}, & t < 0 \end{cases}$$

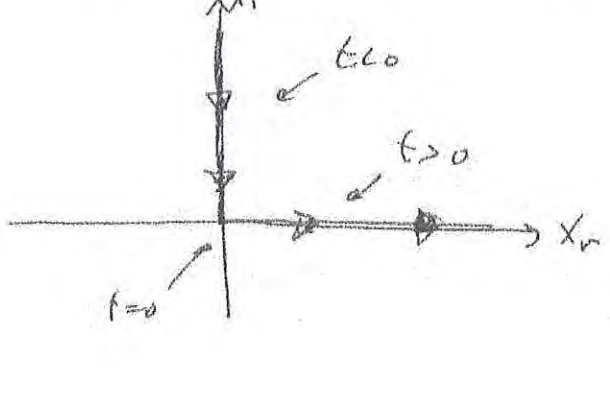
$$x_m^2(t) = \left\{ \begin{matrix} (\sqrt{t})^2 + 0^2, & t \geq 0 \\ 0^2 + (\sqrt{-t})^2, & t < 0 \end{matrix} \right\} = \left\{ \begin{matrix} t, & t \geq 0 \\ -t, & t < 0 \end{matrix} \right\} = |t|$$

$$x_\varphi(t) = \begin{cases} \operatorname{arctg} \dfrac{0}{\sqrt{t}}, & t \geq 0 \\ \operatorname{arctg} \dfrac{\sqrt{-t}}{0}, & t < 0 \end{cases} = \begin{cases} \operatorname{arctg}(0), & t \geq 0 \\ \operatorname{arctg}(+\infty), & t < 0 \end{cases} =$$

$$= \begin{cases} \pi/2, & t < 0 \\ 0, & t \geq 0 \end{cases}$$



NOTA 3: puede verse $x(t)$ como la trayectoria de un punto en $\mathbb{R}^2$ que primero llega al origen por el eje imaginario y luego se aleja de él por el eje real.



(3) $x(t) = e^{-2t} \cdot e^{-j2t}$

En este caso, la forma exponencial nos permite comenzar con módulo y fase.

$$x(t) = (e^{-2t}) \cdot e^{j(-2t)}$$

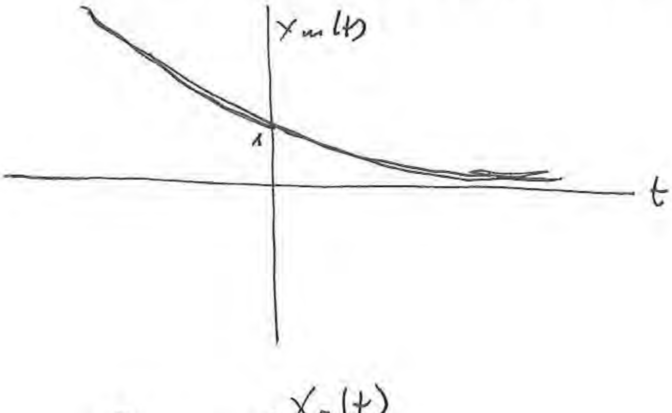
$$x_m(t) = e^{-2t} \qquad x_f(t) = -2t$$

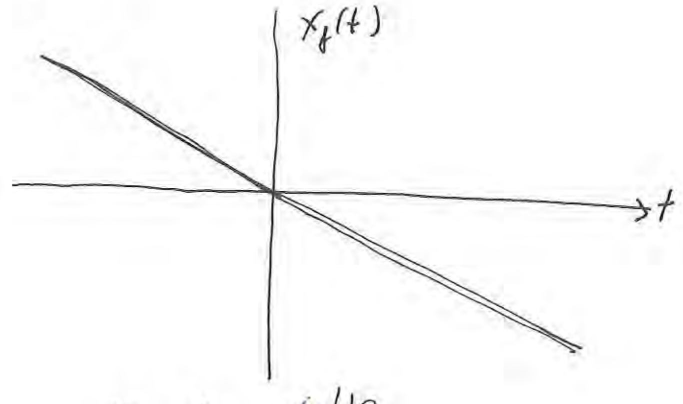
Sabemos por la igualdad de Euler que: $e^{j\theta} = \cos\theta + j\,\mathrm{sen}\,\theta$. Entonces:

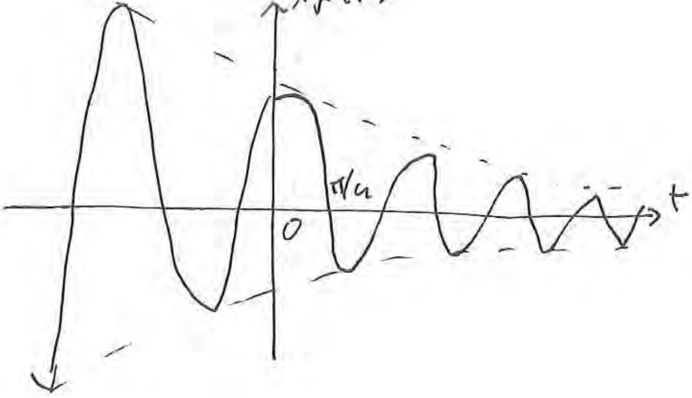
$$x(t) = e^{-2t} e^{-j2t} = e^{-2t}\left(\cos(-2t) + j\,\mathrm{sen}(-2t)\right) =$$

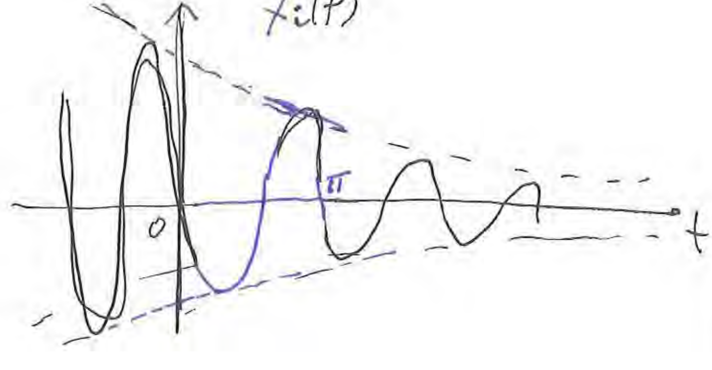
$$= e^{-2t}\cos(2t) - je^{-2t}\,\mathrm{sen}(2t) \Rightarrow$$

$$x_r(t) = e^{-2t}\cos(2t); \qquad x_i(t) = -e^{-2t}\,\mathrm{sen}(2t)$$









Nota 4: puede verse $x(t)$ como la trayectoria espiral de un punto en un plano. ¿Por qué? Explicar cualitativamente.

C3 Estudiar las simetrías de:

(Para estudiar las simetrías de una señal real, calculo $x(-t)$ y la comparo con $x(t)$).

(1) $x(t) = \operatorname{sen}(\pi t)$

$x(-t) = \operatorname{sen}(\pi(-t)) = \operatorname{sen}(-\pi t) = -\operatorname{sen}(\pi t) = -x(t)$.

$\Rightarrow x(t)$ tiene simetría impar.

(2) $y(t) = \cos(2\pi t)$

$y(-t) = \cos(2\pi(-t)) = \cos(-2\pi t) = \cos(2\pi t) = y(t)$

$\Rightarrow y(t)$ tiene simetría par.

(3) $z(t) = e^{-\alpha t}$

$z(-t) = e^{-\alpha(-t)} = e^{\alpha t} \Rightarrow z(t)$ no tiene simetrías.

C4 Calcular la parte par e impar de las señales anteriores.

$x(t) = x_p(t) + x_i(t)$, donde

$$x_p(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_i(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

Lo aplicamos a cada señal del apartado anterior.

(1) $x(t) = \text{sen}(\pi t)$

$x(-t) = -\text{sen}(\pi t)$

$x_p(t) = \frac{1}{2}\left[\text{sen}(\pi t) - \text{sen}(\pi t)\right] = 0$

$x_i(t) = \frac{1}{2}\left[\text{sen}(\pi t) - (-\text{sen}(\pi t))\right] = \text{sen}(\pi t) = x(t)$

(2) $y(t) = \cos(2\pi t)$

$y(-t) = \cos(2\pi t)$

$y_p(t) = \frac{1}{2}\left[\cos(2\pi t) + \cos(2\pi t)\right] = \cos(2\pi t) = y(t)$

$y_i(t) = \frac{1}{2}\left[\cos(2\pi t) - \cos(2\pi t)\right] = 0$

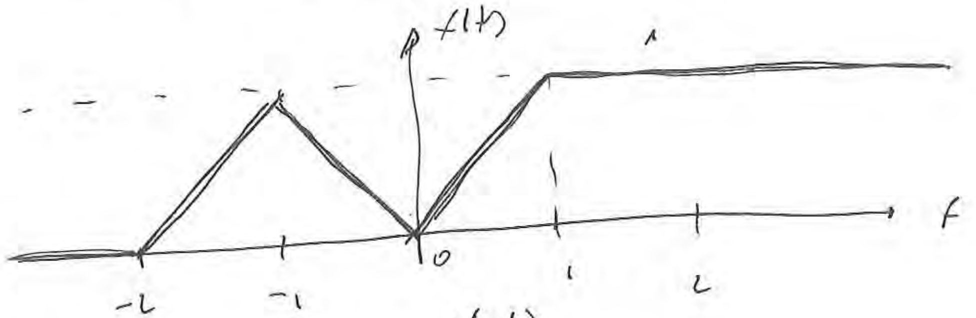
(3) $z(t) = e^{-\alpha t}$

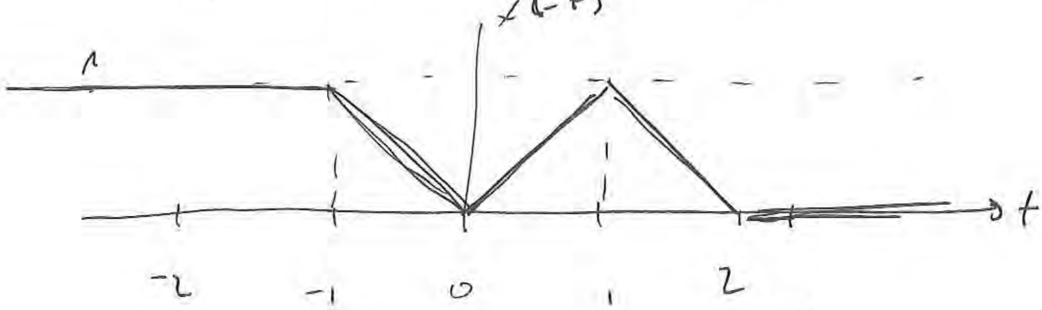
$z(-t) = e^{\alpha t}$

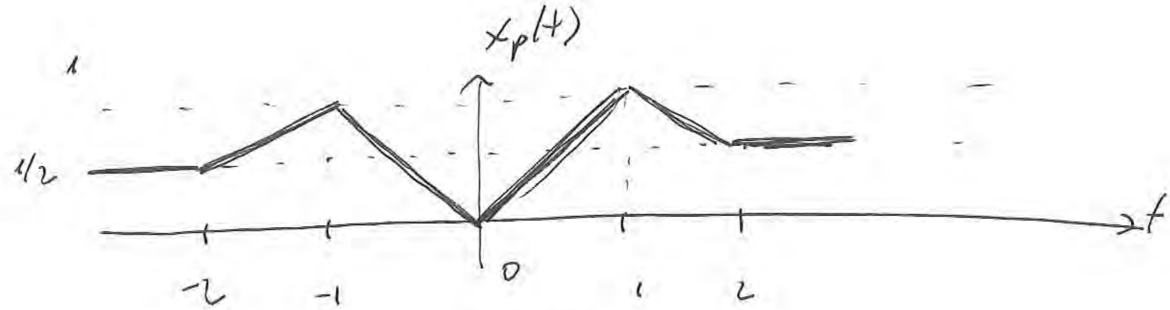
$z_p(t) = \frac{1}{2}\left[e^{-\alpha t} + e^{\alpha t}\right]$

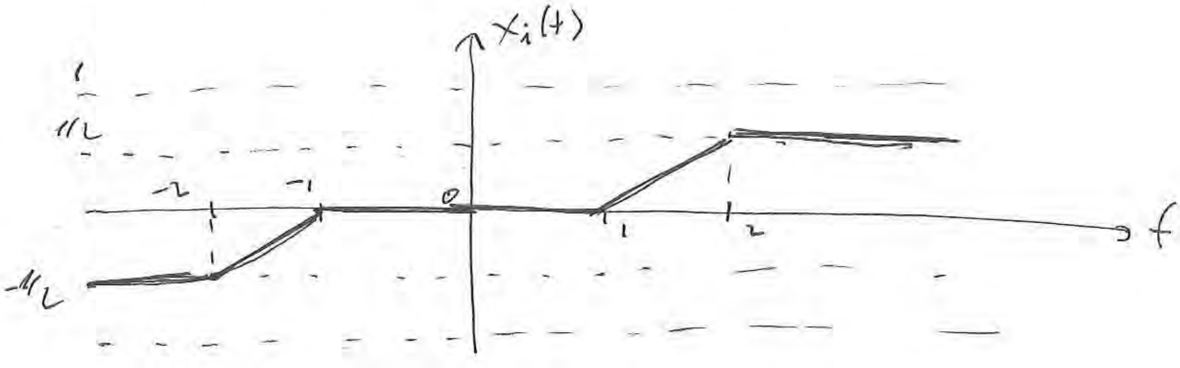
$z_i(t) = \frac{1}{2}\left[e^{-\alpha t} - e^{\alpha t}\right]$

(C5) Calcular la parte par y la parte impar de la señal de la figura.









C6 Encontrar la parte hermítica y antihermítica. Sabemos que

$x(t) = x_h(t) + x_a(t)$, donde

$$x_h(t) = \frac{1}{2}\left(x(t) + x^*(-t)\right)$$

$$x_a(t) = \frac{1}{2}\left(x(t) - x^*(-t)\right)$$

(1) $x(t) = \cos(w_0 t) + j\,\mathrm{sen}(w_0 t) = e^{jw_0 t}$

$x(-t) = e^{-jw_0 t} \Rightarrow x^*(-t) = e^{-jw_0(-t)} = e^{jw_0 t}$

Por tanto, $x_h(t) = \frac{1}{2}\left(e^{jw_0 t} + e^{jw_0 t}\right) = e^{jw_0 t} = x(t)$ //

$x_a(t) = \frac{1}{2}\left(e^{jw_0 t} - e^{+jw_0 t}\right) = 0$ //

Se trata de una señal hermítica.

(2) $y(t) = e^{-2t} e^{+5jt}$

$y(-t) = e^{2t} e^{-5jt} \Rightarrow y^*(-t) = e^{2t} e^{5jt}$.

$y_h(t) = \frac{1}{2}\left(e^{-2t} e^{5jt} + e^{2t} e^{5jt}\right) = \frac{e^{5jt}}{2}\left(e^{-2t} + e^{2t}\right)$ //

$y_a(t) = \frac{1}{2}\left(e^{-2t} e^{5jt} - e^{2t} \cdot e^{5jt}\right) = \frac{e^{5jt}}{2}\left(e^{-2t} - e^{2t}\right)$ //

C7 $x(t) = x_r(t) + j\, x_i(t)$, por ser compleja.

$x(t) = x^*(-t)$, por ser hermítica.

$$x^*(-t) = x_r(-t) - j\, x_i(-t) = x_r(t) + j\, x_i(t)$$

Para que sean iguales, tienen que serlo sus partes real e imaginaria:

$x_r(-t) = x_r(t) \Rightarrow x_r(t)$ es par.

$-x_i(-t) = x_i(t) \Rightarrow x_i(t)$ es impar. //

Si expresamos la señal en forma exponencial:

$$x(t) = x_m(t) \cdot e^{j x_f(t)}$$

$$x(-t) = x_m(-t) \cdot e^{j x_f(-t)}$$

$$x^*(-t) = x_m(-t) \cdot e^{-j x_f(-t)}$$

Para que $x(t) = x^*(-t)$, han de ser iguales en módulo y en fase:

$x_m(t) = x_m(-t) \Rightarrow x_m(t)$ es par

$x_f(t) = -x_f(-t) \Rightarrow x_f(t)$ es impar.

C9

$$\left.\begin{array}{l} x_1(t) = x_1(t+T_0) \\ x_2(t) = x_2(t+T_0) \end{array}\right\} y(t) = x_1(t) + x_2(t)$$

¿Es periódica?

Si lo es, ha de existir T_y tal que $y(t) = y(t+T_y)$ y lo tenemos que encontrar. Ensayamos $T_y = T_0$.

$$y(t+T_0) = x_1(t+T_0) + x_2(t+T_0) =$$
$$= x_1(t) + x_2(t) = y(t) \,//$$

Por tanto, $y(t)$ es periódica y su periodo es T_0.

C10

$$\left.\begin{array}{l} x_1(t) = x_1(t+T_1) \\ x_2(t) = x_2(t+T_2) \end{array}\right\} \Rightarrow y(t) = x_1(t) + x_2(t)$$

es periódica.

