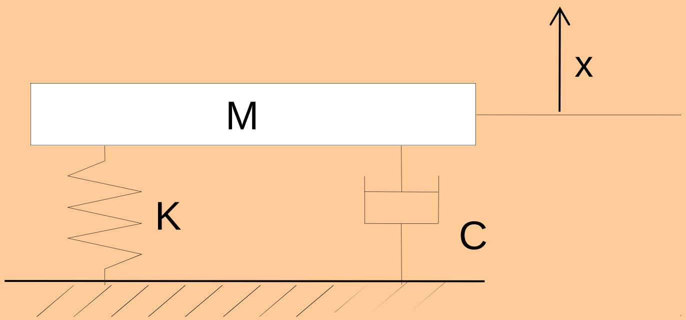
Tema 5

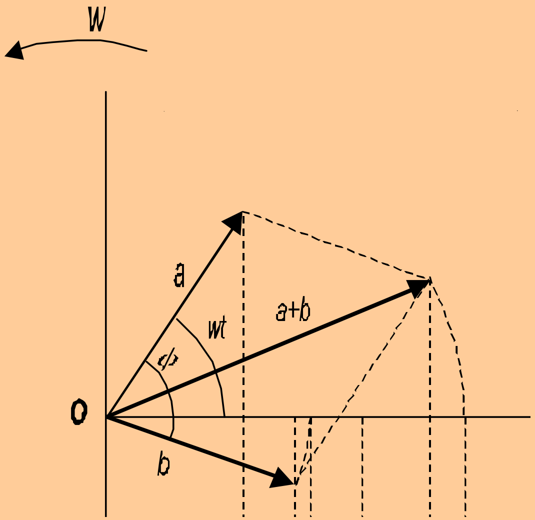
VIBRACIONES EN SISTEMAS MECÁNICOS DE 1 G.D.L.

INDICE

5.2. Formulación del modelo mecánico

Vibración Libre





Ecuación del movimiento vertical:

$$\sum F = -Cv - Kx = Ma$$

$$Ma + Cv + Kx = 0$$

$$M\frac{d^2x}{dt^2} + C\frac{dx}{dt} + Kx = 0$$

$$x = e^{rt}$$

$$\boxed{Mr^2 + Cr + K = 0}$$

$$\boxed{r_{1,2} = \frac{-C \pm \sqrt{C^2 - 4KM}}{2M} = -\frac{C}{2M} \pm \sqrt{\left(\frac{C}{2M}\right)^2 - \frac{K}{M}}}$$

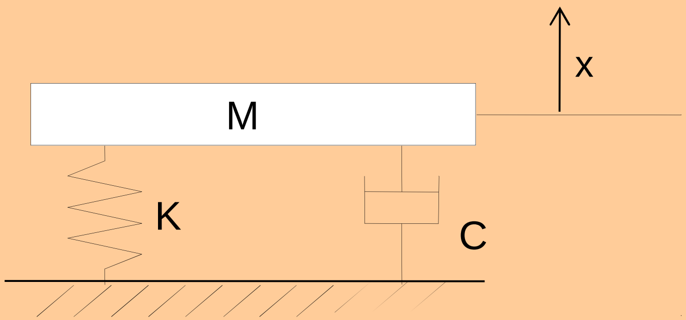
$$x = Ae^{r_1 t} + Be^{r_2 t}$$

$$x = A\,sen(wt) + B\cos(wt)$$

5.2. Formulación del modelo mecánico

Vibración Libre

$$r_{1,2} = \frac{-C \pm \sqrt{C^2 - 4KM}}{2M} = -\frac{C}{2M} \pm \sqrt{\left(\frac{C}{2M}\right)^2 - \frac{K}{M}}$$



$$Cc = 2\sqrt{KM}$$

Amortiguamiento crítico

$$\varsigma = \frac{C}{C_C} = \frac{C}{2M\omega_n} = \frac{C}{2\sqrt{KM}}$$

Factor de amortiguamiento.

$$\omega_d = \sqrt{\frac{K}{M} - \left(\frac{C}{2M}\right)^2} = \sqrt{\omega_n^2 - \varsigma^2\omega_n^2} = \omega_n\sqrt{1-\varsigma^2}$$

Frecuencia natural del sistema amortiguado.

