

PROBLEMAS DE AEROELASTICIDAD

Ejercicio 24

Se desea estudiar la respuesta de un avión a una ráfaga cosenoidal de la forma

$$W_G(s) = \begin{cases} \frac{W_o}{2} \left(1 - \cos\left(\frac{2\pi s}{H}\right) \right) & 0 \leq s \leq H \\ 0 & s > H \end{cases}$$

donde s es la variable tiempo adimensional $s = V_\infty t/b$, H el gradiente adimensional de la ráfaga, $H = 50$ y W_o la intensidad de la ráfaga. Para ello el avión se representa por un ala bidimensional libre de desplazarse verticalmente y las funciones de Kussner y de Wagner se aproximan por

$$\Psi(s) = 1 - \frac{1}{2} e^{-0.130s} - \frac{1}{2} e^{-s}$$

$$\Phi(s) = 1 - 0.165 e^{-0.0455s} - 0.335 e^{-0.300s}$$

Sea λ el parámetro másico definido como $\lambda = m/4\pi\rho b^2$.

Si para los casos de $\lambda = 5$ y de $\lambda = 50$ las raíces de la ecuación cubica

$$p^3 + a_1 p^2 + a_2 p + a_3 = (p + \lambda_1)(p + \lambda_2)(p + \lambda_3) \text{ son}$$

$$\lambda = 5 \quad \lambda_1 = -0.2545 \quad \lambda_2 = -0.0693 + i 0.0174$$

$$\lambda_3 = -0.0693 + i 0.0174$$

$$\lambda = 50 \quad \lambda_1 = -0.2966 \quad \lambda_2 = -0.0433 \quad \lambda_3 = -0.0106$$

Se pide:

- 1) Obtenga la expresión de la respuesta en el dominio del Laplace.
- 2) Determinar el factor de carga definido como

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Solución al problema #24

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$$M \ddot{\xi} = L_G + L_M$$

$$M \frac{U_{\infty}^2}{b^2} \frac{d^2 \xi}{ds^2} = L_G(s) + L_M(s)$$

$$L_M(s) = -\pi \rho_{\infty} U_{\infty}^2 \left[\frac{d^2 \xi}{ds^2} + 2 \int_0^s \phi(s-\sigma) \frac{d^2 \xi}{ds^2} d\sigma \right]$$

$$L_G = 2\pi \rho_{\infty} U_{\infty}^2 b \int_0^s \psi'(s-\sigma) \frac{W_G(\sigma)}{U_{\infty}} d\sigma$$

Tomando la transformada de Laplace

$$M \frac{U_{\infty}^2}{b^2} p^2 \bar{\xi} + \pi \rho_{\infty} U_{\infty}^2 [p^2 \bar{\xi} + 2 \bar{\phi} p^2 \bar{\xi}] =$$

$$2\pi \rho_{\infty} U_{\infty} b \bar{\psi} \cdot p \bar{W}_G$$

$$p^2 \bar{\xi} = \frac{2\pi \rho_{\infty} U_{\infty} b \bar{\psi} p \bar{W}_G / U_{\infty}}{M \frac{U_{\infty}^2}{b^2} + \pi \rho_{\infty} U_{\infty}^2 (1 + 2\bar{\phi})}$$

$$p^2 \bar{\xi} = \frac{2\pi \rho_{\infty} U_{\infty} b \bar{\psi} p \bar{W}_G / U_{\infty}}{M \frac{U_{\infty}^2}{b^2} + \pi \rho_{\infty} U_{\infty}^2 (1 + 2\bar{\phi})}$$

Dividiendo por $4\pi \rho_{\infty}$ obtenemos

$$\boxed{p^2 \bar{\xi} = \frac{b/2 \bar{\psi} p \bar{W}_G / U_{\infty}}{M \frac{U_{\infty}^2}{b^2} + \pi \rho_{\infty} U_{\infty}^2 (1 + 2\bar{\phi})}}$$

