Predictive Modeling Lab 2020-02-03

BSc in Data Science and Engineering

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We finish the zero-course in R from the last lab, introduce the concepts of joint/marginal/conditional distributions, and do some exercises.

Random vectors exercises

- **Exercise 1**. Consider the continuous random vector (X_1, X_2) with (joint) pdf $f : \mathbb{R}^2 \to [0, \infty)$ given by

$$f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & x_1, x_2 > 0, \\ 0, & \text{else.} \end{cases}$$

  a. What is the region where (X_1, X_2) is supported?
  b. Check that f is a proper pdf.
  c. Obtain the (joint) cdf of (X_1, X_2).
  d. Compute $\mathbb{P}[X_1 < 1, X_2 \leq 2]$.
  e. Compute $\mathbb{P}[X_1 > 1 \text{ or } X_2 > 2]$.
  f. Compute $\mathbb{P}[X_1 < 1, X_2 > 2]$.
  g. Obtain the marginal pdfs of (X_1, X_2).
  h. Obtain the marginal cdfs of (X_1, X_2).
  i. Compute $\mathbb{P}[X_1 < 1, X_2 > 2 \text{ or } X_1 > 3]$.
  j. Obtain the conditional pdfs of (X_1, X_2).
  k. Obtain the conditional cdfs of (X_1, X_2).
  l. Compute $\mathbb{P}[X_1 < 1 | X_2 = 3]$.
  m. Compute $\mathbb{P}[1 < X_2 < 2 | X_1 = 2]$.
  n. Are X_1 and X_2 independent?

- **Exercise 2**. Consider the continuous random vector (X_1, X_2) with (joint) pdf $f : \mathbb{R}^2 \to [0, \infty)$ given by

$$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1, \\ 0, & \text{else.} \end{cases}$$

  a. What is the region where (X_1, X_2) is supported?
  b. Check that f is a proper pdf.
  c. Obtain the (joint) cdf of (X_1, X_2).
  d. Obtain the marginal pdfs of X_1 and X_2.
  e. Obtain the marginal cdfs of X_1 and X_2.
  f. Obtain the conditional pdfs of $X_1 | X_2 = x_2$ and $X_2 | X_1 = x_1$.
  g. Are X_1 and X_2 independent?

- **Exercise 3**. Let (X_1, X_2) be a continuous random vector, with uniform density on the square with vertex $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$. Obtain:

  a. The marginal pdfs and cdfs of (X_1, X_2).
  b. The conditional pdfs and cdfs of (X_1, X_2).

- **Exercise 4** (voluntary homework; deadline 2020-02-27). Let (X_1, X_2) be a continuous random vector, with uniform density on the unit sphere $\{(x_1, x_2, x_3) \in \mathbb{R}^2 : x_1^2 + x_2^2 + x_3^2 = 1\}$. Obtain:

  a. The marginal pdf of X_1.
  b. The marginal cdf of X_2.
  c. The expectation and variance of X_1.