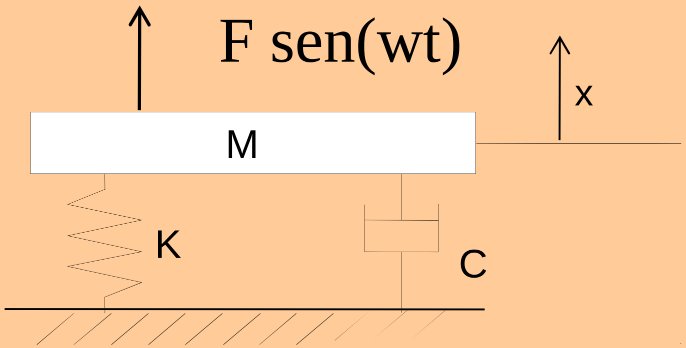
$$\sqrt{\frac{K}{M}} = \omega_n$$

Frecuencia natural del sistema no amortiguado, (C=0)

$$x = e^{-\varsigma\omega_n t}\left[A\operatorname{sen}(\omega_d t) + B\cos(\omega_d t)\right]$$

5.2. Formulación del modelo mecánico

Vibración Forzada



Ecuación del movimiento vertical:

$$Ma + Cv + Kx = Fsen(wt)$$

$$x = e^{rt}$$

$$Mr^2 + Cr + K = Fsen(wt)$$

$$\boxed{r_{1,2} = \frac{-C \pm \sqrt{C^2 - 4KM}}{2M} = -\frac{C}{2M} \pm \sqrt{\left(\frac{C}{2M}\right)^2 - \frac{K}{M}}}$$

Solución de la ecuación completa:

$$x = Ae^{r_1 t} + Be^{r_2 t} + \frac{F}{M\sqrt{(\omega_n^2 - \omega^2)^2 + (2\varsigma\omega_n\omega)^2}} \operatorname{sen}(\omega t - \varphi)$$

$$x = e^{-\varsigma\omega_n t}\left[A\operatorname{sen}(\omega_d t) + B\cos(\omega_d t)\right] + X\operatorname{sen}(\omega t - \varphi)$$

$$\boxed{X = \frac{F}{M\sqrt{(\omega_n^2 - \omega^2)^2 + (2\varsigma\omega_n\omega)^2}}}$$

$$\boxed{\operatorname{tg}\varphi = \frac{2\zeta\omega / \omega_n}{(1 - \omega^2 / \omega_n^2)}}$$