Ensayamos con $T_y = \text{mcm}\{T_1, T_2\} \Rightarrow T_y = k_1 T_1 = k_2 T_2$ (con $k_1, k_2 \in \mathbb{Z}$)

$$y(t+T_y) = x_1(t+T_y) +$$
$$+ x_2(t+T_y) = x_1(t+k_1T_1) + x_2(t+k_2T_2) =$$
$$= x_1(t) + x_2(t) = y(t) \,//$$

C11 • $x_1(t) = \cos(w_0 t)$. Hemos visto que las sinusoidales por sí solas son siempre periódicas, por tanto ésta lo es.

$$w_0 = \frac{2\pi}{T_0} \Rightarrow T_0 = \frac{2\pi}{w_0} \text{ segs.} //$$

• $x_2(t) = \operatorname{sen}(w_0 t + 1/2)$. Como en el caso anterior, lo es:

$$w_0 = \frac{2\pi}{T_0} \Rightarrow T_0 = \frac{2\pi}{w_0} \text{ segs.} //$$

• $x_3(t) = e^{jw_0 t} = \cos(w_0 t) + j\operatorname{sen}(w_0 t)$. Es periódica por ser exponencial imaginaria pura. O bien, es periódica por ser suma de dos señales periódicas con el mismo periodo.

$$T_0 = \frac{2\pi}{w_0} \text{ segs.} //$$

• $x_4(t) = \cos(10\pi t) \rightarrow w_0 = 10\pi = \frac{2\pi}{T_0} \Rightarrow T_0 = \frac{1}{5} \text{ seg.} //$

• $x_5(t) = \underbrace{\operatorname{sen}(10\pi t)}_{(1)} + \underbrace{\cos(20\pi t)}_{(2)}$

$$\left.\begin{array}{l} T_1 = \frac{2\pi}{w_1} = \frac{2\cdot\pi}{10\cdot\pi} = \frac{1}{5} \text{ sg} \\ T_2 = \frac{2\pi}{w_2} = \frac{2\cdot\pi}{20\pi} = \frac{1}{10} \text{ sg} \end{array}\right\} T_5 = \text{m.c.m.}\left\{\frac{1}{5}, \frac{1}{10}\right\} = \frac{1}{5} \text{ sg} //$$

• $x_6(t) = \underbrace{\operatorname{sen}(10\pi t)}_{(1)} + \underbrace{\cos(20t)}_{(2)}$

$$\left.\begin{array}{l} (1)\ T_1 = \frac{2\cdot\pi}{10\pi} = \frac{1}{5} \text{ sg.} \\ (2)\ T_2 = \frac{2\pi}{20} = \frac{\pi}{10} \text{ sg.} \end{array}\right| \begin{array}{l} T_6 = \text{m.c.m.}\{T_1, T_2\} = \\ = \text{m.c.m.}\left\{\frac{1}{5}, \frac{\pi}{10}\right\} \Rightarrow \nexists \end{array}$$

No es periódica.

CUESTIONES NO RESUELTAS:

C.11 (continuación)

- ¿Es periódica la señal $x(t) = \cos(10\pi t)$?

$$\omega_0 = 10\pi = \frac{2\pi}{T_0} \Rightarrow 5 \cdot \cancel{2\pi} = \frac{\cancel{2\pi}}{T_0} \Rightarrow T_0 = \frac{1}{5}$$

- ¿Es periódica la señal $x(t) = \mathrm{sen}(10\pi t) + \cos(20\pi t)$?

  $\mathrm{sen}(10\pi t)$ es periódica de periodo $T_1 = \frac{1}{5}$

  $\cos(2\pi t)$ es periódica de periodo... $\Rightarrow \omega_0 = 20\pi = \frac{2\pi}{T_0} \Rightarrow$

$$10 \cdot 2\pi = \frac{2\pi}{T_0} \Rightarrow T_0 = \frac{1}{10} \Rightarrow$$

  La suma de dos señales $\Rightarrow$ periódicas es periódica de periodo el m.c.m $\{T_1, T_0\} \Rightarrow x(t)$ es periódica de periodo $1/5$.

- Es periódica la señal $x(t) = \mathrm{sen}(10\pi t) + \cos(20t)$?

  $\mathrm{sen}(10\pi t)$ es periódica de periodo $T_1 = \frac{1}{5}$

  $\cos(20t)$ es periódica de periodo... $\Rightarrow \omega_0 = 20 = \frac{2\pi}{T_0}$

$$\Rightarrow 10 \cdot \cancel{2} = \frac{\cancel{2} \cdot \pi}{T_0} \Rightarrow T_0 = \frac{\pi}{10}$$

  m.c.m $\{T_1, T_0\}$ = ??? $\Rightarrow$ **NO EXISTE**, luego no es periódica

- Es periódica la señal $x(t) = \mathrm{sen}(10\pi t) \cdot \cos(20\pi t)$

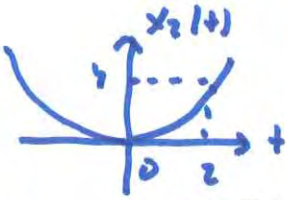
  El producto de dos señales periódicas es periódico con periodo también el mcm de sus periodos, luego $x(t)$ es periódica de periodo $\frac{1}{5}$.

C12 • $x_1(t) = e^{-t}$

$$\langle x_1(t)\rangle_{(2,3)} = \frac{1}{t_f - t_i}\int_{t_i}^{t_f} x_1(t)\,dt = \frac{1}{3-2}\int_2^3 e^{-t}\,dt = \frac{1}{1}\left[\frac{1}{-1}e^{-t}\right]_2^3$$

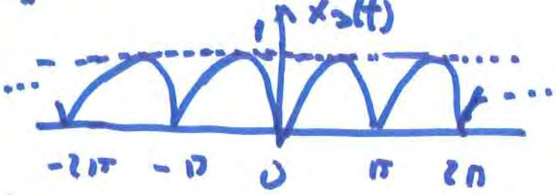
$$= -\left(e^{-3} - e^{-2}\right) =$$

• $x_2(t) = t^2$



$$\langle x_2(t)\rangle_{(1,3)} = \frac{1}{3-1}\int_1^3 t^2\,dt = \frac{1}{2}\left[\frac{t^3}{3}\right]_1^3 = \frac{1}{6}\left(3^3 - 1^3\right) = \frac{26}{6}$$

• $x_3(t) = |\,\mathrm{sen}\ t\,|$

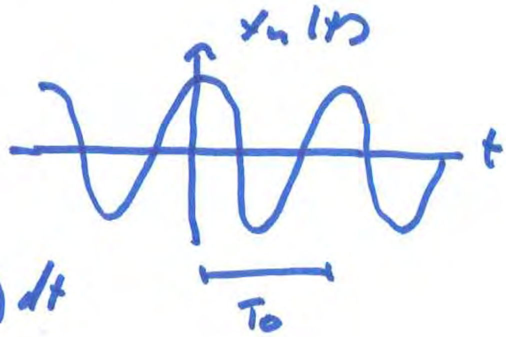


Es periódica. $T_0 = \pi$ (ojo, no 2π)

$$\langle x_3(t)\rangle = \frac{1}{T_0}\int_{\langle T_0\rangle} x_3(t)\,dt = \frac{1}{\pi}\int_0^{\pi} |\,\mathrm{sen}\ t\,|\,dt =$$

$$= \frac{1}{\pi}\int_0^{\pi} \mathrm{sen}\ t\,dt = \frac{1}{\pi}\left[-\cos t\right]_0^{\pi} = -\frac{1}{\pi}\left(\cos\pi - \cos 0\right) = -\frac{1}{\pi}(-1-1) = \frac{2}{\pi}$$

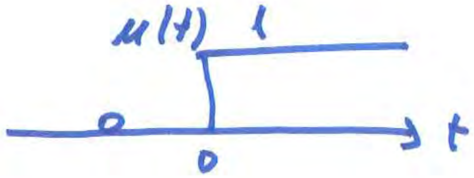
• $x_4(t) = \cos\left(\frac{\pi t}{2}\right)$



$$\langle x_4(t)\rangle = \frac{1}{T_0}\int_{\langle T_0\rangle} \cos\left(\frac{\pi t}{2}\right)dt$$

$= 0$ // (por consideraciones geométricas)

• $x_5(t) = u(t)$



$$\langle x_5(t)\rangle_{(-1,3)} = \frac{1}{3-(-1)}\int_{-1}^{3} u(t)\,dt =$$

$$= \frac{1}{4}\int_{-1}^{0} 0\,dt + \frac{1}{4}\int_{0}^{3} 1\cdot dt = \frac{1}{4}\left[t\right]_0^3 = \frac{3}{4} \;//$$

• $x_6(t) = u(t)$. $\langle x_6(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u(t)\,dt =$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{0}^{T} 1\cdot dt = \lim_{T\to\infty}\frac{T}{2T} = 1/2 \;//$$

• $x_7(t) = e^{j(5\pi t - \frac{1}{3})}$, exp. complejo imaginario puro.

$$\langle x_7(t)\rangle_{(-1,3)} = \frac{1}{3-(-1)}\int_{-1}^{3} e^{j(5\pi t - \frac{1}{3})}\,dt =$$

$$= \frac{1}{4}\left[\frac{1}{j5\pi} e^{j(5\pi t - \frac{1}{3})}\right]_{-1}^{3} = \dots = 0$$

$$\frac{1}{j20\pi}\left(e^{j(15\pi - \frac{1}{3})} - e^{j(-5\pi - \frac{1}{3})}\right) = \frac{1}{j20\pi}\left(e^{j15\pi}\cdot e^{-j/3} - e^{-j5\pi}\cdot e^{-j/3}\right) =$$

$$\frac{e^{-j/3}}{j20\pi}\Big(\underbrace{e^{j15\pi}}_{\substack{\cos(15\pi)+j\,\text{sen}(15\pi)\\ -1+j0=-1}} - \underbrace{e^{-j5\pi}}_{\substack{\cos(-5\pi)+j\,\text{sen}(-5\pi)\\ -1+j0=-1}}\Big) = \frac{e^{-j/3}}{j20\pi}\left[-1-(-1)\right] = 0$$

C13 $x(t)$ definida en energía, entonces:

$$E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty$$. Por tanto, su potencia es:

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \cdot \int_{-T}^{T} |x(\tau)|^2 d\tau = \lim_{T\to\infty} \frac{1}{2T} \cdot \underbrace{\lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt}_{E_\infty} =$$

$$= \lim_{T\to\infty} \frac{E_\infty}{2T} = 0 \ //$$

Si $x(t)$ está definida en potencia:

$$P_\infty = C \Rightarrow \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = C \Rightarrow$$

$$\underbrace{\lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt} = C \cdot \underbrace{\lim_{T\to\infty} 2T} \Rightarrow E_\infty = C \cdot \infty = \infty \ //$$

C14 $E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2 dt \qquad P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$

(1) $x_1(t) = u(t)$.

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{0}^{T} 1^2 dt = \lim_{T\to\infty} \frac{T}{2T} = \frac{1}{2}\,W$$

(2) $x_2(t) = e^{-2t}\cdot u(t)$

$$E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_0^{+\infty} (e^{-2t})^2 dt = \frac{-1}{4}\left[e^{-4t}\right]_0^{+\infty} = \frac{1}{4}\,J$$

(3) $x_3(t) = e^{j(2t+\frac{\pi}{4})}$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |e^{j(2t+\frac{\pi}{4})}|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 1\, dt = 1\,W$$

(4) $x_4(t) = \cos t$

$$\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2}+\frac{1}{2}\cos(2t)\right)dt = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}dt + \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\cos 2t\, dt$$
$$= \frac{1}{4\pi}(2\pi - 0) + \frac{1}{4\pi}\left[\frac{1}{2}\operatorname{sen} 2t\right]_0^{2\pi} = \frac{2\pi}{4\pi} + \frac{1}{8\pi}(\operatorname{sen} 4\pi - \operatorname{sen} 0) = \frac{1}{2} + 0$$

$$P_\infty = P_{T_0} = \frac{1}{T_0}\int_0^{T_0} |\cos t|^2 dt = \frac{1}{2\pi}\int_0^{2\pi}\cos^2 t\, dt = \ldots = \frac{1}{2}\,W$$

(5) $x_5(t) = \left(\frac{1}{2}\right)^t\cdot u(t)$

$$E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_0^{+\infty}\left(\frac{1}{2}\right)^{2t} dt = \int_0^{+\infty}\left(\frac{1}{4}\right)^t dt = \frac{1}{\ln 4}\,J$$

$$\int a^t dt = \frac{a^t}{\ln a} \Rightarrow \int_0^\infty \left(\frac{1}{4}\right)^t dt = \left[\frac{\left(\frac{1}{4}\right)^t}{\ln\frac{1}{4}}\right]_0^\infty = \frac{1}{\underbrace{\ln 1}_{0} - \ln 4}\cdot\left[\underbrace{\left(\tfrac{1}{4}\right)^\infty}_{0} - \underbrace{\left(\tfrac{1}{4}\right)^0}_{1}\right] =$$

$$= \frac{-1}{-\ln 4} = \frac{1}{\ln 4}$$

C.15 • ¿Cómo afecta el orden para $v(t) = x(-t+b)$?

Primer intento (y correcto) $\Rightarrow$ $z(t) = x(t+b)$

$$s(t) = z(-t) = x(-t+b) = v(t) \quad (ok)$$

Segundo intento (mal) $\Rightarrow$ $z(t) = x(-t)$

$$s(t) = z(t+b) = x(-(t+b)) = x(-t-b) \neq v(t)$$

• ¿Cómo afecta el orden para $v(t) = x(-at)$?

Primer intento (y correcto) $\Rightarrow$ $z(t) = x(at)$

$$s(t) = z(-t) = x(a(-t)) = x(-at) \quad ok$$

Segundo intento (y tb. correcto $\Rightarrow$ $z(t) = x(-t)$

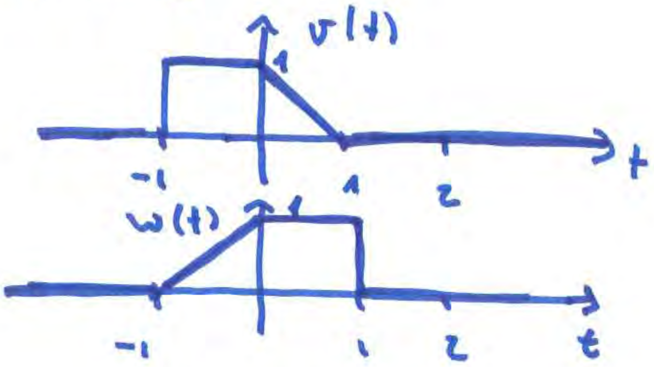
$$s(t) = z(at) = x(-(at)) = x(-at) \quad ok$$

• ¿Cuál es el orden que se debe seguir cuando hay varias transformaciones de la variable independiente?

Primero siempre el desplazamiento, luego lo que se quiera.

C17

- $y_1(t) = x(-t+1)$



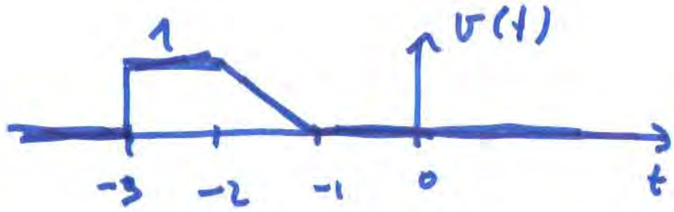
$v(t) = x(t+1)$

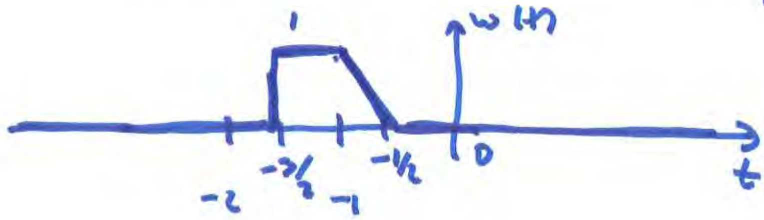
$w(t) = v(-t) =$

$= x(-t+1) = y_1(t)$

- $y_2(t) = x(2t+3)$

$v(t) = x(t+3)$

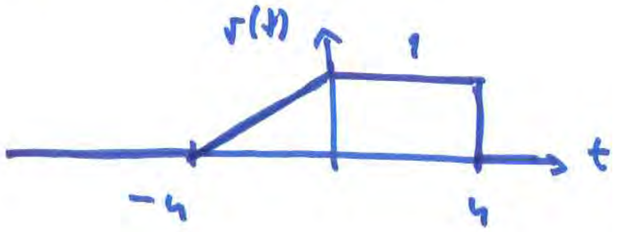




$w(t) = v(2t) =$

$x(2t+3) = y_2(t)$

- $y_4(t) = -2\ x\left(-\frac{t}{4}+1\right) + 3$

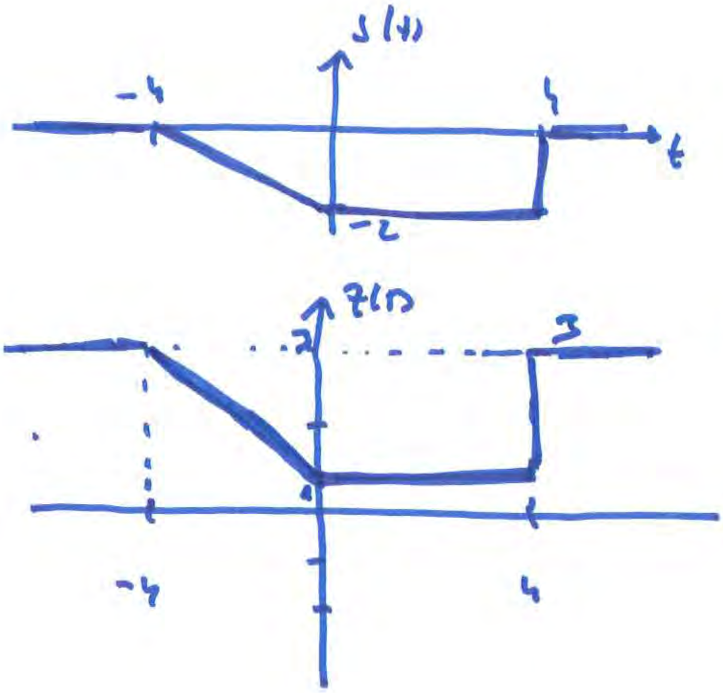


$v(t) = x(t+1)$ (ya hecho)

$w(t) = v(-t)$ (ya hecho)

$r(t) = w(t/4) =$

$= x\left(-\frac{t}{4}+1\right)$



$$s(t) = -2\,v(t) =$$
$$= -2\,x\left(-\frac{t}{4}+1\right)$$

$$z(t) = s(t) + 3 =$$
$$= -2\,x\left(-\frac{t}{4}+1\right) + 3 = y_4(t)$$

C19. Considerar la sinusoide $x(t) = 100 \cdot \cos(400\pi t + 60^\circ)$

① ¿Cuál es la amplitud máxima de la señal?

