

# Segunda parte (ex.3)

De EjerciciosLMF2014

```
header {* Examen 3 *}
```

```
theory ex3_sol
```

```
imports Main  
begin
```

```
text {*
```

Reglas básicas de deducción natural de la lógica proposicional,  
de los cuantificadores y de la igualdad:

```
• conjI:      [[P; Q]] ⇒ P ∧ Q  
• conjunct1:  P ∧ Q ⇒ P  
• conjunct2:  P ∧ Q ⇒ Q  
• notnotD:    ¬¬ P ⇒ P  
• mp:        [[P → Q; P]] ⇒ Q  
• impI:      (P ⇒ Q) ⇒ P → Q  
• disjI1:     P ⇒ P ∨ Q  
• disjI2:     Q ⇒ P ∨ Q  
• disjE:      [[P ∨ Q; P ⇒ R; Q ⇒ R]] ⇒ R  
• FalseE:    False ⇒ P  
• notE:       [[¬P; P]] ⇒ R  
• notI:       (P ⇒ False) ⇒ ¬P  
• iffI:       [[P ⇒ Q; Q ⇒ P]] ⇒ P = Q  
• iffD1:      [Q = P; Q] ⇒ P  
• iffD2:      [P = Q; Q] ⇒ P  
• ccontr:     (¬P ⇒ False) ⇒ P  
  
• allI:       [[∀x. P x; P x ⇒ R]] ⇒ R  
• allE:       (∧x. P x) ⇒ ∀x. P x  
• exI:        P x ⇒ ∃x. P x  
• exE:        [[∃x. P x; ∧x. P x ⇒ Q]] ⇒ Q  
  
• refl:      t = t  
• subst:     [[s = t; P s]] ⇒ P t  
• trans:     [[r = s; s = t]] ⇒ r = t  
• sym:       s = t ⇒ t = s  
• not_sym:   t ≠ s ⇒ s ≠ t  
• ssubst:    [[t = s; P s]] ⇒ P t  
• box_equals: [[a = b; a = c; b = d]] ⇒ a = d  
• arg_cong:  x = y ⇒ f x = f y  
• fun_cong:  f = g ⇒ f x = g x  
• cong:      [[f = g; x = y]] ⇒ f x = g y
```

```
*}
```

The logo for Cartagena99 features the text 'Cartagena99' in a stylized, blue, serif font. The '99' is significantly larger and more prominent than the 'Cartagena' part. The text is set against a light blue background with a subtle gradient and a soft shadow effect.

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```
∀x. P x → (∀y. Q y → R x y), ∃x. P x ∧ (∃y. ¬(R x y)) ⊢ ¬(∀x. Q x)
----- *
```

lemma ej\_2\_c:

```
assumes "∀x. P x → (∀y. Q y → R x y)"
        "∃x. P x ∧ (∃y. ¬(R x y))"
shows "¬(∀x. Q x)"
```

proof

```
assume "∀x. Q x"
obtain a where 1: "P a ∧ (∃y. ¬(R a y))" using assms (2) ..
hence "∃y. ¬(R a y)" by (rule conjunct2)
then obtain b where "¬(R a b)" ..
have "P a" using 1 by (rule conjunct1)
have "P a → (∀y. Q y → R a y)" using assms (1) ..
hence "∀y. Q y → R a y" using `P a` by (rule mp)
hence "Q b → R a b" ..
have "Q b" using `∀x. Q x` ..
with `Q b → R a b` have "R a b" ..
with `¬(R a b)` show False ..
qed
```

text {\*

-----  
Ejercicio . Definir la función

suma :: "nat list ⇒ nat"

tal que (suma xs) es la suma de los elementos de la lista de números naturales xs. Por ejemplo,

suma [3::nat,2,4] = 9

-----  
\*}

```
fun suma :: "nat list ⇒ nat" where
  "suma [] = 0"
| "suma (x#xs) = x + suma xs"
```

```
value "suma [6::nat,2,4]" -- "= 12"
```

text {\*

-----  
Ejercicio. Demostrar o refutar

suma (xs @ ys) = suma xs + suma ys

-----  
\*}

lemma suma\_append:

```
"suma (xs @ ys) = suma xs + suma ys"
```

proof (induct xs)

```
show "suma ([] @ ys) = suma [] + suma ys" by simp
```

next

```
fix a xs
```

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qed

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