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Tomando las aproximaciones

$$\Psi(s) = 1 - \frac{1}{2} e^{-0.130s} - \frac{1}{2} e^{-s}$$

$$\phi(s) = 1 - 0.165 e^{-0.0455s} - 0.335 e^{-0.3s}$$

obtenemos la TxF de Laplace

$$\bar{\Psi}(p) = \frac{1}{p} - \frac{1/2}{p+0.130} - \frac{1/2}{p+1}$$

$$\bar{\phi}(p) = \frac{1}{p} - \frac{0.165}{p+0.0455} - \frac{0.335}{p+0.3}$$

Tenemos que calcular ahora la Tx de Laplace de W_G

$$W_G(s) = \frac{W_0}{2} \left(1 - \cos\left(\frac{2\pi s}{H}\right) \right) \quad 0 \leq s \leq H$$

$$0 \quad s > H$$

$$\bar{W}_G(p) = \int_0^{\infty} e^{-ps} W_G(s) ds = \int_0^H e^{-ps} \frac{W_0}{2} \left(1 - \cos\left(\frac{2\pi s}{H}\right) \right) ds$$

$$= \frac{W_0}{2} \left[\frac{e^{-ps}}{-p} \Big|_0^H - \frac{e^{-ps}}{p^2 + \left(\frac{2\pi}{H}\right)^2} \left(-p \cos\left(\frac{2\pi s}{H}\right) - \frac{2\pi}{H} \sin\left(\frac{2\pi s}{H}\right) \right) \Big|_0^H \right]$$

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$$\bar{W}_g(p) = \frac{W_0}{z} \left[\frac{1}{p} (1 - e^{-pH}) - \frac{p}{p^2 + \left(\frac{2\pi}{H}\right)^2} (1 - e^{-pH}) \right]^{3/8}$$

$$\bar{W}_g(p) = \frac{W_0}{2p} \frac{(1 - e^{-pH}) \left(\frac{2\pi}{H}\right)^2}{p^2 + \left(\frac{2\pi}{H}\right)^2}$$

Sustituyendo obtenemos

$$p^2 \bar{g} = \frac{bW_0}{4U_{00}} \left(\frac{2\pi}{H}\right)^2 \frac{(1 - e^{-pH})}{p^2 + \left(\frac{2\pi}{H}\right)^2} \left(\frac{1}{p} - \frac{1/2}{p+0.130} - \frac{1/2}{p+1} \right)$$

$$\lambda + \frac{1}{4} + \frac{1}{2} \left(\frac{1}{p} - \frac{0.165}{p+0.0455} - \frac{0.335}{p+0.3} \right)$$

Reordenando términos obtenemos

$$p^2 \bar{g} = \frac{bW_0}{U_{00}} \left(\frac{\pi}{H}\right)^2 \frac{0.1412}{1+0.250} \frac{p^3 + 0.5756p^2 + 0.09315p + 0.003141}{(p+0.13)(p+1)(p^3 + a_1p^2 + a_2p + a_3)}$$

$$\frac{1}{p^2 + \left(\frac{2\pi}{H}\right)^2} (1 - e^{-pH})$$

$$\text{Denominador } v^p(p) = p^3 + 0.5756p^2 + 0.09315p + 0.003141$$

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$$\ddot{\xi}(s) = \frac{bW_0}{U_0} \left(\frac{\pi}{H}\right)^2 \frac{0.1412}{\lambda + 0.25} \mathcal{L}^{-1} \left[\frac{W(p)}{\mathcal{D}(p)} (1 - e^{-pH}) \right]$$

Sea $f(s) = \mathcal{L}^{-1} \frac{W(p)}{\mathcal{D}(p)}$ por el teorema del retardo de la transformada de Laplace

$$\mathcal{L}^{-1} \left[\frac{W(p)}{\mathcal{D}(p)} (1 - e^{-pH}) \right] = f(s) - f(s-H)$$

donde $f(s-H)$ solo esta definida para $s > H$.

$$\ddot{\xi}(s) = \frac{bW_0}{U_0} \left(\frac{\pi}{H}\right)^2 \frac{0.1412}{\lambda + 0.25} [f(s) - f(s-H)]$$

Però

$$f(s) = A_1 e^{-0.130s} + A_2 e^{-s} + B_1 e^{-\gamma_1 s} + B_2 e^{-\gamma_2 s} + B_3 e^{-\gamma_3 s} + B_4 e^{-i \frac{2\pi}{H} s} + B_5 e^{i \frac{2\pi}{H} s}$$