$X_{max} = 100.$

② ¿Cuál es la frecuencia en Hertzios? ¿Cuál es la frecuencia en rad/s?

$$w_0 = 400\pi = 2\pi \cdot f \Rightarrow f = \frac{400\pi}{2\pi} = \frac{\cancel{2}\cdot\cancel{\pi}\cdot 200}{\cancel{2\pi}} = 200$$

$f = 200\,[Hz] = 200\left[\frac{1}{s}\right]$. Eso son 200 vueltas por segundo.

$$\left.\begin{array}{l} 1 \text{ vuelta/s} \longrightarrow 2\pi \text{ radianes/s} \\ 200 \text{ vueltas/segundo} \longrightarrow x \text{ rad/s} \end{array}\right\} \Rightarrow x = \frac{200 \cdot 2\pi}{1} = 400\pi \text{ rad/s}$$

③ ¿Cuál es el ángulo de fase en radianes? ¿Cuál es el ángulo de fase en grados?

$\varphi = 60^\circ$

$$\left.\begin{array}{l} 360^\circ \longrightarrow 2\pi \text{ radianes} \\ 60 \longrightarrow x \text{ radianes} \end{array}\right\} x = \frac{60 \cdot 2\pi}{360} = \frac{\pi}{3} \text{ rad}$$

④ ¿Cuál es el periodo en milisegundos?

$$T_0 = \frac{1}{f} = \frac{1}{200}\,[s] = \frac{1}{200} \cdot 1000\,[ms] = 5\ ms$$

⑤ ¿Cuándo es la primera vez después de $t=0$ en que $x = 100$?

$$x(t_1) = 100 = 100 \cdot \cos\left(400\pi t_1 + \frac{\pi}{3}\right)$$

Lógicamente eso ocurrirá cuando $\cos\left(400\pi t_1 + \frac{\pi}{3}\right) = 1$.

Será cuando $\left(400\pi t_1 + \frac{\pi}{3}\right) = 2\pi \Rightarrow 400\pi t_1 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \Rightarrow$

$$400\, t_1 = \frac{5}{3} \Rightarrow t_1 = \frac{5}{1200}\,[s]$$

Comprobamos:

$$100 \cdot \cos\left(400\pi \cdot \frac{5}{1200} + \frac{\pi}{3}\right) = 100\cos\left(\frac{5\pi}{3} + \frac{\pi}{3}\right) = 100 \cdot \cos\left(\frac{6\pi}{3}\right) = 100 \cdot \cos(2\pi) = 100$$

C20 Exponencial imaginaria pura de tiempo continuo:

$$x(t) = A\, e^{j(\omega_0 t)}, \quad A \in \mathbb{C}$$

¿Es periódica? Lo será si algún T puede garantizar que

$$x(t) = x(t+T) \quad \forall t$$

$$x(t+T) = A\, e^{j\omega_0 (t+T)} = \underbrace{A\, e^{j\omega_0 t}}_{x(t)} \cdot \underbrace{e^{j\omega_0 T}}_{¿1?}$$

$$e^{j\omega_0 T} = 1 \iff \omega_0 T = k \cdot 2\pi, \quad k = 1, 2, 3, \ldots$$

$$\iff T = \frac{k \cdot 2\pi}{\omega_0}, \text{ que para } k=1 \text{ da } \boxed{T_0 = \frac{2\pi}{\omega_0}}$$

Por tanto, basta elegir $T = k \cdot \frac{2\pi}{\omega_0}$ sg. y es periódica.

C21 Estudiar la periodicidad.

(1) $x_1(t) = j \cdot e^{10jt}$. Exponencial imaginaria pura, periódica con periodo fundamental $\boxed{T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10} \text{ sg}}$

(2) $x_2(t) = e^{(-1+j)t} = \underbrace{e^{-t}}_{\text{no periódica}} \cdot \underbrace{e^{jt}}_{\text{periódica}}$, No es periódica

(3) $x_3(t) = \underbrace{2\cos(10t+1)}_{\text{señal (a)}} - \underbrace{\text{sen}\,(4t-1)}_{\text{señal (b)}}$

$x_3(t)$ es la suma de 2 señales periódicas (sinusoidales).

Señal (a): $w_a = 10 = \frac{2\pi}{T_a} \Rightarrow T_a = \frac{2\pi}{10}$ seg $= \frac{\pi}{5}$ seg

Señal (b): $w_b = 4 = 2\pi/T_b \Rightarrow T_b = \frac{2\pi}{4}$ seg $= \frac{\pi}{2}$ seg

Por tanto, es periódica, y $\boxed{T_x = \text{m.c.m.}\left(\frac{\pi}{5}, \frac{\pi}{2}\right) = \pi \text{ seg}}$

(4) $x_4(t) = \underbrace{1}_{\text{constante}} + \underbrace{e^{j\frac{4\pi}{7}t}}_{\text{periódica}} - \underbrace{e^{j\frac{2\pi}{5}t}}_{\text{periódica.}}$

$$\left.\begin{aligned} T_a &= \frac{2\pi}{w_a} = \frac{2\pi}{4\pi/7} = \frac{14}{4} = \frac{7}{2} \text{ seg} \\ T_b &= \frac{2\pi}{w_b} = \frac{2\pi}{2\pi/5} = 5 \text{ seg.} \end{aligned}\right\}$$

Periódica.

$T_x = \text{m.c.m.}\left\{\frac{7}{2}, 5\right\}$

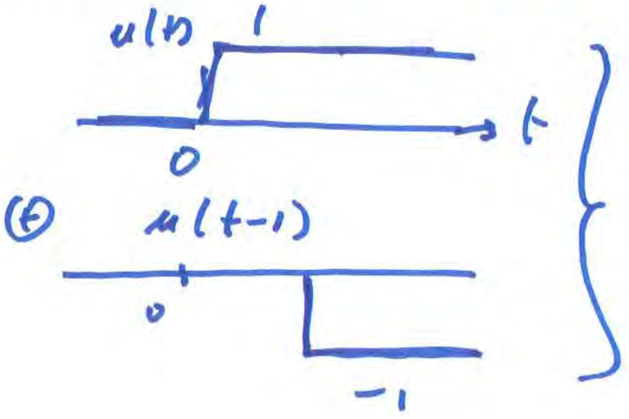
$\boxed{T_x = 35 \text{ seg}}$

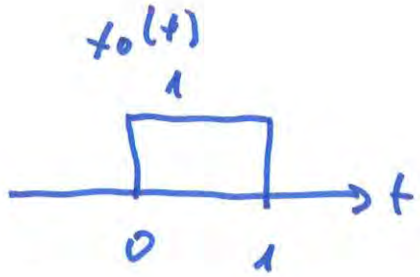
(5) $x_5(t) = \cos^2(2t - \pi/3)$. Periódica, por serlo el coseno.

Para el coseno: $T_0 = \frac{2\pi}{w_0} = \frac{2\pi}{2} = \pi$ seg.

Al cuadrado, $\boxed{T_x = T_0/2 = \pi/2 \text{ seg}}$

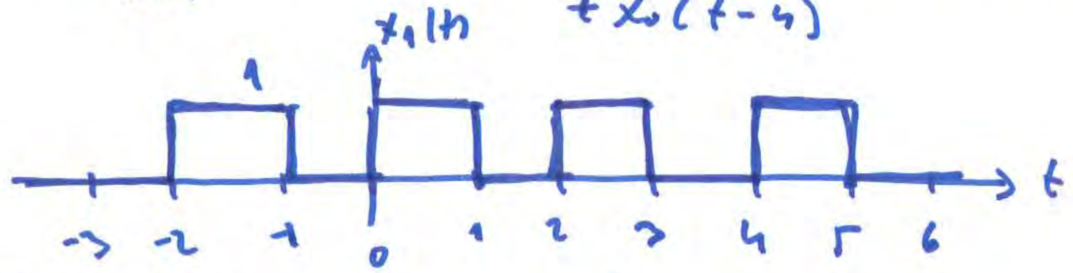
C22 • $x_0(t) = u(t) - u(t-1)$





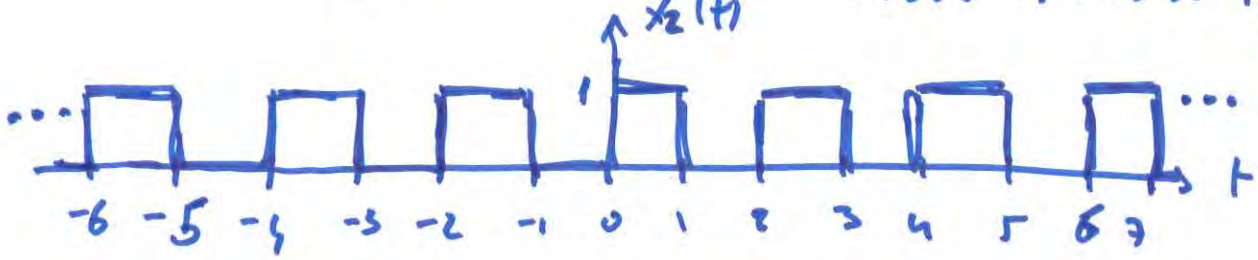
No es periódica (no se repite)

• $x_1(t) = \sum_{k=-1}^{2} x_0(t-2k) = x_0(t+2) + x_0(t) + x_0(t-2) + x_0(t-4)$

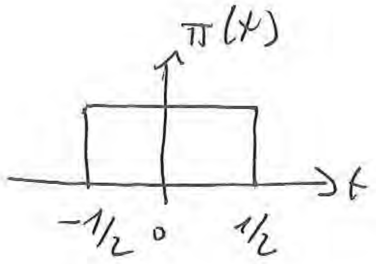


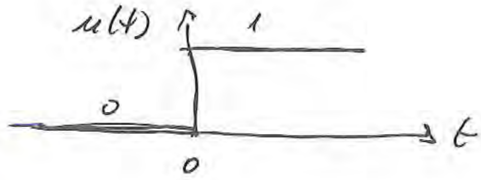
No es periódica (no se repite siempre).

• $x_2(t) = \sum_{k=-\infty}^{+\infty} x_0(t-2k) = \dots + x_0(t+2) + x_0(t) + x_0(t-2) + x_0(t-4) + \dots$



Es periódica, $\boxed{T_0 = 2 \text{ sg.}}$

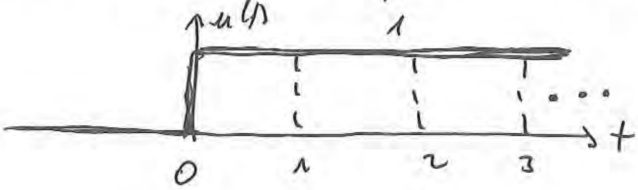




(C23) Expresar π(t) como suma de escalones unidad desplazados y escalados.

$$\boxed{\pi(t) = u\left(t+\tfrac{1}{2}\right) - u\left(t-\tfrac{1}{2}\right)}$$

(C24) Expresar u(t) como suma de pulsos π(t) desplazados y escalados.



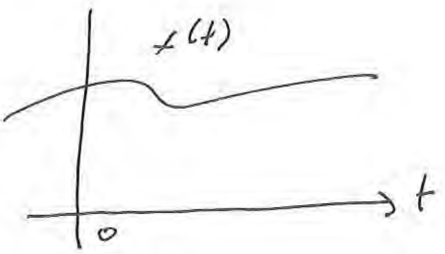
$$v(t) = \pi(t - 1/2)$$

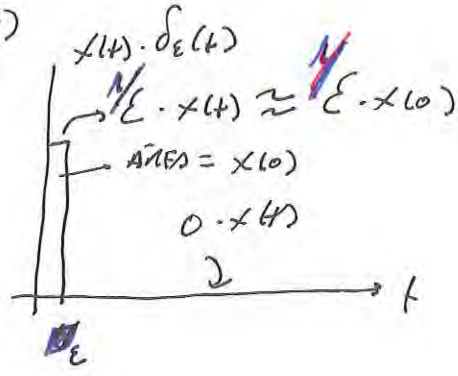
$$u(t) = v(t) + v(t-1) + v(t-2) + v(t-3) + \dots$$

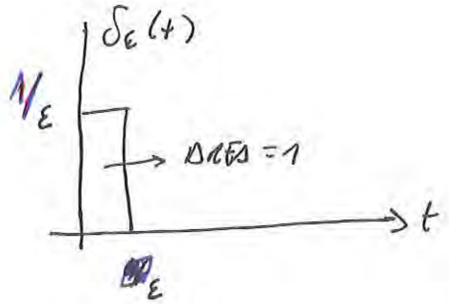
$$= \sum_{k=0}^{+\infty} v(t-k) = \sum_{k=0}^{+\infty} \pi\left(t-k-\tfrac{1}{2}\right)$$

C27 Demostrar que $x(t)\cdot\delta(t) = x(0)\cdot\delta(t)$

Partimos de $x(t)\cdot\delta_\varepsilon(t)$

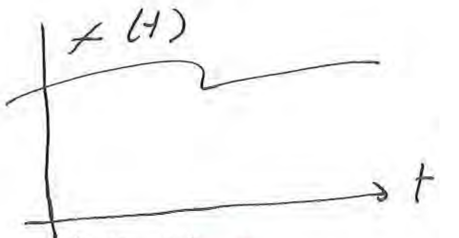


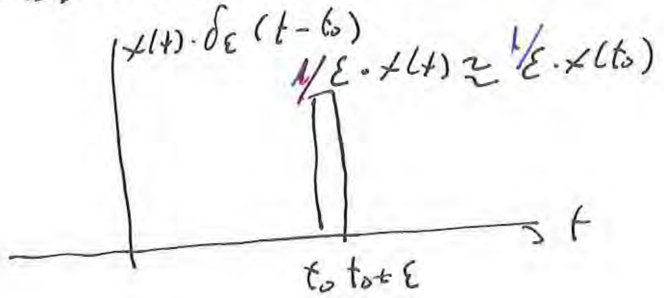


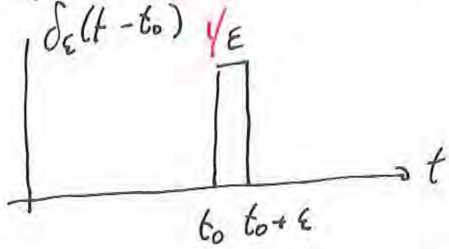


$$\lim_{\varepsilon\to 0} x(t)\,\delta_\varepsilon(t) = x(0)\cdot\delta(t)$$

Es similar para $x(t)\cdot\delta(t-t_0)$. Partimos de $x(t)\cdot\delta_\varepsilon(t-t_0)$







$$\lim_{\varepsilon\to 0} x(t)\,\delta_\varepsilon(t-t_0) = x(t_0)\cdot\delta(t-t_0)$$

Para demostrar (3) partimos de las propiedades anteriores.

$$\int_{-\infty}^{+\infty} x(z)\,\underbrace{\delta(t_0-z)}_{\substack{t_0-z=0\\ t_0=z}}\,dz = \int_{-\infty}^{+\infty} x(t_0)\,\delta(t_0-z)\,dz =$$

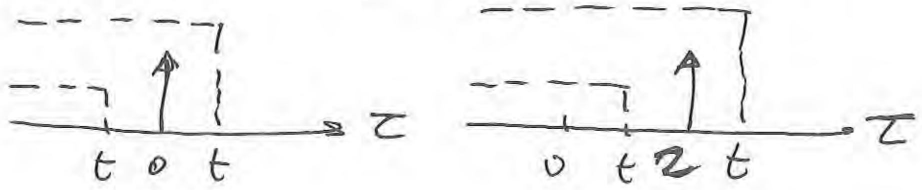
$$= x(t_0)\underbrace{\int_{-\infty}^{+\infty}\delta(t_0-z)\,dz}_{1} = x(t_0)$$

(C29) Sabemos que:

$$\delta(t) = \frac{du(t)}{dt}; \quad u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

(1) $$y(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau = \int_{-\infty}^{t} \left(\delta(\tau) - \delta(\tau-2)\right) d\tau =$$

$$= \underbrace{\int_{-\infty}^{t} \delta(\tau)\, d\tau} - \underbrace{\int_{-\infty}^{t} \delta(\tau-2)\, d\tau} =$$



$$= u(t) - u(t-2) //$$

(2) Integral de $x_2(t) = -\delta(t+3) + \delta(t-1) + 3\delta(t-3)$.
Podemos aplicar directamente las propiedades de integración:

$$y(t) = -u(t+3) + u(t-1) + 3u(t-3) //$$

¿Puedes representar gráficamente $y(t)$?

(C30) • Derivada de $x_1(t) = u(t+3) - 2u(t+3) + u(t+6)$

$$y(t) = \frac{dx_1(t)}{dt} = \delta(t+3) - 2\delta(t+3) + \delta(t+6) =$$

$$= -\delta(t+3) + \delta(t+6) //$$

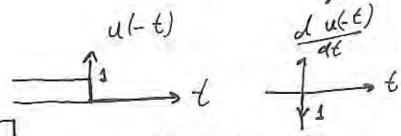
- Derivada de $x_2(t) = 3u(t) - 2'5\,u(t-3)$

$$y(t) = \frac{dx_2(t)}{dt} = 3\,\delta(t) - 2'5\,\delta(t-3)$$

- Derivada de $x_3(t) = u(t-t_0) + e^{t}\cdot u(t-3) - 2u(t)$

$$y(t) = \frac{dx_3(t)}{dt} = \delta(t-t_0) + e^{t}\cdot u(t-3) + \underbrace{e^{t}\delta(t-3)} -$$

$$- 2\delta(t) = \delta(t-t_0) + e^{t}u(t-3) + e^{3}\delta(t-3) - 2\delta(t) \;//$$



¿POR QUÉ?