5.2. Formulación del modelo mecánico

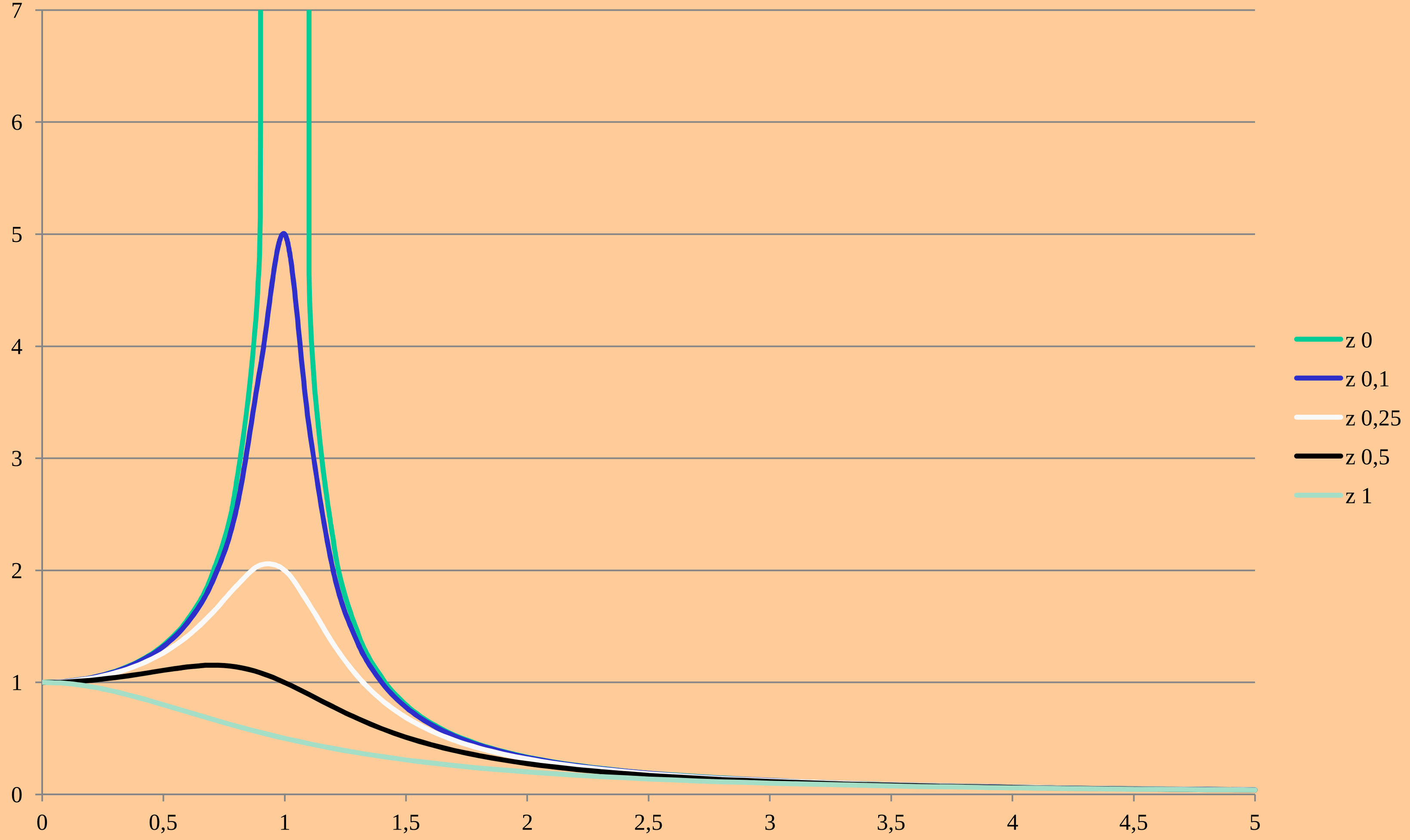
Vibración Forzada

Factor de Amplificación, A:

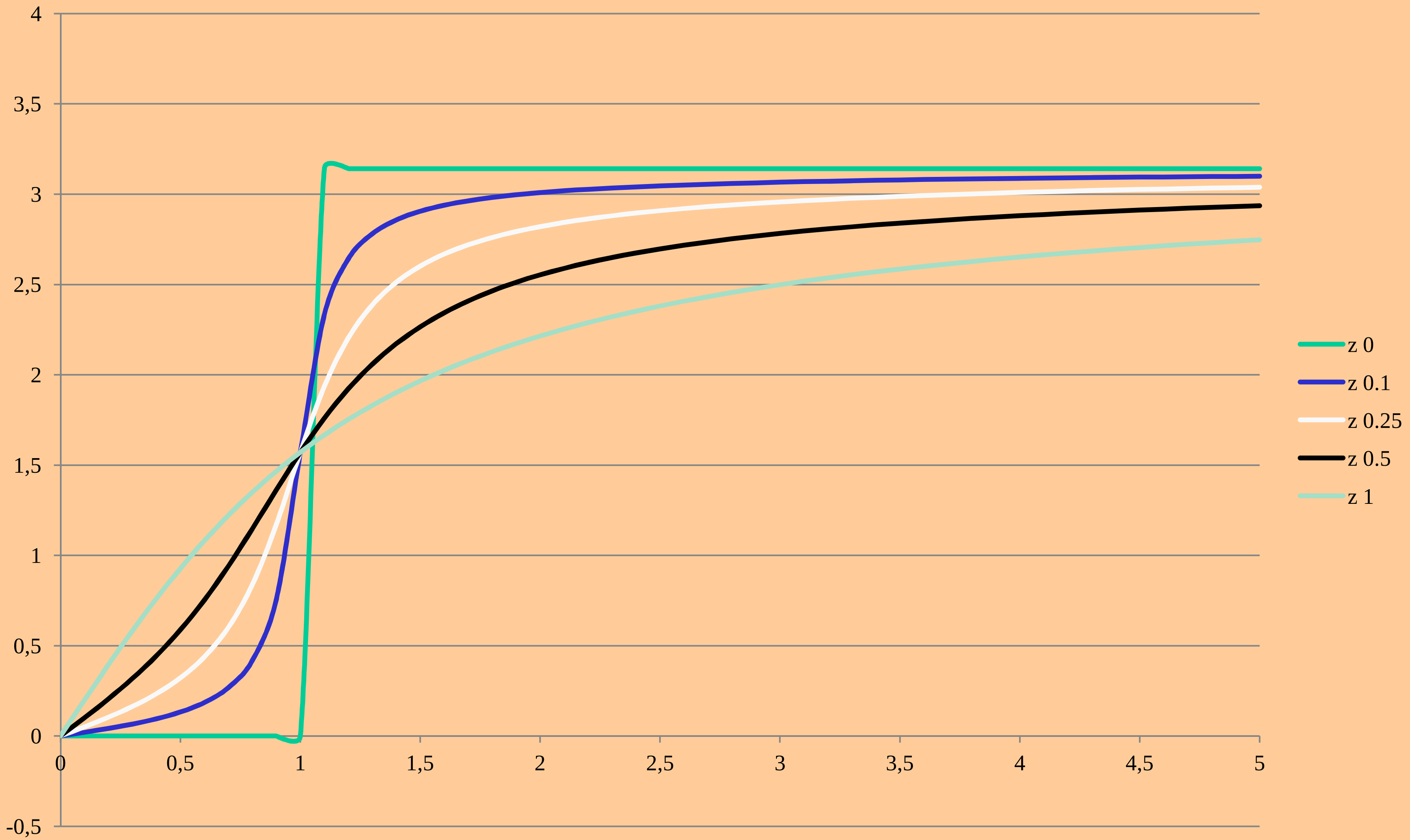
$$\boxed{A = \frac{X}{\delta_{est}}} = \frac{X}{F/K} = \frac{\dfrac{F}{M\sqrt{(\omega_n^2 - \omega^2)^2 + (2\varsigma\omega_n\omega)^2}}}{\dfrac{F}{K}}$$

