GLOBAL FINAL EXAM

The use of computer is necessary. The solution of the problems has to be a unique file (pdf, word or something similar). The file has to include the solution, the codes used and the necessary explanations.

PROBLEM 1 (4 points)

Let V be a function $\mathbb{R}^n \to \mathbb{R}$ with gradient $\nabla V(\mathbf{x}) = (\frac{\partial V}{\partial x_1}, \cdots, \frac{\partial V}{\partial x_n})$. A simple iterative algorithm to find a local minimum of V, the steepest descent, consists on defining a succession of $\mathbf{x}$ values until the gradient $\nabla V(\mathbf{x}) = 0$. The scheeme of the algorithm is

Algorithm 1 Aim: For the function $V(\mathbf{x})$ $\mathbb{R}^n \to \mathbb{R}$, calculate $\mathbf{x} = \text{argmin}_x V(\mathbf{x})$

Define an initial $\mathbf{x} = \mathbf{x}_0$
Compute initial descending direction: $\mathbf{p} = -\nabla V(\mathbf{x}) = -(\frac{\partial V}{\partial x_1}, \cdots, \frac{\partial V}{\partial x_n})$
Check intial error: $error = \|\mathbf{p}\|$
while $error > tol$ **do**
  $\mathbf{x} \leftarrow \mathbf{x} + \alpha \mathbf{p}$
  $\mathbf{p} = -\nabla V(\mathbf{x}) = -(\frac{\partial V}{\partial x_1}, \cdots, \frac{\partial V}{\partial x_n})$
  $error = \|\mathbf{p}\|$
end while

being α a fix parameter defining the size of each iteration and tol the tolerance.

1. write in matlab/octave the algorithm

   INPUTS: The functions $V(\mathbf{x})$, $\nabla V(\mathbf{x})$, α and tol

   OUTPUT: The resulting $\mathbf{x}$ that minmizes $V(\mathbf{x})$

2. write a matlab/octave function of

$$V(x_1, x_2) = \sin^2(x_1 * x_2) + (x_1 - 3)^2 + (x_2 + 2)^2$$

   INPUT: The vector $\mathbf{x}$

   OUTPUT: The value of the function $V(\mathbf{x})$

3. write a matlab/octave that evaluates de gradient of V. Derivates should be done by hand!

   INPUT: The vector $\mathbf{x}$

   OUTPUT: The value of the gradient of the function $\nabla V(\mathbf{x})$ (a vector)

4. Obtain the minimum of the function defined in point (2) using the algorithm developed. Use $\alpha = 1E-3$ and $tol = 1E-7$