- Derivada de $x_4(t) = \text{sen}(\pi t)\cdot \overbrace{u(-t)}$

$$y(t) = \frac{dx(t)}{dt} = \pi\cos(\pi t)\cdot u(-t) + \text{sen}(\pi t)\cdot(-\delta(t)) =$$

$$= \pi\cdot\cos\pi t\cdot u(-t) - \cancel{\text{sen}(0)\cdot\delta(t)} = \pi\cdot\cos(\pi t)\,u(-t) \;//$$

¿Eres capaz de representar gráficamente?

C31 Expresión analítica de $x(t)$ y su derivada.

QUIERO TENER

TENÍA

$$x(t) = 2\cdot u(t+2) + (1-2)\,u(t+1) +$$

$$+ (-1-(+1))\,u(t) +$$

$$+ (0-(-1))\,u(t-1) +$$

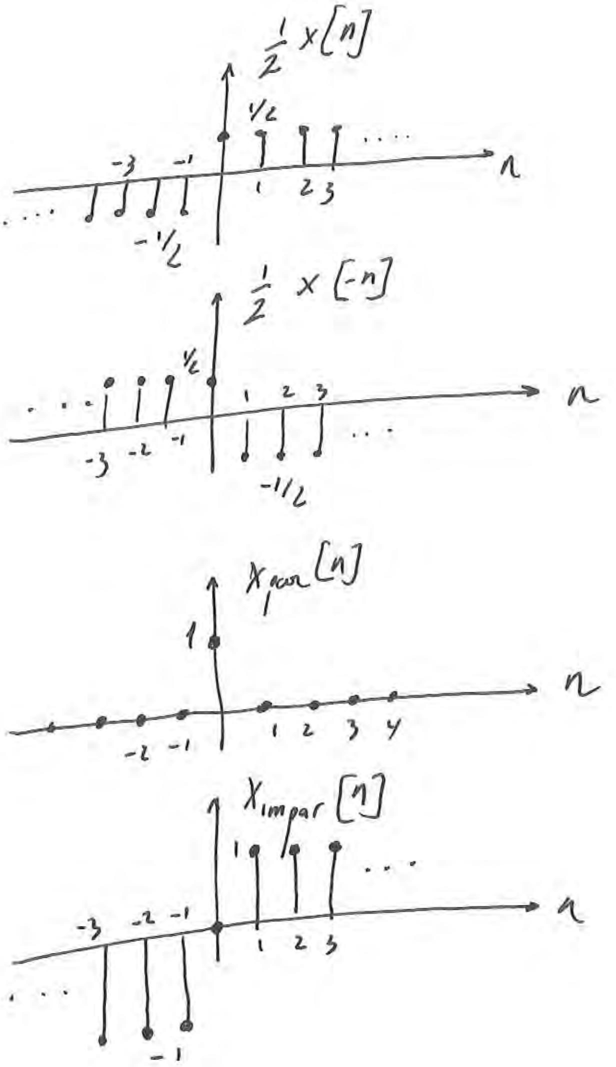
$$+ (1-0)\,u(t-2) + (-1)\,u(t-3) =$$

$$= 2u(t+2) - u(t+1) - 2u(t) + u(t-1) +$$
$$+ u(t-2) - u(t-3)$$

$$y(t) = 2\delta(t+2) - \delta(t+1) - 2\delta(t) + \delta(t-1) +$$
$$+ \delta(t-2) - \delta(t-3) \; //$$

¿Puedes verificar gráficamente que $y(t) = \frac{dx(t)}{dt}$?

(34)



(37) $< x[n] >_{(1,3)}$ con $x[n] = n^2$

$$< x[n] >_{(1,3)} = \frac{1}{3-1+1} \sum_{k=1}^{3} k^2 = \frac{1}{3}\left(1^2 + 2^2 + 3^2\right) = \frac{14}{3}$$

$M_\infty\left(x[n]\right)$ con $x[n] = \cos\left(\frac{n\pi}{2}\right)$

Esa señal es periódica $\boxed{\frac{\pi/2}{2\pi} = \frac{1}{4} = \frac{k}{N} \in \mathbb{Q}}$ de periodo 4.

Por lo tanto $< x[n] > = \frac{1}{4} \sum_{k=0}^{3} \cos\left(\frac{\pi}{2} k\right) = \frac{1}{4}\left(1 + 0 + (-1) + 0\right) = 0$

38 Calcular potencia y energía de

$$x[n] = \left(\frac{1}{2}\right)^n \cdot u[n]$$

$$E_\infty\{x[n]\} = \sum_{k=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^k \cdot u[k]\right|^2 = \sum_{k=0}^{\infty}\left(\left(\frac{1}{2}\right)^k\right)^2 = \sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k =$$

$$= \frac{0 \cdot \frac{1}{4} - 1}{\frac{1}{4} - 1} = \frac{-1}{-3/4} = \frac{4}{3}$$

suma de progresión geométrica de razón $\frac{1}{4}$

$$\frac{\text{último} \cdot \text{razón} - \text{primero}}{\text{razón} - \text{uno}}$$

$$P_{m_{TOTAL}}\{x[n]\} = \lim_{N\to\infty}\left\{\frac{1}{2N+1} \cdot \sum_{k=-N}^{N} |x[k]|^2\right\}$$

$$= \lim_{N\to\infty}\left\{\frac{1}{2N+1} \sum_{k=0}^{N}\left(\frac{1}{4}\right)^k\right\} = \lim_{N\to\infty}\left\{\frac{1}{2N+1} \frac{\cancel{\left(\frac{1}{4}\right)^N \cdot \frac{1}{4}}^{\,0} - 1}{\frac{1}{4} - 1}\right\}$$

$$= \lim_{N\to\infty}\left\{\frac{4/3}{2N+1}\right\} = 0$$

DEFINIDA EN ENERGÍA.

$$x[n] = e^{j\left(\frac{\pi}{2} \cdot n + \frac{\pi}{8}\right)}$$

Es una señal periódica, de periodo $\left(\frac{\pi/2}{2\pi} = \frac{k}{N} \in \mathbb{Q}\right)$ $N = 4$.

$$P_{m_{TOTAL}}\{x[n]\} = \frac{1}{4} \cdot \sum_{k=0}^{3} \left| e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)}\right|^2 = \frac{1}{4}\sum_{k=0}^{3} 1 = \frac{1}{4} \cdot 4 = 1$$

módulo 1

$E_\infty = \infty$ y está definida en potencia.

$$x[n] = \cos\left(\frac{\pi}{4}\cdot n\right)$$

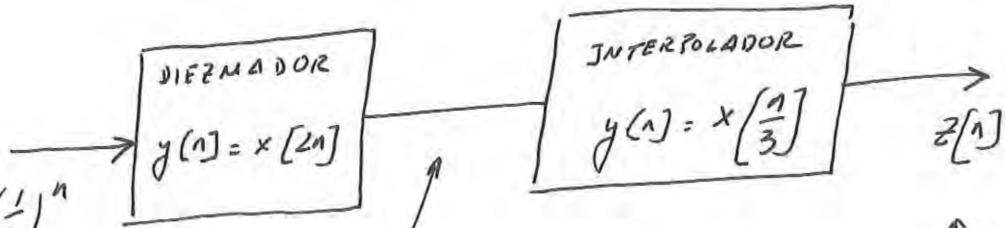
Es una señal periódica $\left(\frac{\pi/4}{2\pi} = \frac{1}{8} = \frac{k}{N}\right)$, con $N = 8$

$$P_{m\,TOTAL}\{x[n]\} = \frac{1}{8}\sum_{k=0}^{7}\left|\cos\left(\frac{\pi}{4}\cdot k\right)\right|^2 = \frac{1}{8}\left(1^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + 0 + \left(\frac{-\sqrt{2}}{2}\right)^2 + (-1)^2 + \right.$$

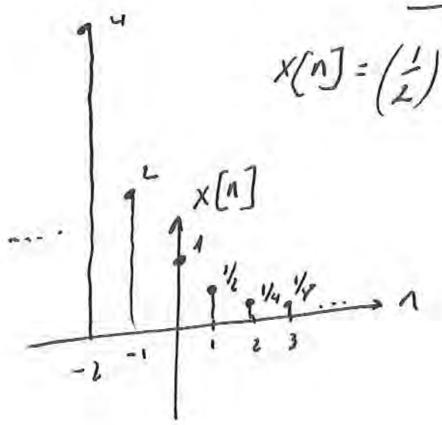
$$\left. + \left(-\frac{\sqrt{2}}{2}\right)^2 + 0 + \left(\frac{\sqrt{2}}{2}\right)^2\right) = \frac{1}{8}\left(1 + \frac{1}{2} + \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{2}\right) = \frac{4}{8} = \frac{1}{2}$$

$E_\infty = \infty$. ESTÁ DEFINIDA EN POTENCIA

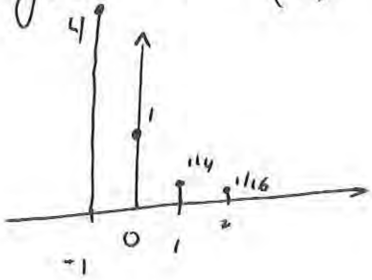
39



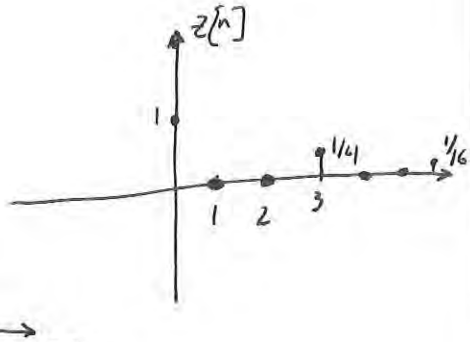
$x[n] = \left(\frac{1}{2}\right)^n$



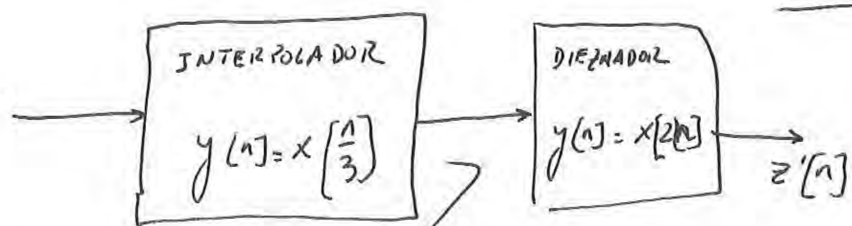
$y[n] = x[2n] = \left(\frac{1}{2}\right)^{2n} = \left(\frac{1}{4}\right)^n$

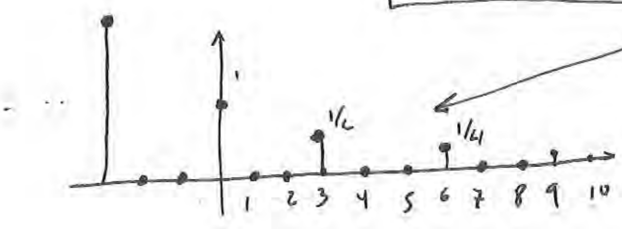


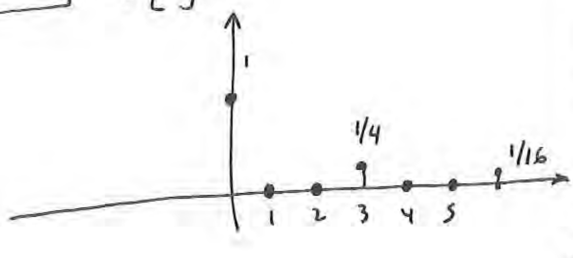
$z[n] = y\left[\frac{n}{3}\right]$



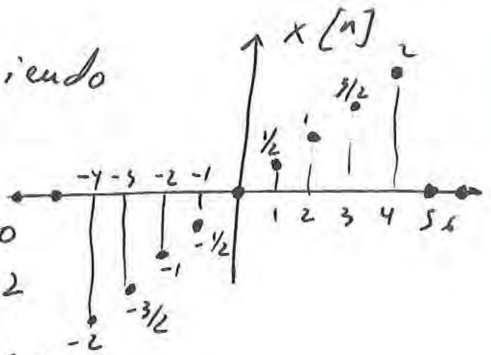
$x[n] = \left(\frac{1}{2}\right)^n$







(40) Apdo 1 $\Rightarrow$ $y_1[n] = x[2n]$ siendo



$\vdots$

$y_1[n=-3] = x[-6] = 0$

$y_1[n=-2] = x[-4] = -2$

$y_1[n=-1] = x[-2] = -1$

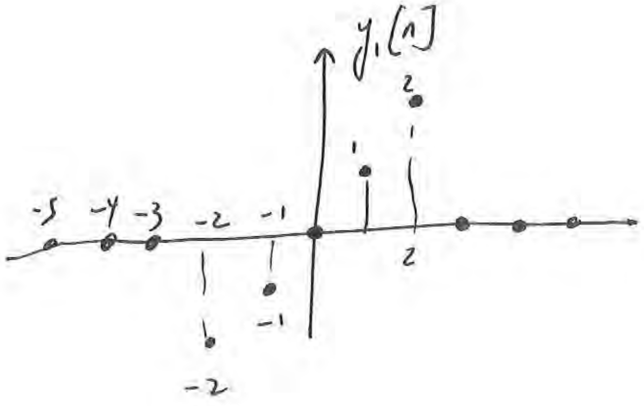
$y_1(n=0) = x[0] = 0$

$y_1(n=1) = x[2] = 1$

$y_1[n=2] = x[4] = 2$

$y_1[n=3] = x[6] = 0$

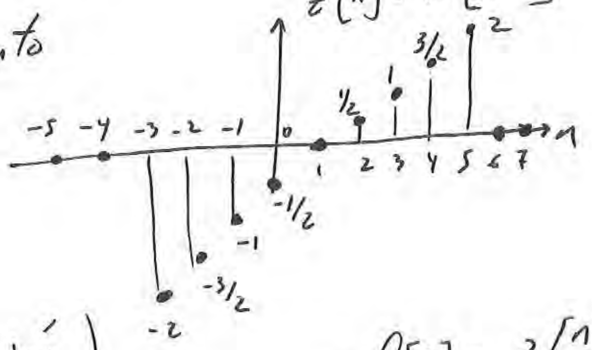
$\vdots$



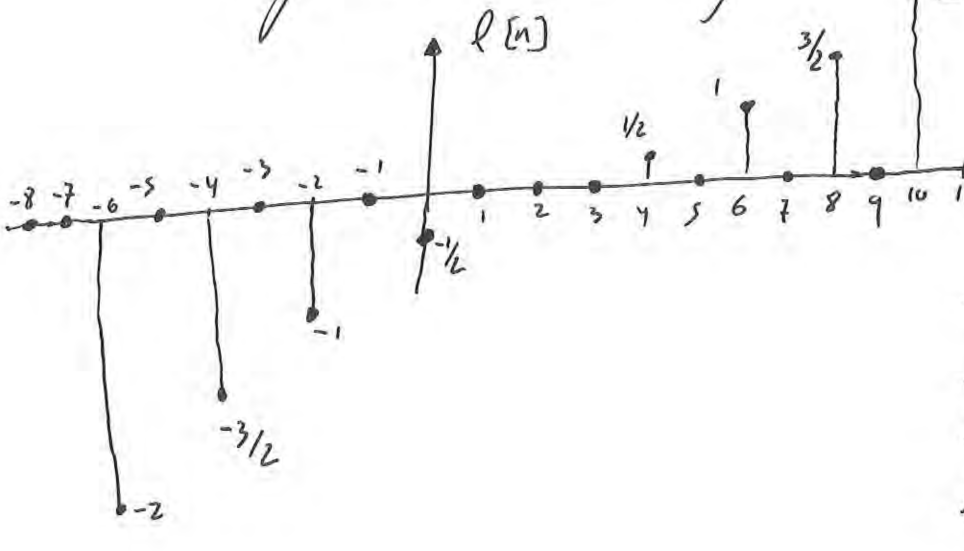
Apdo 2 $\Rightarrow$ $y_2[n] = -x\left[\frac{n}{2} - 1\right] + 2$

Primero el desplazamiento

$z[n] = x[n-1]$



Luego escalado (interpolación) $\longrightarrow$ $l[n] = z\left[\frac{n}{2}\right]$



$l[n=0] = z[0] = -\frac{1}{2}$

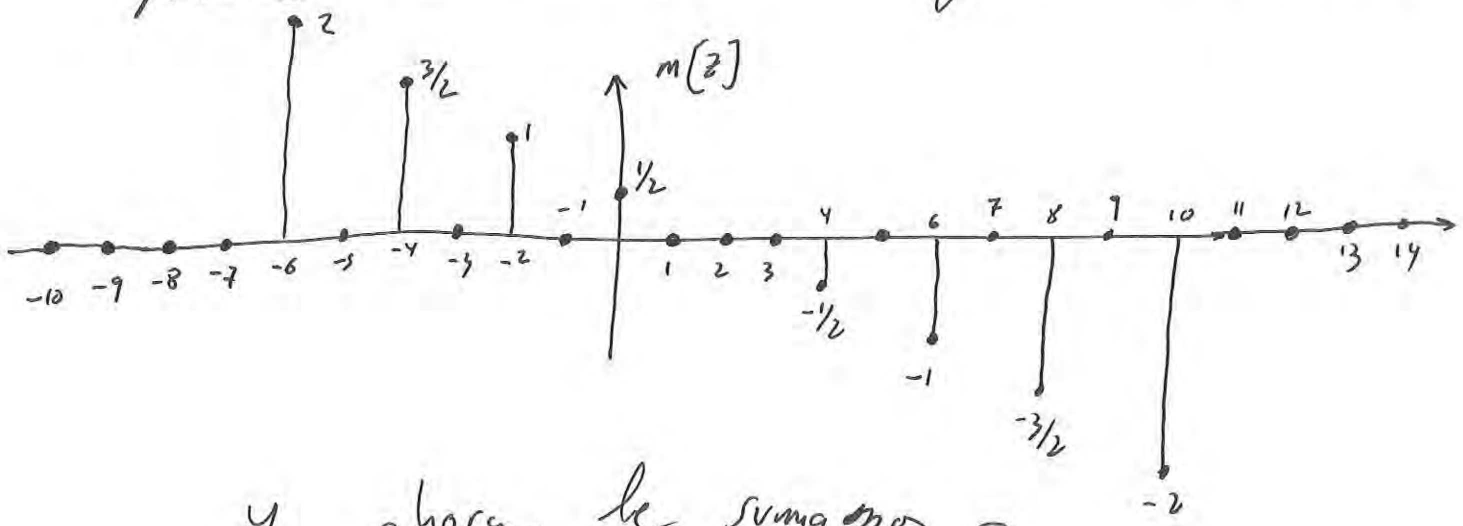
$l[n=1] = z\left[\frac{1}{2}\right] = ? \Rightarrow 0$

