UNED

International Master's in Theoretical & Practical Application of Finite Element Method and CAE Simulation

Dynamic Analysis of Structures for the Finite Element Method

Handout 2 (20 points)

Exercise. [Chapter 1]. The structure of the figure consists on two columns, one floor and a damper. The stiffness of the floor is very large when compared to that of the columns, whereas the mass of the columns may be neglected when compared to that of the floor. Hence, the system can be considered as a single degree of freedom system (SDOF), where the displacement of that SODF is the horizontal movement of the floor. The compounded stiffness of the columns is $k = 2 \times 12EI/L^3$, the mass of the floor is m and the effective horizontal damping of the damper is $c = C\cos^2\alpha$, where $\alpha = \arctan(L/d)$.

1. (1 point) Compute the undamped natural circular frequency ω_n of the system (i.e. when $C = 0$) in terms of k, m ($c = 0$). Obtain such frequency in terms of E, I, L, C, m, α. Verify that the units of the stiffness are N/m. Obtain those of the circular frequency ω_n. Compute the natural frequency f_n in Hz. Compute the period of the system T in seconds.

2. (1 point) Compute the value of C that gives the critical damping $\zeta = C/C_{cr} = 1$ of the system.

3. (1 point) Compute the damped natural frequency of the system as a function of the parameters E, I, L, C, m, α

4. (1 point) Use the following values: $EI/L^3 = 10^5\,\mathrm{N}/\mathrm{m}$, $d = L$, $m = 1000\,\mathrm{kg}$. Plot the damped natural frequency as a function of $C \in [0,\ C_{cr}]$

5. (1 point) Assume zero forces and an initial condition of a prescribed displacement $u_0 = 0.1\,\mathrm{m}$. Plot in the same graph (use MATLAB[1], SciLAB, GNU Octave or even Excel) the response of the system up to 5 periods for the following damping ratios: $\zeta = 0\%$, $\zeta = 10\%$; $\zeta = 20\%$; $\zeta = 100\%$; $\zeta = 200\%$. Use three graphs, one for the displacements of all systems, one for the velocities and one for the accelerations. Which are the damping constants c for those values?

6. (1 point) From the obtained graph, verify for the cases of $\zeta = 10\%$ and $\zeta = 20\%$ that the obtained damping is effectively the prescribed one (measuring the amplitude decay). For these frequency values, which is the number of cycles needed to damp the response to only a 1% of the maximum one? Verify also that the obtained damped frequency is given by $\omega_d = \omega_n\sqrt{1-\zeta^2}$ (you will need to make a zoom because note that $\omega_n \approx \omega_d$)

7. (1 point) The basement of the structure suffers an acceleration given by a_g so in an inertial reference system we can consider that the load applied to the mass m is $-ma_g$. Draw the forces acting on the mass using an inertial system of reference and using an absolute system of reference. Comment on the difference in the kinematic variables (displacements and velocities) between both systems of reference (absolute and inertial). You may consult Section 10.2 of the notes.

8. (1 point) The system with a damping of with $\zeta = 10\%$ is subjected to a forced ground motion of the type $a_g = F_0/m\ \sin\omega t$, where $a_0 = F_0/m$ is the amplitude, which is equal to the force F_0 needed to obtain a static displacement of $0.1\,\mathrm{m}$ divided by the mass. Plot the response of the system up to 10 periods of the forcing frequency, where the forcing frequencies are (use the same plot, scale conveniently) $\omega = 0.5\omega_n$; $\omega = 0.95\omega_n$; $\omega = \omega_n$; $\omega = 1.05\omega_n$; $\omega = 2\omega_n$. Repeat the plot for the following damping ratios: $\zeta = 0\%$; $\zeta = 0.1\%$; $\zeta = 1\%$.

9. (1 point) Repeat for this particular system the plot of Figure 14 of the notes, plotting the maximum response amplitudes against circular frequency for different damping ratios. Normalize also to the static value.