Donde γ_1, γ_2 y γ_3 son las raíces del polinomio cúbico

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$$\underline{\lambda=5} \quad a_1 = 0.3931238; a_2 = 0.0403905; a_3 = 0.013000 \quad 5/8$$

$$\frac{1}{p} - \frac{1/2}{p+0.13} - \frac{1/2}{p+1}$$

$$\lambda + \frac{1}{4} + \frac{1}{2} \left(\frac{1}{p} - \frac{0.165}{p+0.0455} - \frac{0.335}{p+0.3} \right) \equiv$$

$$\frac{0.1412}{\lambda + \frac{1}{4}} \cdot \frac{p^3 + 0.5756p^2 + 0.09315p + 0.003141}{(p+0.13)(p+1)(p^3 + a_1p^2 + a_2p + a_3)}$$

$$\frac{(p+0.13)(p+1) - \frac{1}{2}p(p+1) - \frac{1}{2}(p)(p+0.13)}{\cancel{(p+0.13)(p+1)}}$$

$$\frac{p(p+0.0455)(p+0.3) \left(\lambda + \frac{1}{4} \right) + \frac{1}{2} \left((p+0.0455)(p+0.3) - 0.165p \cdot (p+0.3) - 0.335p(p+0.0455) \right)}{\cancel{p(p+0.0455)(p+0.3)}}$$

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$$p^2 + 1.13p + 0.13 - \frac{p^2}{2} - 0.5p - 0.5p^2 - 0.065p$$

$$= 0.565p + 0.13$$

$$N^p(p) = (p+0.3)(p+0.0455)(0.565p+0.13)$$

Denominador

$$p(p+0.0455)(p+0.3) = p^3 + 0.3455p^2 + 0.0137p$$

$$\frac{1}{2}(p+0.0455)(p+0.3) = \frac{1}{2}p^2 + 0.1728p + 0.0069$$

$$-\frac{0.165}{2}p(p+0.3) = -0.0825p^2 - 0.0248p$$

$$-\frac{0.335}{2}p(p+0.0455) = -0.1675p^2 - 0.0076p$$

Sumando

$$\left(1 + \frac{1}{4}\right)(p^3 + 0.3455p^2 + 0.0137p) + 0.25p^2 + 0.1404p + 0.0069$$

$$\left(1 + \frac{1}{4}\right) \left[p^3 + \left(0.3455 + \frac{0.25}{1 + \frac{1}{4}}\right)p^2 + \left(0.0137 + \frac{0.1404}{1 + \frac{1}{4}}\right)p + \frac{0.0069}{1 + \frac{1}{4}} \right]$$

$$\oplus (p) = (p+1)(p+0.12)(p+0.2725)$$

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$$\lambda = 5$$

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$$\gamma_1 = -0.2546, \quad \gamma_2 = -0.0693 + 0.0181i$$

$$\gamma_3 = -0.0693 - 0.0181i$$

$$B_1 = -0.0004$$

$$B_2 = 0.0592 + 0.0747i$$

$$B_3 = 0.0592 - 0.0747i$$

$$A_2 = -0.0004$$

$$A_1 = -0.0573$$

$$B_5 = -0.0302 - 0.0165i$$

$$B_4 = -0.0302 + 0.0165i$$

~~$\frac{d^2g}{dt^2} = \frac{W_0 U_0}{b}$~~
 $\frac{d^2g}{dt^2} = \frac{W_0 U_0}{b}$

$$\frac{d^2g}{ds^2} = \frac{b^2}{U_0^2} \frac{d^2g}{dt^2}$$

$$s = \frac{U_0 t}{b}$$

$$\frac{d^2g}{dt^2} = \frac{U_0^3}{b^2} \frac{d^2g}{ds^2} = \frac{W_0 U_0}{b} \left(\frac{\pi}{H}\right)^2 \frac{0.1412}{1+0.25s} [f(s) - f(s-H)]$$

$$\frac{d^2g}{dt^2} = \frac{0.28241}{1+0.25s} [f(s) - f(s-H)]$$

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$$\lambda = 50$$

$$\gamma_1 = -0.2966 \quad \gamma_2 = -0.0433 \quad \gamma_3 = -0.0106$$

$$B_1 = 0$$

$$B_2 = -0.0007$$

$$B_3 = 0.0111$$

$$A_2 = 0$$

$$A_1 = -0.0017$$

$$B_5 = -0.0043 + 0.0010i$$

$$B_4 = -0.0043 - 0.0010i$$

$$dn = 0.0162 \quad \text{máximo ocurre en } \underline{s = 37}$$

γ vale

$$\frac{\xi}{\frac{10000}{216}} = 0.8126$$



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