$l[n=2] = z[1] = 0$

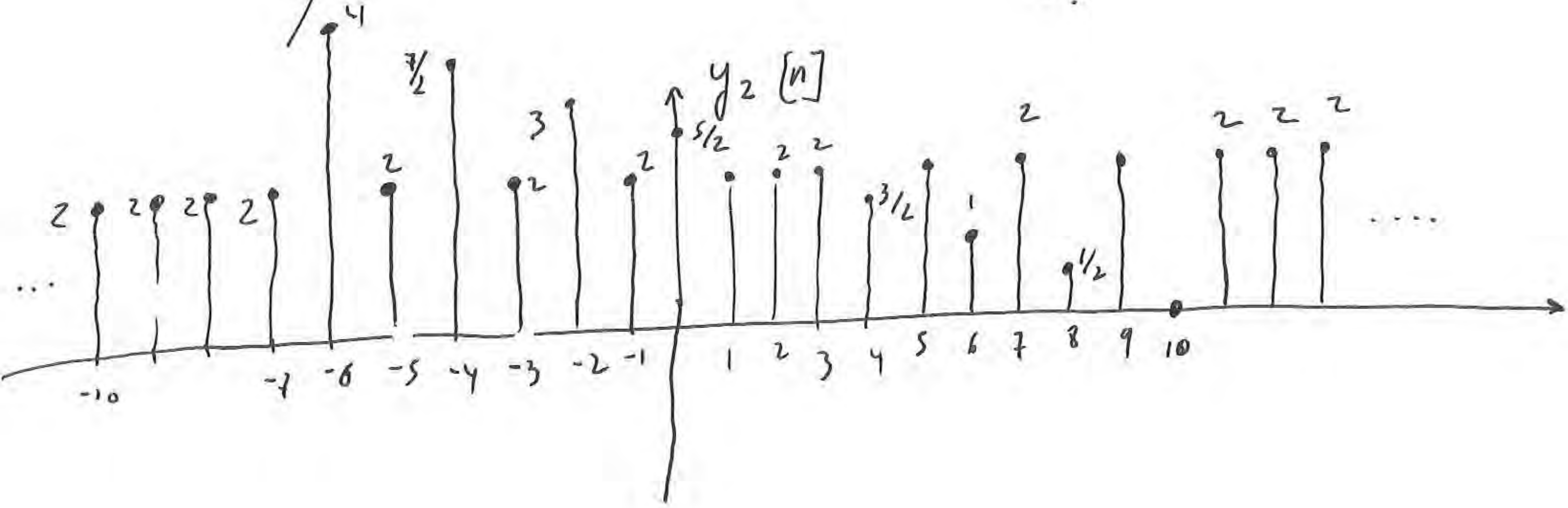
$l[n=3] = z\left[\frac{3}{2}\right] = ? \Rightarrow 0$

$l[n=4] = z\left[\frac{4}{2}\right] = \frac{1}{2}$

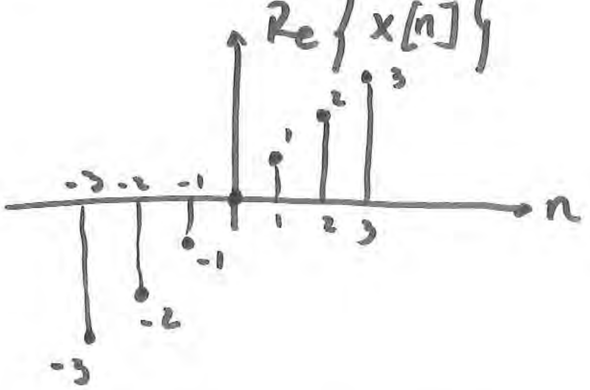
Ahora cambiamos de signo: $m[z] = -l[z]$

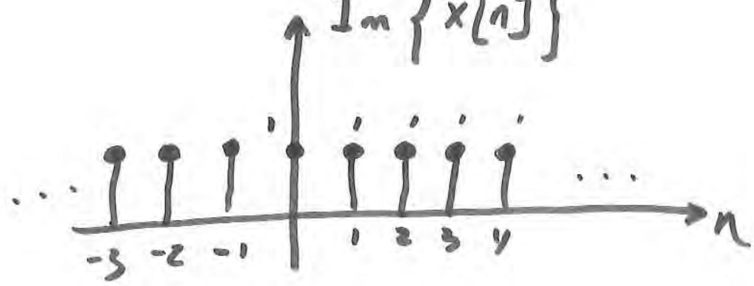


Y ahora le sumamos 2.



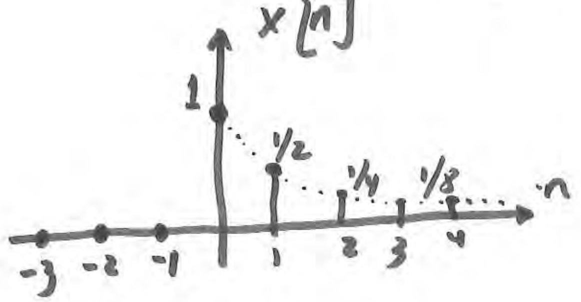
C41 Represente la parte real y la parte imaginaria de $x[n] = n + j$.





C42 Si tenemos la secuencia $x[n] = \begin{cases} \left(\frac{1}{2}\right)^n & \text{si } n \geq 0 \\ 0 & \text{si } n < 0 \end{cases}$

represente $y[n] = x[2n]$ y $z[n] = x\left[\frac{n}{2}\right]$.



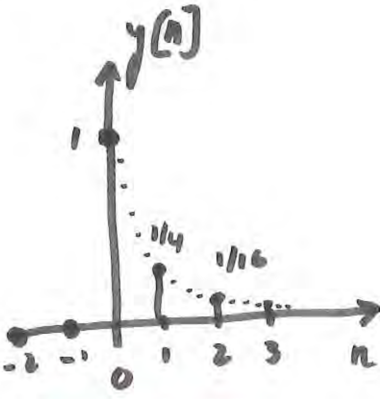
$\Rightarrow$ $y[-2] = x[-4] = 0$

$y[-1] = x[-2] = 0$

$y[0] = x[0] = 1$

$y[1] = x[2] = 1/4$

$y[2] = x[4] = 1/16$



DIEZMADO

$z[-2] = x[-1] = 0$

$z[-1] = x\left[-\frac{1}{2}\right] = ?$

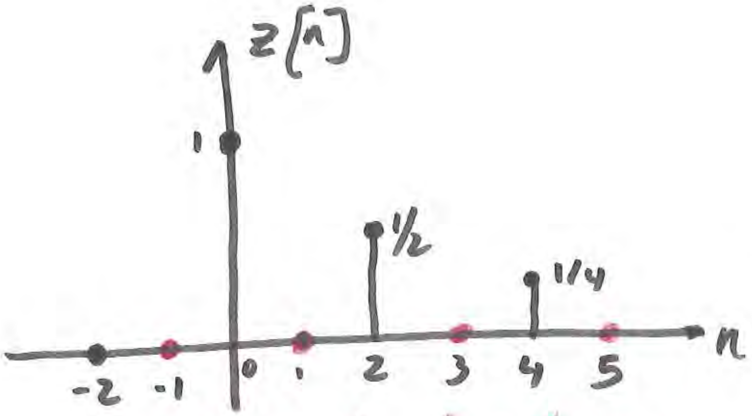
$z[0] = x[0] = 1$

$z[1] = x\left[\frac{1}{2}\right] = ?$

$z[2] = x[1] = 1/2$

$z[3] = ?$

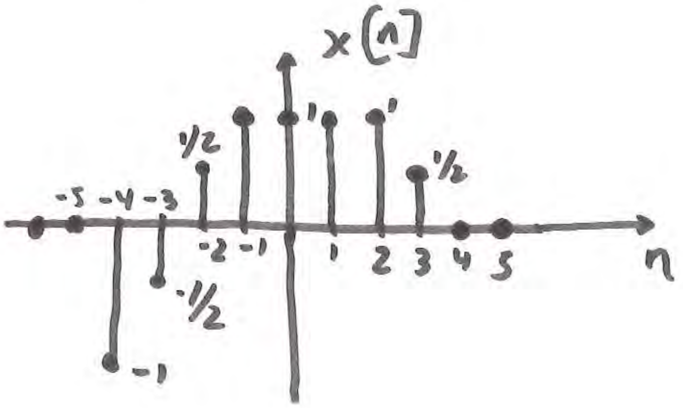
$z[4] = 1/4$

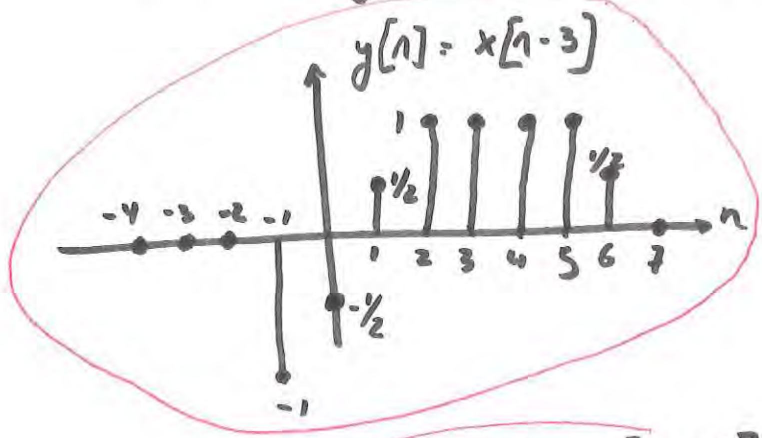


INTERPOLACIÓN ¿criterio?

(C43) Si $x[n]$ tiene esta forma represente

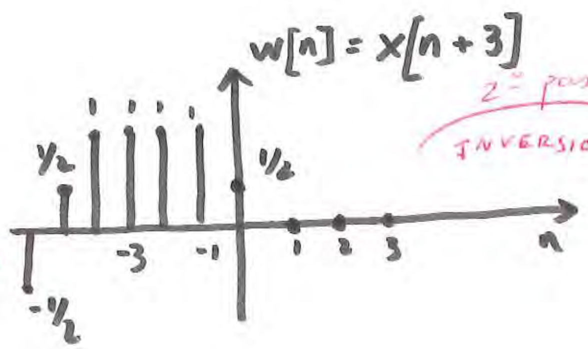
$y[n] = x[n-3]$; $z[n] = x[3-n]$ y $\ell[n] = x[n] \cdot u[3-n]$





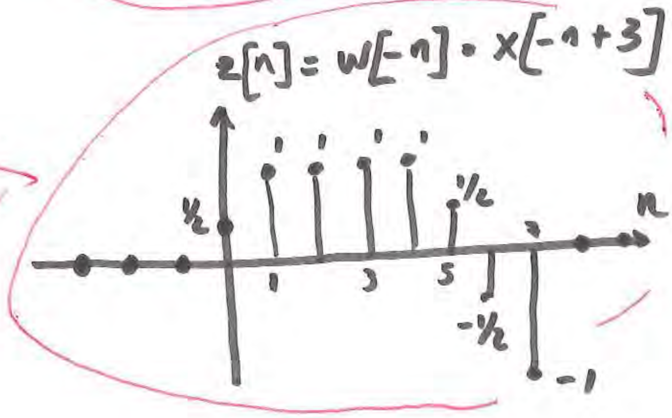
1er paso

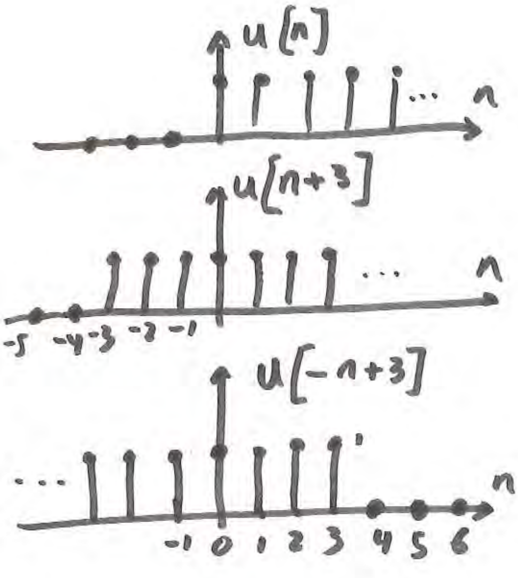
DESPLAZAMIENTO

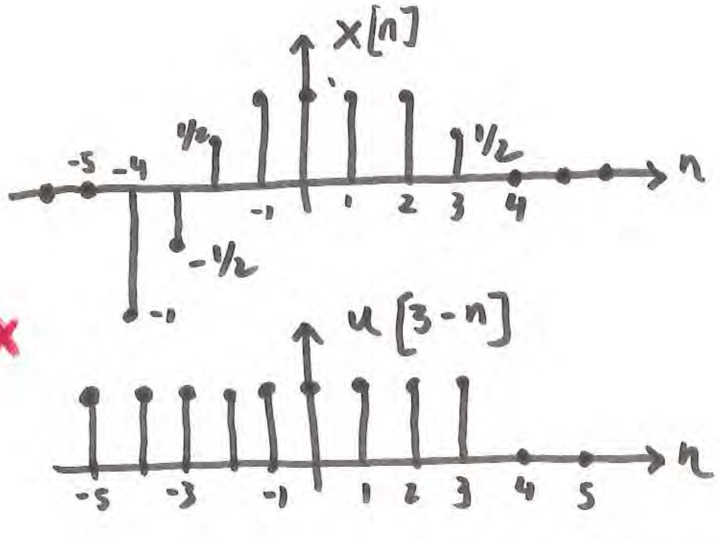


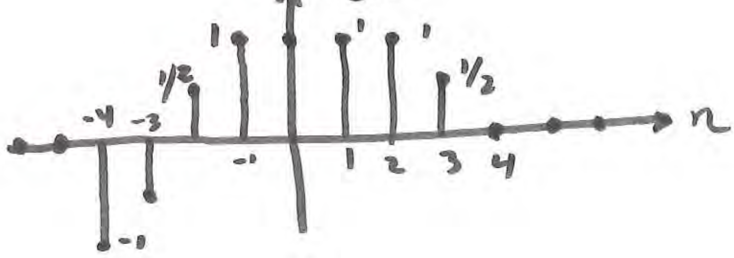
2º paso

INVERSION









$\ell[n] = x[n]$

C44 Demuestre que si $x[n]$ es impar, entonces $\sum_{n=-\infty}^{\infty} x[n] = 0$.

Descomponemos en 3 partes.

$$\sum_{n=-\infty}^{\infty} x(n) = \sum_{n=-\infty}^{-1} x[n] + x[0] + \sum_{n=1}^{\infty} x[n]$$

$=0$ por ser impar, por ser simétrica respecto al origen de coordenadas.

Si aplico el cambio de variable $m=-n$ al primer sumando...

$\left\{\begin{array}{l} \text{para } n=-1 \Rightarrow m=1 \\ \text{para } n=-\infty \Rightarrow m=\infty \end{array}\right\} \Rightarrow \sum_{m=1}^{\infty} x[-m]$, re-escribiendo otra vez con variable independiente $n \Rightarrow \sum_{n=1}^{\infty} x[-n]$. Por ser impar $x[n]=-x[-n]$

luego $\sum_{n=1}^{\infty} -x[n] = -\sum_{n=1}^{\infty} x[n]$.

Entonces $\sum_{n=-\infty}^{\infty} x[n] = -\sum_{n=1}^{\infty} x[n] + 0 + \sum_{n=1}^{\infty} x[n] = 0$

C45 Calcule la potencia y la energía de las secuencias $x[n] = \left(\frac{1}{2}\right)^n \cdot u[n]$ e $y[n] = \frac{j^{-1}}{\sqrt{2}} \cdot e^{-10nj}$

$$E_\infty\{x[n]\} = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n \cdot u[n]\right|^2 = \sum_{n=0}^{\infty}\left(\left(\frac{1}{2}\right)^n\right)^2 = \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n =$$

$$= \frac{0 \cdot \frac{1}{4} - 1}{\frac{1}{4} - 1} = \frac{-1}{-3/4} = \frac{4}{3}$$

SUMA DE PROGRESIÓN GEOMÉTRICA DE RAZÓN $\frac{1}{4}$

$$\frac{\text{último} \cdot \text{razón} - \text{primero}}{\text{razón} - 1}$$

$$P_{m_{TOTAL}}\{x[n]\} = \lim_{N\to\infty}\left\{\frac{1}{2N+1} \cdot \sum_{n=-N}^{N} |x[n]|^2\right\} =$$

$$= \lim_{N\to\infty}\left\{\frac{1}{2N+1}\sum_{n=0}^{N}\left(\frac{1}{4}\right)^n\right\} = \lim_{N\to\infty}\left\{\frac{1}{2N+1}\frac{\left(\frac{1}{4}\right)^{N}\cdot\frac{1}{4} - 1}{\frac{1}{4} - 1}\right\}$$

$$= \lim_{N\to\infty}\left\{\frac{4/3}{2N+1}\right\} = 0$$

Es una señal definida en energía. Aunque no la hemos pintado, seguro que es convergente.

$$E_\infty\{y[n]\} = \sum_{n=-\infty}^{\infty} |y[n]|^2 = \sum_{n=-\infty}^{\infty} \left\{ \left|\frac{j-1}{\sqrt{2}}\right| \cdot \underbrace{\left|e^{-10nj}\right|}_{1} \right\}^2$$

$$= \sum_{n=-\infty}^{\infty} \left\{ \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} \right\}^2 = \sum_{n=-\infty}^{\infty} 1 = \infty$$

$$P_{m\,TOTAL}\{y[n]\} = \lim_{N\to\infty} \left\{ \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \right\} =$$

$$= \lim_{N\to\infty} \left\{ \frac{1}{2N+1} \cdot (2N+1) \right\} = 1$$

Es una señal definida en potencia.