10. (1 point) Repeat for this particular case the plot of Figure 15 of the notes, plotting the displacement, velocity and acceleration response factors.

[1] MATLAB is usually available at most universities, but it is a pay-program. SciLAB and Octave are freeware, mostly MATLAB compatible and freely available over the internet. Code written in MATLAB usually runs on both SciLAB and Octave without (or with minor) changes. These type of programs are tools you should be used to (otherwise, this is the chance to get used to!).

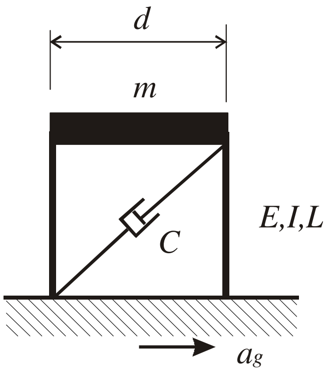

Figure of Exercise 1

Exercise. [Chapter 2]. The structure of the figure (left) consists on two floors, one with mass $M = 1000\,\mathrm{kg}$ and another with mass $2M = 2000\,\mathrm{kg}$. The stiffness of the floors is very large when compared to that of the columns, whereas the mass of the columns may be neglected when compared to that of the floors. Hence, the system can be considered as a TWO degree of freedom system (2DOF), where the displacements of the 2DOF are the horizontal movements of the floor. The compounded stiffness of the columns at each level is $k = 2 \times 12EI/L^3$, where $EI/L^3 = 10^5\,\mathrm{N\,/\,m}$

1. (2 points) Compute by hand the undamped natural frequencies of the system ω_1 and ω_2 (ω_1 being the smallest one) and the modes of vibration ϕ_1, ϕ_2. Verify the results with MATLAB or similar program.

2. (2 points). The structure is under a ground harmonic movement with acceleration amplitude A_g (assume $A_g = 1$) and frequency ω_g, i.e.

$$a_g = A_g \sin(\omega_g t)$$

Let $\mathbf{J}_g = [1, 1]^T$. Compute the participation factor $b_i = \phi_i^T \mathbf{M}\mathbf{J}_g$ (Section 10.2 may help) and the modal mass b_i^2 for both modes.

3. (2 points). The damping is $\zeta_1 = \zeta_2 = 2\%$ of the critical damping. Compute using modal superposition the response of the structure for forcing frequencies of $\omega_g = 0.1\omega_1$, $\omega_g = \omega_1$, $\omega_g = (\omega_1 + \omega_2)/2$, $\omega_g = \omega_2$, $\omega_g = 10\omega_2$. Plot the results for both degrees of freedom in MATLAB or similar. Use a plot for each ω_g value.

4. (4 points) Write down the uncoupled system of equations. Assume that the ground movement has the frequency of $\omega_g = \omega_1$. Heuristically, design a TMD to damp the first mode in a setting like the one shown in the right of the figure.

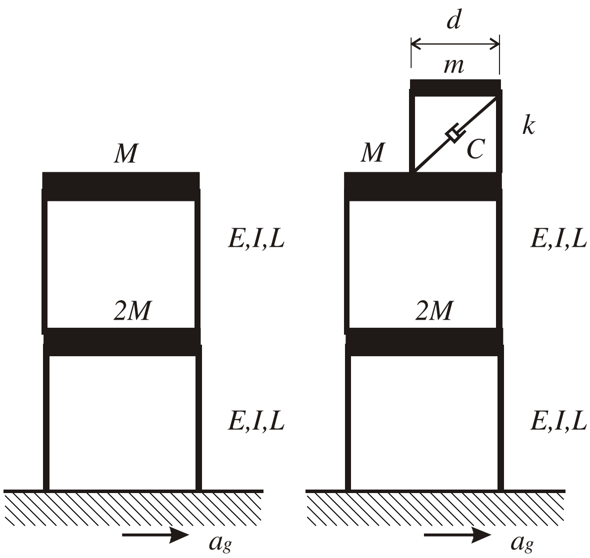

Figure of Exercise 2