$$\boxed{\mathbf{A} = \frac{\mathbf{1}}{\sqrt{(\mathbf{1} - \omega^2/\omega_n^2)^2 + (\mathbf{2}\varsigma\omega/\omega_n)^2}}}$$

Factor de Amplificación, A:



Ángulo de desfase, φ:



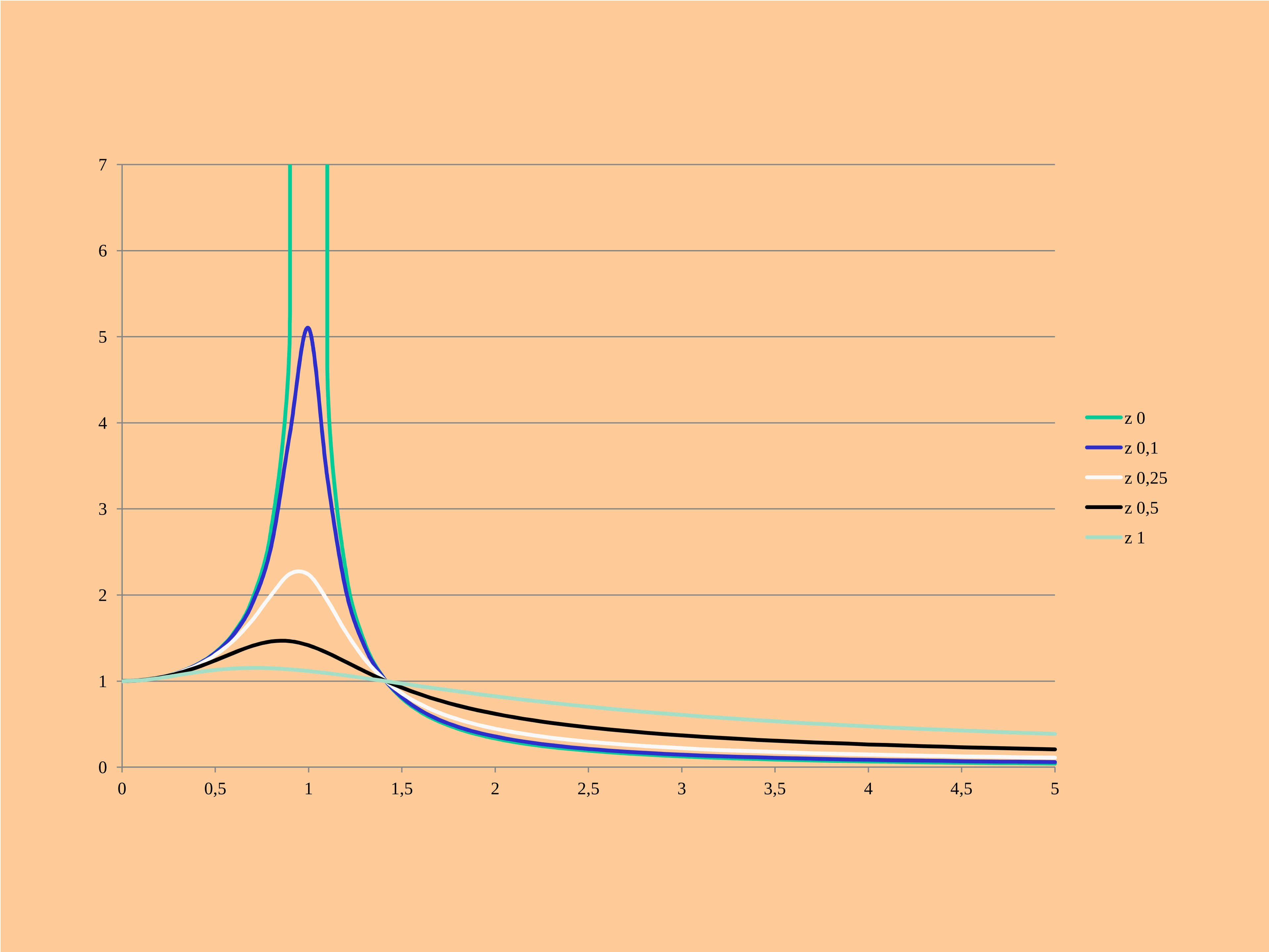
5.2. Formulación del modelo mecánico

Vibración Forzada

Función de Transmisibilidad

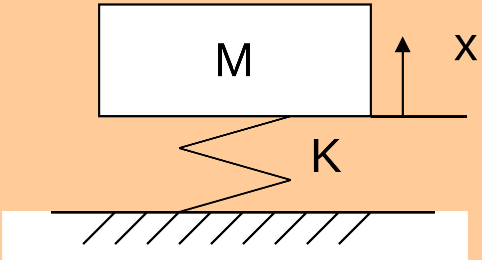
$$\boxed{\mathrm{T.R.} = \frac{\text{Fuerza transmitida}}{\text{Fuerza aplicada}}}$$

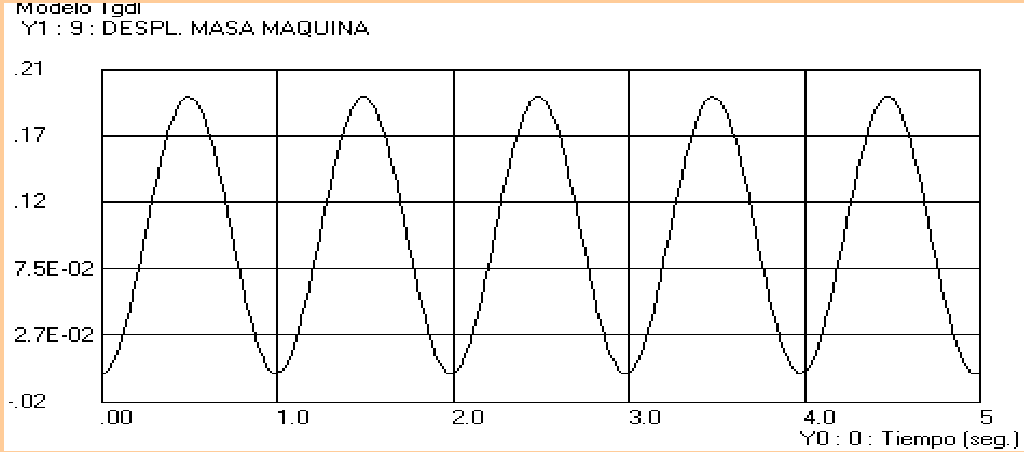
$$\boxed{\mathrm{T.R.} = \sqrt{\frac{1 + \left(2\varsigma\omega / \omega_n\right)^2}{(1 - \omega^2 / \omega_n^2)^2 + \left(2\varsigma\omega / \omega_n\right)^2}}}$$