C46 Diga si la secuencia $x[n] = e^{j\frac{5\pi}{4}\cdot n}$ es periódica y en tal caso calcule el periodo.

$$w_0 = \frac{5\pi}{4}$$

Es periódica si $\frac{w_0}{2\pi} = \frac{k}{N} \in \mathbb{Q}$.

$$\frac{5\pi/4}{2\pi} = \frac{5}{8} = \frac{k}{N} \in \mathbb{Q} \Rightarrow$$ Si es periódica

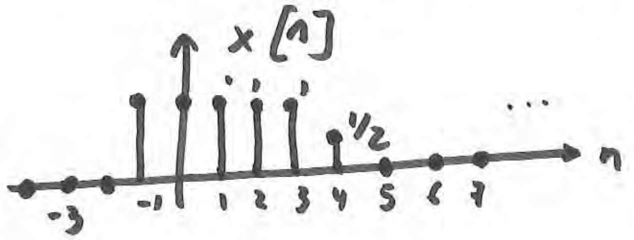
y el periodo es $N = 8$

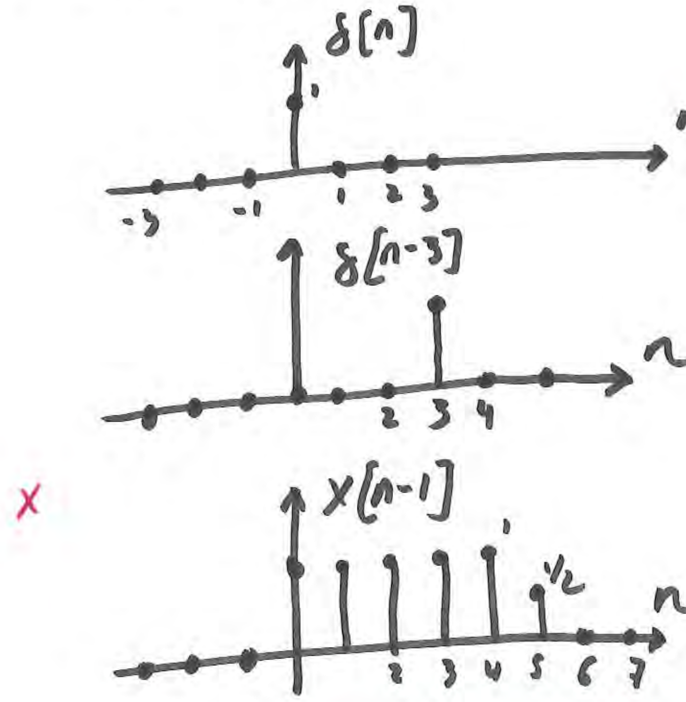
C47 Diga si la secuencia $x[n] = e^{j\frac{3}{8}n}$ es periódica y en tal caso calcule su periodo.

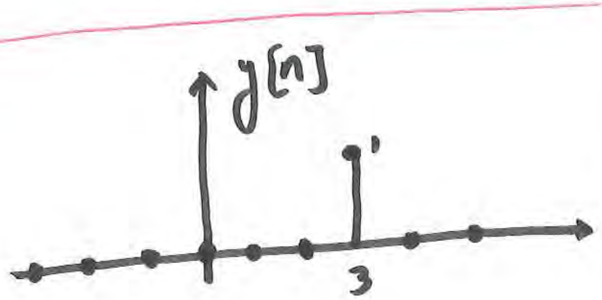
$$w_0 = \frac{3}{8}$$

$$\frac{3/8}{2\pi} = \frac{3}{16\pi} \neq \frac{K}{N} \in \mathbb{Q} \Rightarrow \text{No es periódica.}$$

C48 Dada la secuencia $x[n]$ dibuje la señal $y[n] = x[n-1] \cdot \delta[n-3]$







Más rápido:

$x[n-1] \cdot \delta[n-3] = x[3-1] \cdot \delta[n-3] =$
$= x[2] \cdot \delta[n-3] = 1 \cdot \delta[n-3] = \delta[n-3]$

Tema 1 - Señales en el Dominio Temporal

Problemas resueltos

PROBLEMAS TEMA 1.

P1 Expresar en forma de exponenciales complejas.

(a) $x(t) = 2\cdot\cos\left(2\pi\cdot 60t + \frac{\pi}{4}\right)$

$$x(t) = 2\cdot\frac{1}{2}\left[e^{j(2\pi\cdot 60t+\pi/4)} + e^{-j(2\pi\cdot 60t+\pi/4)}\right]$$

$$x(t) = e^{j(120\pi t+\pi/4)} + e^{-j(120\pi t+\pi/4)}$$

(b) $x(t) = 2\cdot\cos\left(t+\frac{\pi}{6}\right) + 4\cdot\operatorname{sen}\left(t-\frac{\pi}{3}\right)$

$$x(t) = 2\cdot\frac{1}{2}\left[e^{j(t+\pi/6)} + e^{-j(t+\pi/6)}\right] + 4\cdot\frac{1}{2j}\left[e^{j(t-\frac{\pi}{3})} - e^{-j(t-\frac{\pi}{3})}\right] =$$

$\frac{1}{j} = -j$ $\quad -\pi/2$

$$= e^{jt}\cdot e^{j\frac{\pi}{6}} + e^{-jt}\cdot e^{-j\frac{\pi}{6}} + 2\cdot e^{-j\frac{\pi}{2}}\cdot\left[e^{jt}\cdot e^{-j\frac{\pi}{3}} - e^{-jt}\cdot e^{j\frac{\pi}{3}}\right]$$

$$= e^{jt}\cdot e^{j\pi/6} + e^{-jt}\cdot e^{-j\pi/6} + 2\cdot e^{jt}\cdot e^{-j\left(\frac{3\pi+2\pi}{6}\right)} - 2\cdot e^{-jt}\cdot e^{-j\cdot\frac{(3\pi-2\pi)}{6}}$$

$$= e^{jt}\cdot e^{j\frac{\pi}{6}} + e^{-jt}\cdot e^{-j\frac{\pi}{6}} + 2\cdot e^{jt}\cdot e^{-j\frac{5\pi}{6}} - 2\cdot e^{-jt}\cdot e^{-j\frac{\pi}{6}} =$$

$\pi/6 \quad -\frac{5\pi}{6}$

$e^{-j\frac{5\pi}{6}} = -e^{j\frac{\pi}{6}}$

$$= e^{jt}\cdot e^{j\pi/6} + e^{-jt}\cdot e^{-j\pi/6} - 2\cdot e^{jt}\cdot e^{j\pi/6} - 2\cdot e^{-jt}\cdot e^{-j\pi/6} =$$

$$= \boxed{-e^{jt}\cdot e^{j\pi/6} - e^{-jt}\cdot e^{-j\pi/6} = -e^{j(t+\frac{\pi}{6})} - e^{-j(t+\frac{\pi}{6})}}$$

P2 Módulo, fase, potencia y energía.

(1) $x(t) = e^{j(2t+\pi/4)}$

$$|x(t)| = |e^{j(2t+\pi/4)}| = 1 \quad \forall t$$

$$\measuredangle\{x(t)\} = 2t + \pi/4$$

Como la exponencial imaginaria pura es periódica, calculo su potencia en un periodo:

$$P_x = \frac{1}{T}\int_{\langle T \rangle} |x(t)|^2 dt = \frac{1}{\pi}\int_0^{\pi} 1^2 dt = 1\ W$$

$$\left[\omega_0 = \frac{2\pi}{T} = 2 \Rightarrow T = \pi \text{ seg}\right] \qquad E_x = \infty \text{ (def. en potencia)}.$$

<u>(2) $x(t) = \cos t$.</u> $\left[\omega_0 = \frac{2\pi}{T} = 1 \rightarrow T = 2\pi \text{ seg}\right]$

$$|x(t)| = |\cos t|$$

$$\measuredangle\{x(t)\} = \text{arctg}\,\frac{\text{im}\{x(t)\}}{\text{re}\{x(t)\}}$$

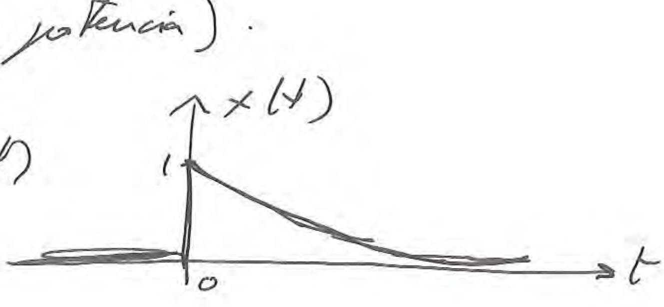
. Al ser una señal real, pero que cambia de signo, su fase será 0 (cuando es positiva) o π (cuando es negativa). Podemos expresarlo:

$$\measuredangle\{x(t)\} = \begin{cases} 0, & |t| \le \frac{\pi}{2} \\ \pi, & \frac{\pi}{2} \le |t| < \pi \end{cases} \quad \text{(Periódica de periodo } T\text{)}$$

$$P_x = \frac{1}{T}\int_{\langle T \rangle} |x(t)|^2 dt = \frac{1}{2\pi}\int_0^{2\pi} \cos^2 t\, dt = \dots = \frac{1}{2}\ W$$

$E_x = \infty$ (definida en potencia).

(3) $x(t) = e^{-2t} \cdot u(t)$



$$|x(t)| = |e^{-2t}\cdot u(t)| = e^{-2t}\cdot u(t)$$

$x(t)$ real, siempre ≥ 0

$$\measuredangle\{x(t)\} = \arctan \frac{\mathrm{im}\{x(t)\}}{\mathrm{re}\{x(t)\}} = \arctan 0 = 0 \quad \forall t$$

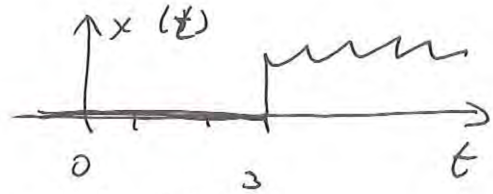
No es periódica, es de duración infinita, pero convergente a cero. Calculo antes su energía:

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} (e^{-2t})^2 \cdot (u(t))^2 dt =$$

$$= \int_{0}^{+\infty} e^{-4t} dt = \dots = \frac{1}{4}\ J \qquad P_x = 0$$

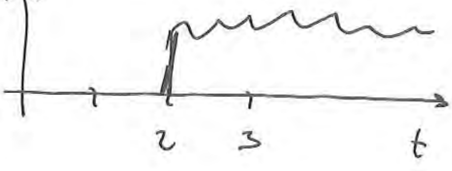
(definida en energía)

P3 $x(t) = 0, \quad t < 3$. ¿Para qué valores de t es nula?:

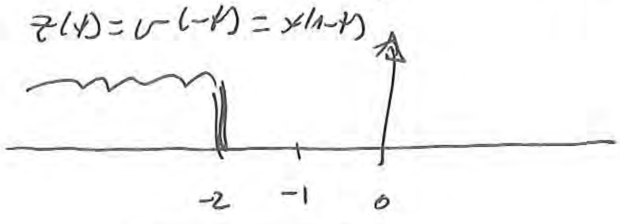


(1) $y(t) = x(1-t)$

$v(t) = x(t+1)$



$z(t) = v(-t) = x(1-t)$



$$\boxed{t > -2}$$

Analíticamente, podemos decir que $x(t) = g(t)\cdot u(t-3)$

$\Rightarrow y(t) = x(1-t) = g(1-t)\cdot \underbrace{u(1-t-3)} \Rightarrow$

$u(\underbrace{1-t-3}) = 0 \iff 1-t-3 < 0 \Rightarrow \boxed{-2 < t}$

(P4) Expresar la parte real como $A e^{-\alpha t} \cos(\omega t + \phi)$

$A, \alpha, \omega, \phi \in \mathbb{R}$, $\quad A > 0$, $\quad -\pi \le \phi \le \pi$

(1) $x(t) = -2 = \cos(\pi) \cdot 2 \Rightarrow \boxed{A=2,\ \phi=\pi,\ \alpha=0,\ \omega=0}$

(2) $x(t) = \sqrt{2}\, e^{j\frac{\pi}{4}} \cos(3t + 2\pi)$

$\mathrm{Re}\{x(t)\} = \mathrm{Re}\{\sqrt{2}\, e^{j\pi/4} \cdot \cos(3t)\} =$

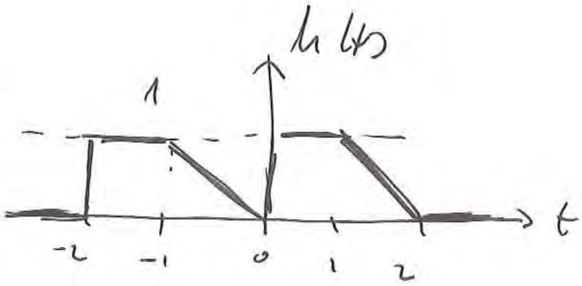
$= \sqrt{2} \cdot \cos(3t) \cdot \mathrm{Re}\{e^{j\pi/4}\} = \sqrt{2} \cdot \underbrace{\cos(\tfrac{\pi}{4})}_{\sqrt{2}/2} \cos(3t) =$

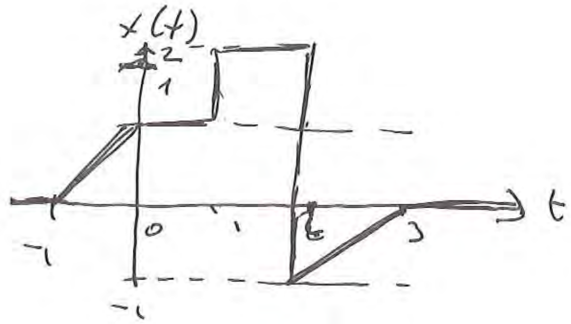
$= \cos(3t) \Rightarrow$

$\Rightarrow \boxed{A=1;\ \alpha=0;\ \omega=3\ r/s;\ \phi=0}$

(3) (4) Completar.

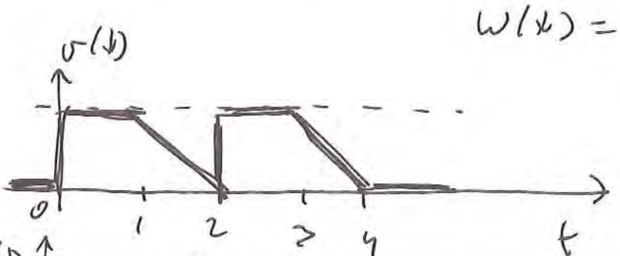
(P5)

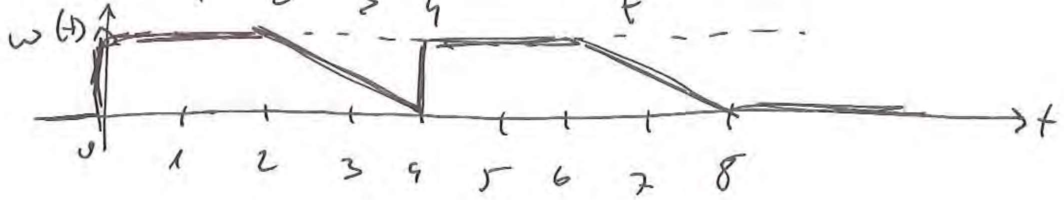




(1) $h(t/2 - 2)$. $\quad v(t) = h(t-2)$;

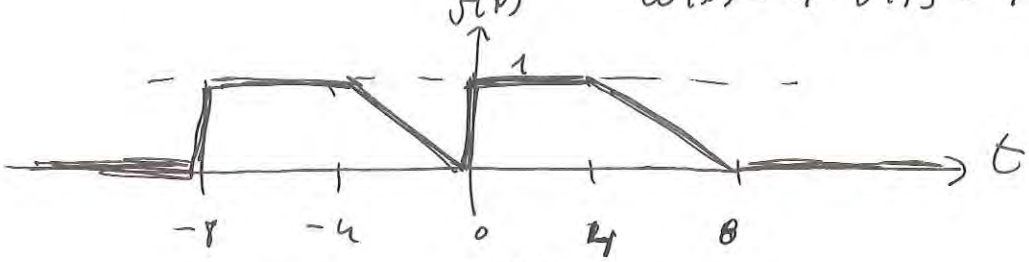
$w(t) = v(t/2) = h(t/2 - 2)$

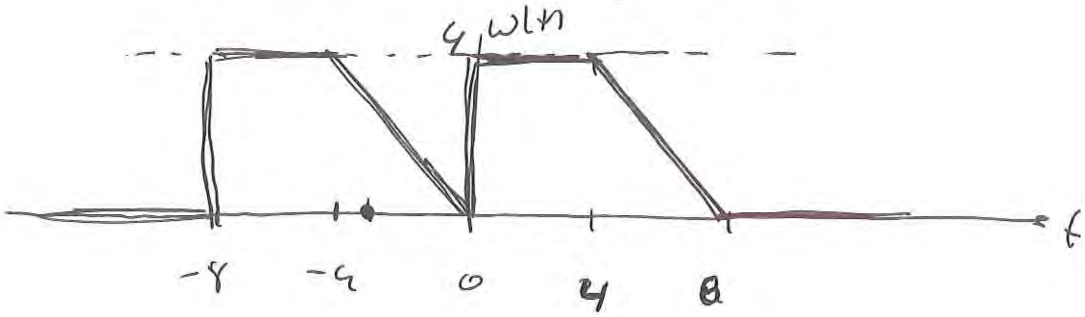




(4) $y(t) = 4\, h(t/4)$. $v(t) = h(t/4)$

$w(t) = 4 \cdot v(t) = 4\, h(t/4)$

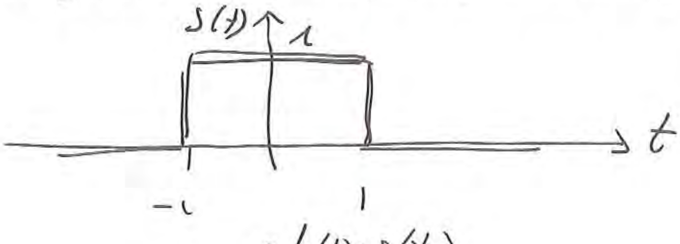


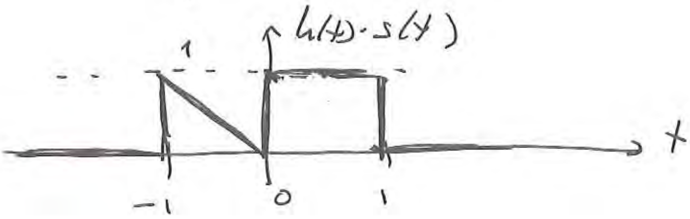


(5) $y(t) = h(t/2) \cdot \underbrace{\delta(t+1)}_{\neq 0 \text{ en } t+1=0,\ t=-1} = h(-1/2)\, \delta(t+1) = \boxed{\frac{1}{2}\, \delta(t+1)}$

[Comprobar gráficamente]

(6) $h(t) \cdot [u(t+1) - u(t-1)] = h(t) \cdot s(t)$

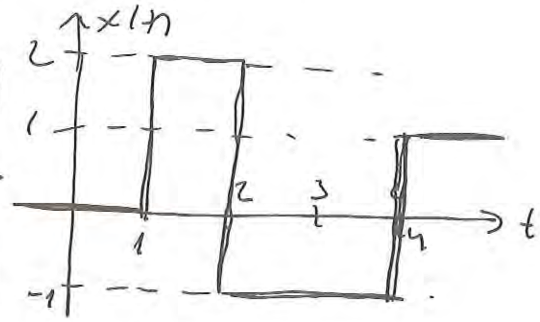




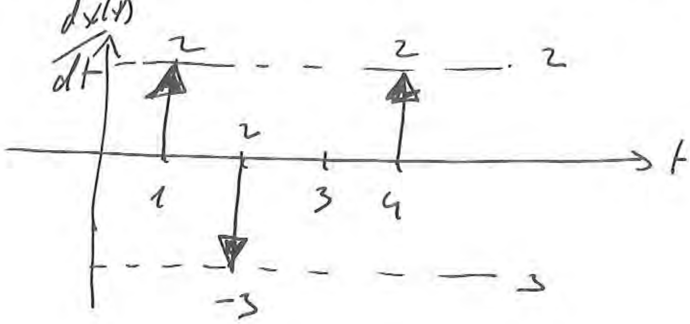
(7) (8) Completar.

P7 Calcular la derivada de las siguientes señales.

(1)
$$x(t) = \begin{cases} 0, & \text{si } t \leq 1 \\ 2, & \text{si } 1 < t \leq 2 \\ -1, & \text{si } 2 < t \leq 4 \\ 1, & \text{si } t > 4 \end{cases}$$



$$\frac{dx(t)}{dt} = 2\,\delta(t-1) - 3\,\delta(t-2) + \delta(t-4) \;//$$



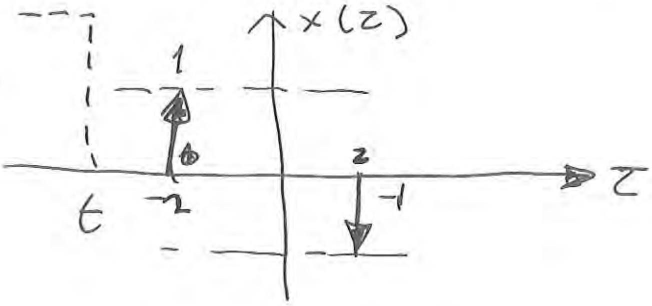
(2) $x(t) = u(t+2) - u(t-2) \Rightarrow \dfrac{dx(t)}{dt} = \delta(t+2) - \delta(t-2) \;//$

(3) $x(t) = e^{j\pi t} \cdot u(t) \Rightarrow$

$$\frac{dx(t)}{dt} = j\pi\, e^{j\pi t} \cdot u(t) + e^{j\pi t}\,\delta(t) = \pi e^{j(\pi t + \pi/2)} + \delta(t) \;//$$

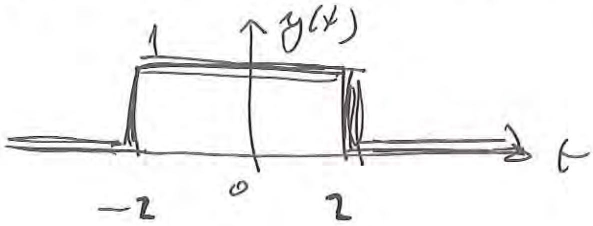
P8 Integrar las siguientes señales calculando $y(t)=\int_{-\infty}^{t} x(\tau)\,d\tau$

(a) $x(t)=\delta(t+2)-\delta(t-2)$

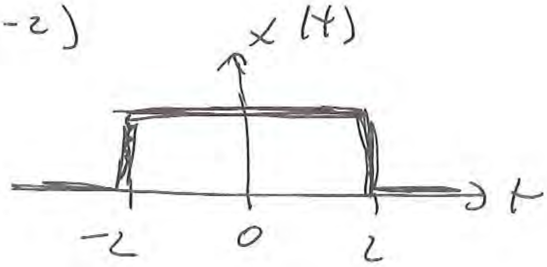


$$y(t)=\int_{-\infty}^{t} x(\tau)\,d\tau =$$

$$=\int_{-\infty}^{t} \left(\delta(\tau+2)-\delta(\tau-2)\right)d\tau=\begin{cases}0, & t<-2\\ 1, & -2\le t<2\\ 0, & t\geq 2\end{cases}=u(t+2)-u(t-2)$$



(b) $x(t)=u(t+2)-u(t-2)$

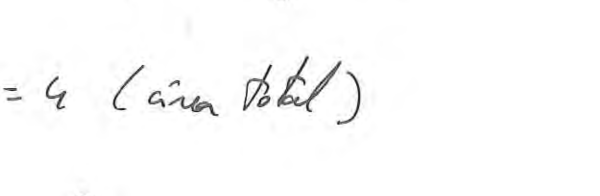


$$y(t)=\int_{-\infty}^{t} x(\tau)\,d\tau = ⊛$$

(a) $t<-2 \Rightarrow y(t)=0$

(b) $t\geq -2,\ t<2 \Rightarrow$

$$y(t)=\int_{-2}^{t} 1\,d\tau = t+2$$



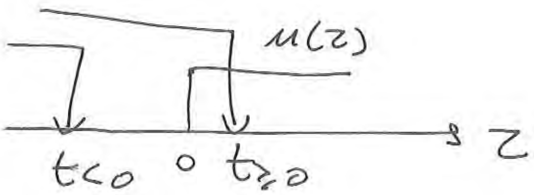
(c) $t\geq 2 \Rightarrow y(t)=\int_{-\infty}^{t} x(\tau)\,d\tau = 4$ (área total)

$$y(t)=\begin{cases}0, & t<-2\\ t+2, & -2\le t<2\\ 4, & t\geq 2\end{cases}$$

NOTA: podemos expresar $y(t)$ de forma alternativa:

$$y(t) = (t+2)\,u(t+2) + \left[4 - (t+2)\right] u(t-2) \quad (\text{¿Por qué?})$$

(c) $x(t) = e^{j\pi t} \cdot u(t)$



$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau =$$

$$= \int_{-\infty}^{t} e^{j\pi\tau} u(\tau)\,d\tau \;\Big<\; \begin{array}{l} t<0;\ y(t) = \int_{-\infty}^{t} 0\,d\tau = 0. \\ t\geq 0;\ y(t) = \int_{0}^{t} e^{j\pi\tau}\,d\tau = \end{array}$$

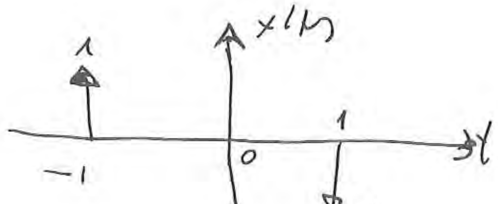
$$= \frac{1}{j\pi}\left[e^{j\pi\tau}\right]_0^t = \frac{1}{j\pi}\left(e^{j\pi t} - 1\right)$$

Por tanto: $\boxed{y(t) = \frac{1}{j\pi}\left(e^{j\pi t} - 1\right) \cdot u(t)}$

P9

$x(t) = \delta(t+2) - \delta(t-2)$

$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$.



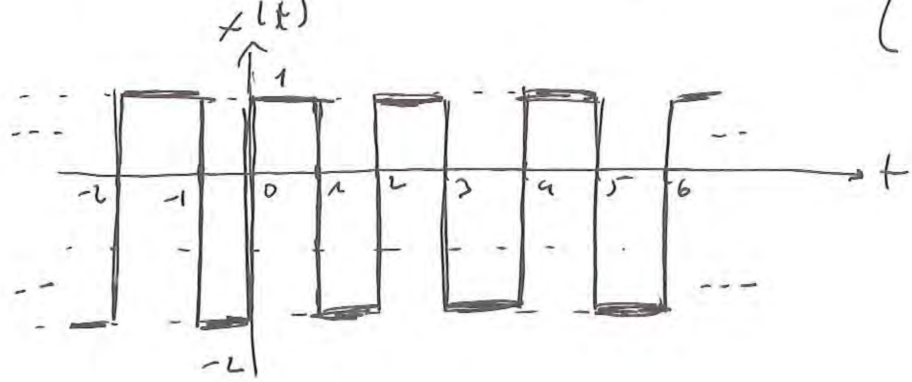
¿E_y? Una forma que se puede complicar es sustituir sin más.

$$E_y = \int_{-\infty}^{+\infty} |y(t)|^2 dt = \int_{-\infty}^{+\infty} \left| \int_{-\infty}^{+\infty} (\delta(\tau+2) - \delta(\tau-2))\,d\tau \right|^2 dt \ldots$$

Pero teniendo en cuenta cómo es $y(t)$

$$\boxed{E_y = \int_{-2}^{2} 1^2\,dt = 4}$$

P10 $x(t)$ periódica de periodo $T = 2$ seg. $x(t) = \begin{cases} 1, & 0 \le t < 1 \\ -2, & 1 \le t < 2 \end{cases}$

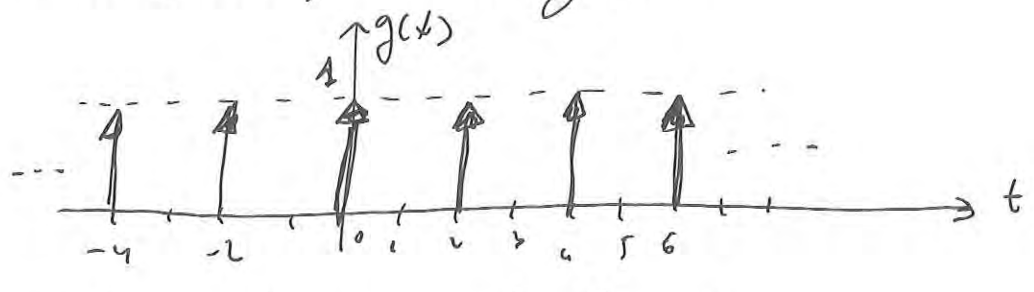


Encontrar A_1, t_1, A_2, t_2, para que:

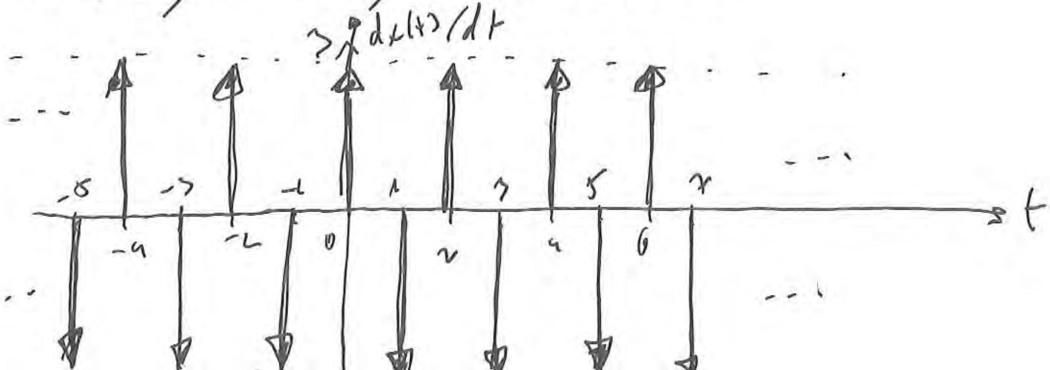
$$\frac{dx(t)}{dt} = A_1\, g(t-t_1) + A_2\, g(t-t_2), \text{ donde}$$

$$g(t) = \sum_{k=-\infty}^{+\infty} \delta(t-2k).$$

Comenzamos representando $g(t)$:



También podemos representar $dx(t)/dt$:



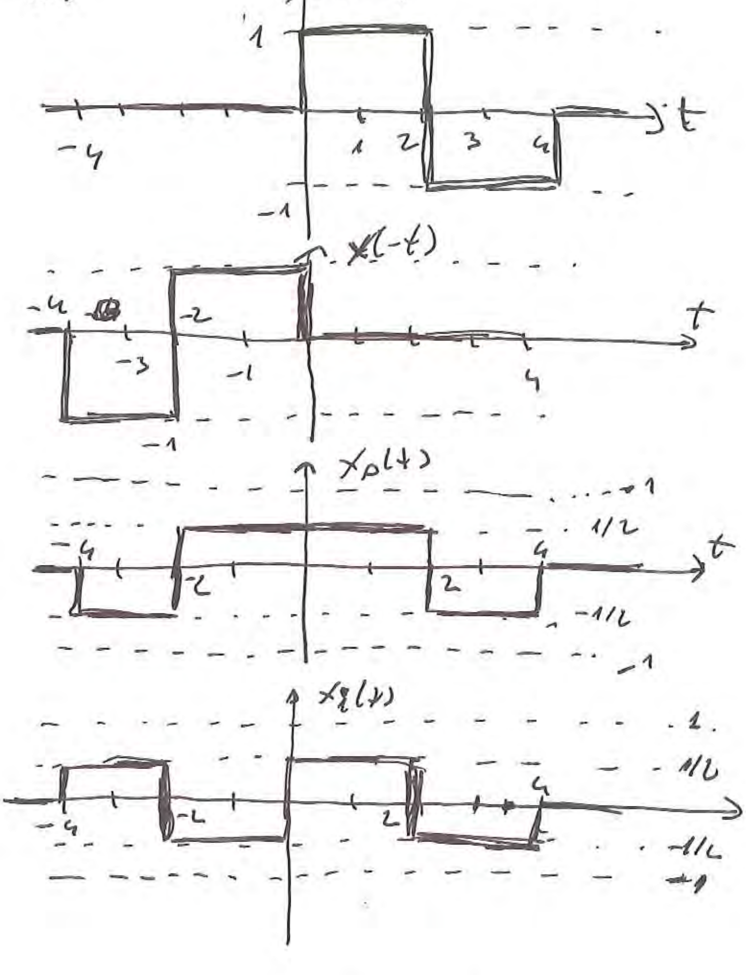
$\Rightarrow$ $\boxed{A_1 = 3,\ t_1 = 0}$ (nos proporciona los "δ" positivos de $x'(t)$)

$\boxed{A_2 = -3,\ t_2 = 1}$ (" " negativos ")

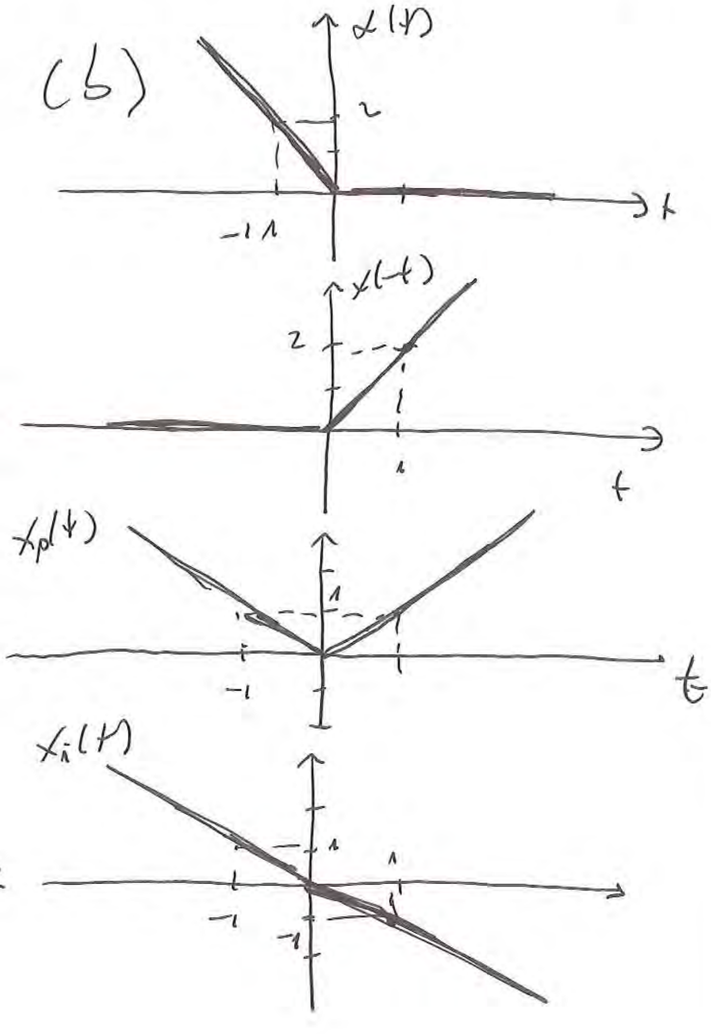
P11 Dibujar las partes par e impar de las señales dadas.

$$x_p(t) = \frac{1}{2}\left(x(t) + x(-t)\right); \quad x_i(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$

(a)

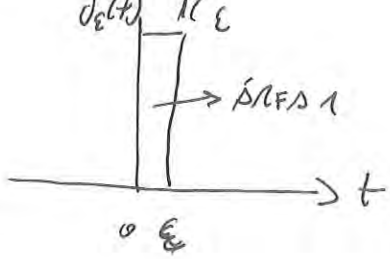


(b)

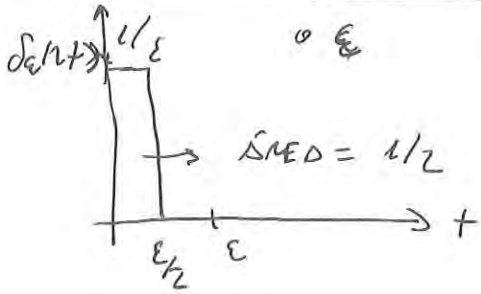


(c) (d) Completar.