7
6
5
4
3
2
1
0
0
0,5
1
1,5
2
2,5
3
3,5
4
4,5
5
z 0
z 0,1
z 0,25
z 0,5
z 1

Método de Rayleigh





$$x = X sen\,(\omega_n t)$$

$$\dot{x} = X\omega_n \cos(\omega_n t)$$

$$\boxed{\tau = 2\pi/\omega_n \;\Longrightarrow\; \mathrm{T}_{\mathrm{prom}} = \mathrm{V}_{\mathrm{prom}}}$$

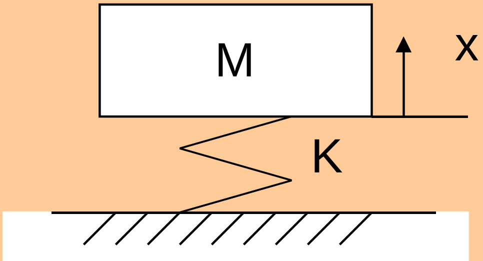
$$T_{prom} = \frac{1}{\tau}\int_0^\tau \frac{1}{2} M\dot{x}^2 dt = \frac{1}{2} M \frac{X^2\omega_n^2}{\tau}\int_0^\tau \cos^2(\omega_n t)dt = \frac{1}{4} MX^2\omega_n^2$$

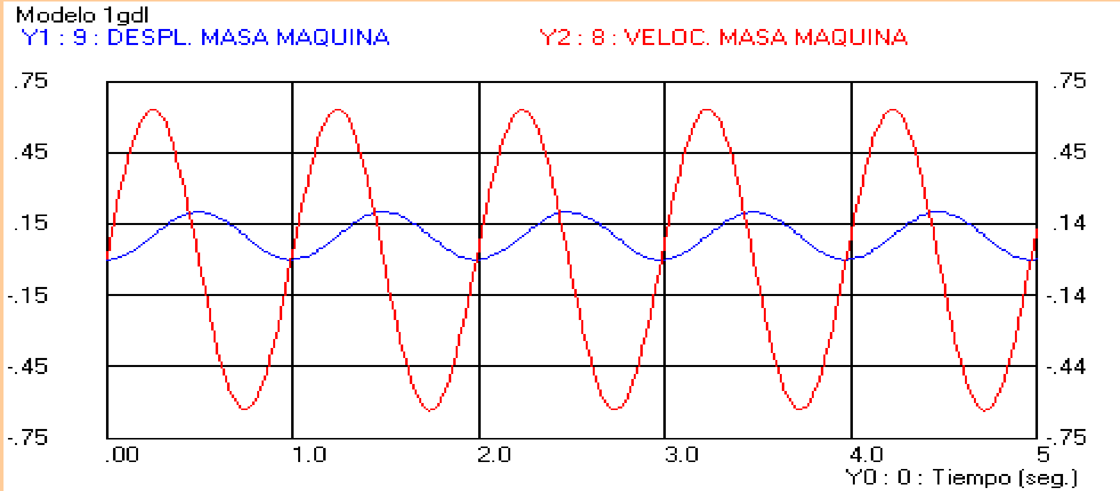
$$V_{prom} = \frac{1}{\tau}\int_0^\tau \frac{1}{2} Kx^2 dt = \frac{1}{2}\frac{KX^2}{\tau}\int_0^\tau sen^2(\omega_n t)dt = \frac{1}{4} KX^2$$

$$\frac{1}{4} MX^2\omega_n^2 = \frac{1}{4} KX^2$$

$$\boxed{\omega_n = \sqrt{\frac{K}{M}}}$$

Método de Rayleigh





$$\boxed{\Delta \mathbf{T}_{\mathbf{max}} = \Delta \mathbf{V}_{\mathbf{max}}}$$

$$\left.\begin{aligned} x &= X sen(\omega_n t) \\ \dot{x} &= X\omega_n \cos(\omega_n t) \end{aligned}\right\} \quad \frac{1}{2} M X^2 \omega_n^2 = \frac{1}{2} K X^2 \quad \Rightarrow \quad \boxed{\omega_n = \sqrt{\frac{K}{M}}}$$

Tres premisas importantes del Método de energía de Rayleigh:

a) ***Forma del modo.***

b) ***Movimiento armónico simple.***

c) ***Sólo considera dos formas de energía, la cinética y la potencial.***