(P12) Demostrar que $\delta(2t) = \frac{1}{2}\,\delta(t)$.



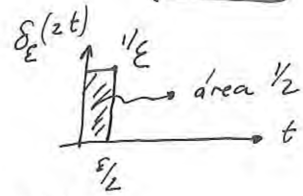
$$\delta_\varepsilon(t) = \frac{1}{\varepsilon}\left(u(t) - u(t-\varepsilon)\right)$$



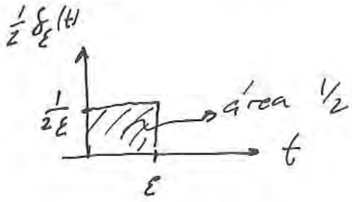
$$\delta_\varepsilon(2t) = \frac{1}{\varepsilon}\left(u(t) - u(t-2\varepsilon)\right)$$

$$\boxed{\delta(2t) = \lim_{\varepsilon\to 0} \delta_\varepsilon(2t) = \frac{1}{2}\lim_{\varepsilon\to 0} \delta_\varepsilon(t) = \frac{1}{2}\,\delta(t)}$$

Es lo mismo llevar al límite $\varepsilon\to 0$ esto



que esto



P13 $\phi_{xy}(t) = \int_{-\infty}^{+\infty} x(t+\tau)\, g(\tau)\, d\tau$

(a) Relación entre $\phi_{yx}(t)$ y $\phi_{xy}(t)$.

$$\phi_{yx}(t) = \int_{-\infty}^{+\infty} y(t+\tau)\, x(\tau)\, d\tau =$$

Cambio de variable: $\left.\begin{array}{l} t+\tau=\sigma \\ \tau=\sigma-t \\ d\tau=d\sigma \end{array}\right\}$ $\left\{\begin{array}{l} \tau\to+\infty \Rightarrow \sigma\to+\infty \\ \tau\to-\infty \Rightarrow \sigma\to-\infty \end{array}\right.$

$$= \int_{-\infty}^{+\infty} x(\sigma-t)\, g(\sigma)\, d\sigma = \phi_{xy}(-t)$$ //

(b) $x(t)$ periódica: $x(t) = x(t+T_0)$, $T_0 \in \mathbb{R}^+$

$$\phi_{xx}(t) = \int_{-\infty}^{+\infty} x(t+\tau)\, x(\tau)\, d\tau$$

$$\phi_{xx}(t+T) = \int_{-\infty}^{+\infty} \underbrace{x(t+T+\tau)}_{\substack{\| \\ x(t+\tau),\ \text{si } T=T_0}}\, x(\tau)\, d\tau =$$

$$= \int_{-\infty}^{+\infty} x(t+\tau)\, x(\tau)\, d\tau = \phi_{xx}(t) \Rightarrow \text{periódica } (T_0)$$

(c) Sabemos $\phi_{yx}(t) = \phi_{xy}(-t)$

Por tanto, $\phi_{xx}(t) = \phi_{xx}(-t) \Rightarrow$ Par //

y la parte impar de $\phi_{xx}(t)$ es 0 //

P14 Demostrar que

$$\int_{-\infty}^{+\infty} x^2(t)\,dt = \int_{-\infty}^{+\infty} x^2_{par}(t)\,dt + \int_{-\infty}^{+\infty} x^2_{impar}(t)\,dt .$$

Sabemos que siempre $x(t) = x_{par}(t) + x_{impar}(t)$.

$$\int_{-\infty}^{+\infty} x^2(t)\,dt = \int_{-\infty}^{+\infty} (x_{par}(t) + x_{impar}(t))^2\,dt =$$

$$= \underbrace{\int_{-\infty}^{+\infty} x^2_{par}(t)\,dt + \int_{-\infty}^{+\infty} x^2_{impar}(t)\,dt}\, + \underbrace{2\int_{-\infty}^{+\infty} x_{par}(t)\cdot x_{impar}(t)\,dt}_{¿I=0?}$$

$$I = \underbrace{\int_{-\infty}^{0} x_{par}(t)\, x_{impar}(t)\,dt}_{\substack{s=-t \\ ds=-dt}} + \int_{0}^{+\infty} x_{par}(t)\, x_{impar}(t)\,dt =$$

$t=0 \to s=0$

$t \to -\infty \to s \to +\infty$

$$= \int_{+\infty}^{0} x_{par}(-s)\, x_{impar}(-s)\,(-ds) + \int_{0}^{+\infty} x_{par}(t)\, x_{impar}(t)\,dt =$$

$$= \int_{0}^{+\infty} x_{par}(s)\cdot(-x_{impar}(s))\,ds + \int_{0}^{+\infty} x_{par}(t)\, x_{impar}(t)\,dt = 0 //$$

Problemas de secuencias TEMA 1.

(P1) Determinar el módulo y la fase, así como la potencia y la energía (P_∞ y E_∞) de las siguientes secuencias.

a) $$x[n] = e^{j\left(\frac{\pi}{2}\cdot n + \frac{\pi}{8}\right)}$$

b) $$x[n] = \cos\left[\frac{\pi}{4} n\right]$$

SOLUCIÓN:

$$x[n] = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)}$$

a) $|x[n]| = 1$

$$\measuredangle x[n] = \frac{\pi}{2}\cdot n + \frac{\pi}{8}$$

$$P_\infty = \lim_{N\to\infty}\left\{\frac{1}{2N+1}\sum_{k=-N}^{N}|x[k]|^2\right\} = \lim_{N\to\infty}\left\{\frac{1}{2N+1}\sum_{k=-N}^{N}1^2\right\} =$$

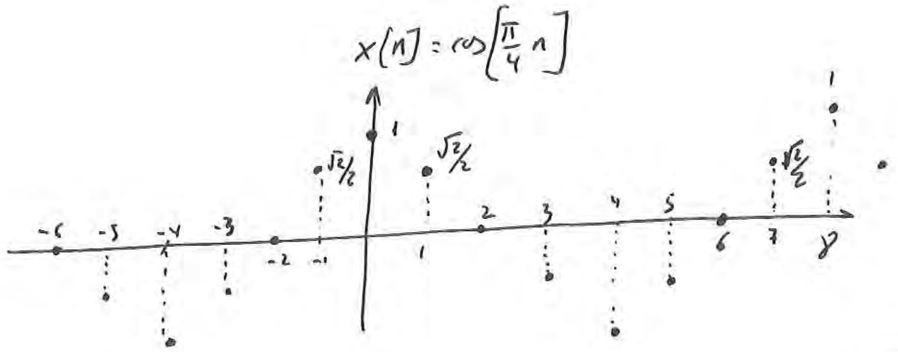
$$= \lim_{N\to\infty}\left\{\frac{1}{2N+1}\cdot 2N+1\right\} = \lim_{N\to\infty} 1 = 1$$

$$E_\infty = \sum_{k=-\infty}^{\infty}|x[k]|^2 = \sum_{k=-\infty}^{\infty}1 = \infty$$

Es una señal definida en potencia.

b) $$x[n] = \cos\left[\frac{\pi}{4} n\right]$$

Es una señal real, pero que va cambiando de valores positivos a negativos, por lo que:



Creemos que es periódica. Veamos $¿\frac{w_0}{2\pi} = \frac{k}{N} \in \mathbb{Q}?$

$$\frac{\pi/4}{2\pi} = \frac{1}{8} = \frac{k}{N} \in \mathbb{Q} \Rightarrow \text{Sí es periódica}$$

y el periodo es $N = 8$. Por lo tanto:

$$|x[n]| = \begin{cases} \cos\left(\frac{\pi}{4} n\right) & \text{si } (-2 \pm k \cdot N) \leq n \leq (2 \pm k \cdot N) \\ -\cos\left(\frac{\pi}{4} n\right) & \text{si } (2 \pm kN) < n < (6 \pm kN) \end{cases}$$

con $k = \ldots, -3, -2, -1, 0, 1, 2, 3 \ldots$

$$\measuredangle x[n] = \begin{cases} 0 & \text{si } (-2 \pm kN) \leq n \; (2 \pm kN) \\ \pi & \text{si } (2 \pm kN) < n \; (6 \pm kN) \end{cases}$$

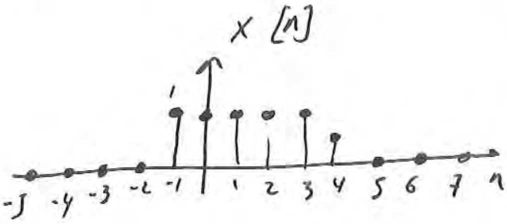
$$P_\infty = \text{Por ser periódica} = \frac{1}{N_0} \cdot \sum_{K=0}^{7} |x[K]|^2 = \frac{1}{8}\left(1 + \frac{1}{2} + 0 + \right.$$

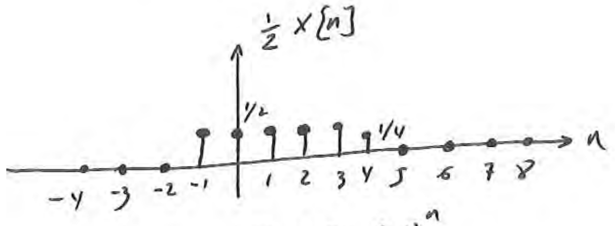
$$\left. + \frac{1}{2} + 1 + \frac{1}{2} + 0 + \frac{1}{2}\right) = \frac{1}{8}(4) = \frac{1}{2}$$

$E_\infty = \infty$ por estar definida en potencia.

P16 Dada la secuencia $x[n]$, dibuje la señal

$$y[n] = \frac{1}{2} \cdot x[n] + \frac{1}{2}(-1)^n \cdot x[n]$$





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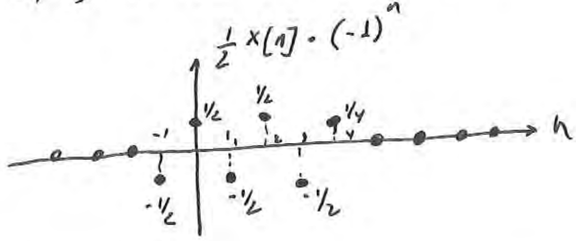
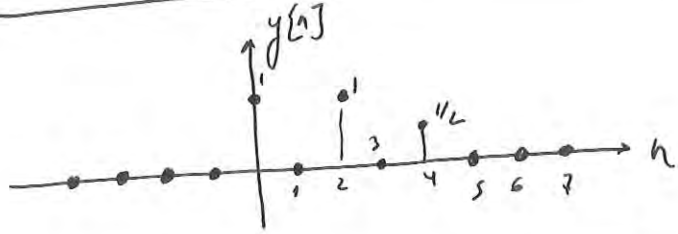




P17 Determine si las siguientes secuencias son o no periódicas y en su caso encuentre el periodo.

$$x[n] = \cos\left(\frac{8\pi n}{7} + 2\right)$$

Lo será si $\frac{\omega_0}{2\pi} = \frac{k}{N} \in \mathbb{Q}$

$$\frac{8\pi/7}{2\pi} = \frac{8}{14} = \frac{4}{7} = \frac{k}{N} \in \mathbb{Q} \Rightarrow$$ Es periódica de periodo $\underline{N = 7}$

$$x[n] = e^{j7\pi n}$$

$$w_0 = 7\pi \Rightarrow \frac{7\pi}{2\pi} = \frac{7}{2} = \frac{k}{N} \in \mathbb{Q} \Rightarrow$$ Sí es periódica y $N = 2$.

$$x[n] = e^{j\left[\frac{n}{8} - \pi\right]}$$

$$w_0 = \frac{1}{8} \Rightarrow \frac{1/8}{2\pi} = \frac{1}{16\pi} \neq \frac{k}{N} \in \mathbb{Q} \Rightarrow$$ No es periódica.

$$x[n] = 1 + e^{j\frac{4\pi n}{7}} + e^{j\frac{2\pi n}{5}}$$

El primer sumando es una cte y no no influye.

$e^{j\frac{4\pi}{7}\cdot n}$ es periódica $\left(\frac{4\pi/7}{2\pi} = \frac{4}{14} = \frac{2}{7} = \frac{k}{N_1} \in \mathbb{Q}\right)$ de periodo $N_1 = 7$

$e^{j\frac{2\pi}{5}\cdot n}$ es periódica $\left(\frac{2\pi/5}{2\pi} = \frac{1}{5} = \frac{k}{N_2} \in \mathbb{Q}\right)$ de periodo $N_2 = 5$

Por lo tanto $x[n]$ será periódica de periodo $N = \text{mcm}\{7, 5\} = 35$.

$$x[n] = 3 \cdot e^{j\frac{3}{5}\left(n + \frac{1}{2}\right)}$$

$$x[n] = 3 \cdot e^{j\frac{3}{10}} \cdot e^{j\frac{3}{5}n}$$

$$w_0 = \frac{3}{5} \Rightarrow \frac{3/5}{2\pi} = \frac{3}{10\pi} \neq \frac{k}{N} \in \mathbb{Q} \Rightarrow$$ No es periódica

$$x[n] = 3 \cdot e^{\frac{j3\pi\left(n+\frac{1}{2}\right)}{5}}$$

$$x[n] = 3 \cdot e^{j\frac{3\pi}{10}} \cdot e^{j\frac{3\pi}{5} \cdot n}$$

$$w_0 = \frac{3\pi}{5} \Rightarrow \frac{3\pi/5}{2\pi} = \frac{3}{10} = \frac{k}{N} \in \mathbb{Q} \Rightarrow$$

Sí es periódica y $N = 10$

$$x[n] = 2 \cdot \cos\left(\frac{\pi n}{4}\right) + \operatorname{sen}\left(\frac{\pi n}{8}\right) - 2 \cdot \cos\left(\frac{\pi n}{2} + \frac{\pi}{6}\right)$$

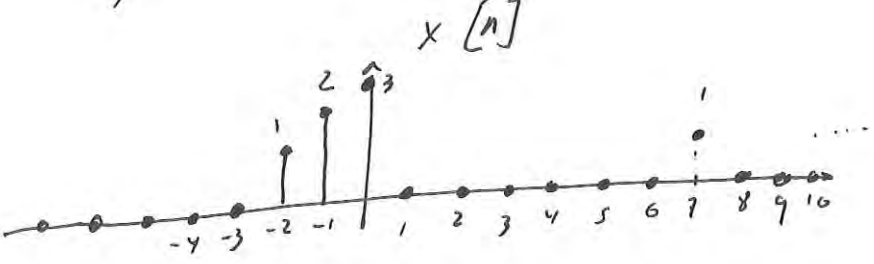
$$w_1 = \frac{\pi}{4} \Rightarrow \frac{\pi/4}{2\pi} = \frac{1}{8} = \frac{k}{N_1} \in \mathbb{Q} \Rightarrow N_1 = 8$$

$$w_2 = \frac{\pi}{8} \Rightarrow \frac{\pi/8}{2\pi} = \frac{1}{16} = \frac{k}{N_2} \in \mathbb{Q} \Rightarrow N_2 = 16$$

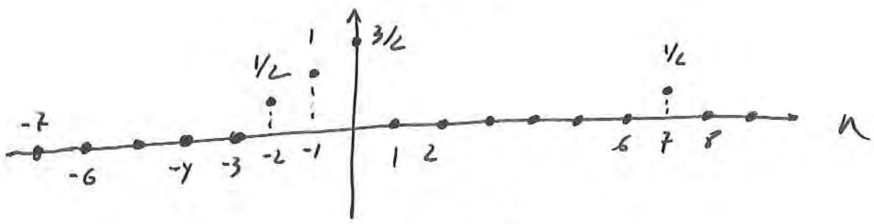
$$w_3 = \frac{\pi}{2} \Rightarrow \frac{\pi/2}{2\pi} = \frac{1}{4} = \frac{k}{N_3} \in \mathbb{Q} \Rightarrow N_3 = 4$$

$x[n]$ es periódica con $N = \text{m.c.m}\{4, 8, 16\} = 16$

P18 Calcular la parte par y la parte impar de $x[n]$.



$\frac{1}{2} \cdot x[n]$



$\frac{1}{2}\, x[-n]$