CEU
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UNIT 6: Linear prediction

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(based on “Biomedical Signal Analysis”, 2nd edition, © Willey 2015)

Linear prediction

- Linear prediction is based on using filtering to predict the values of a signal, given its past values
- It can be used for
  - Data compression
  - System modeling
- In this course we focus on the second application
  - Given a signal, model the LTI system that produced it
  - If an *all-pole* filter is used, then the **spectrum** of the signal can be accurately estimated

Autoregressive (AR) or All-Pole Modeling

Problem: Obtain an AR model when the input to the system **x(n)** is unknown. We only know the output **y(t)**

Solution (1/4): An AR model requires the current input sample **x(n)** and the las **P** values of the signal **y(n)** (output of the system)

IIR filter

gain

$$y(n) = -\sum_{k=1}^{P} a_k y(n-k) + G \cdot x(n)$$

x[n] ------ ? --------- y[n]

I

COEFFICIENTS???

- The transfer function is equal to

$$H(z) = \frac{G}{1+\sum_{k=1}^{P} a_k Z^{-k}} = \frac{G}{A(Z)}$$

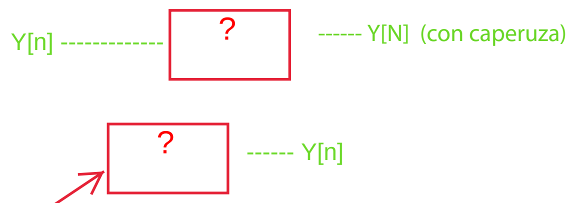
polinomio en el denominador

Autoregresive (AR) or All-Pole Modeling

Solution (2/4):

- Since we do not know the input **x(n)** we resort to the following equation:

$$\tilde{y}(n) = -\sum_{k=1}^{P} a_k y(n-k)$$



where $\tilde{y}(n)$ is an estimation of $y(n)$

- If can be proven that finding the set of coefficients that minimizes the MSE between the signal $y(n)$ and its estimation $\tilde{y}(n)$ leads to the following: E
  - The minimization of the error is an estimation of the spectrum of **y(n)** given that the following formula is used:

$$\widetilde{S_{YY}}(\omega) = |H(\omega)|^2 = \frac{G^2}{|A(Z)|^2} = \frac{G^2}{|1+\sum_{k=1}^{P} a_k \exp(-jk\omega)|^2}$$

Autoregresive (AR) or All-Pole Modeling

Solution (3/4):

$$\widetilde{S_{YY}}(\omega) = |H(\omega)|^2 = \frac{G^2}{|A(Z)|^2} = \frac{G^2}{|1+\sum_{k=1}^{P} a_k \exp(-jk\quad)|^2}$$

- If the estimation of the power spectrum is used only to analyze the frequency response of **y(n)**, with disregard of the magnitude, it is not necessary to compute the value of **G**, thus

$$\widetilde{S_{YY}}(\omega) \propto \frac{1}{|1+\sum_{k=1}^{P} a_k \exp(-jk\omega)|^2}$$

- For $P \to \infty$ the estimation is perfect

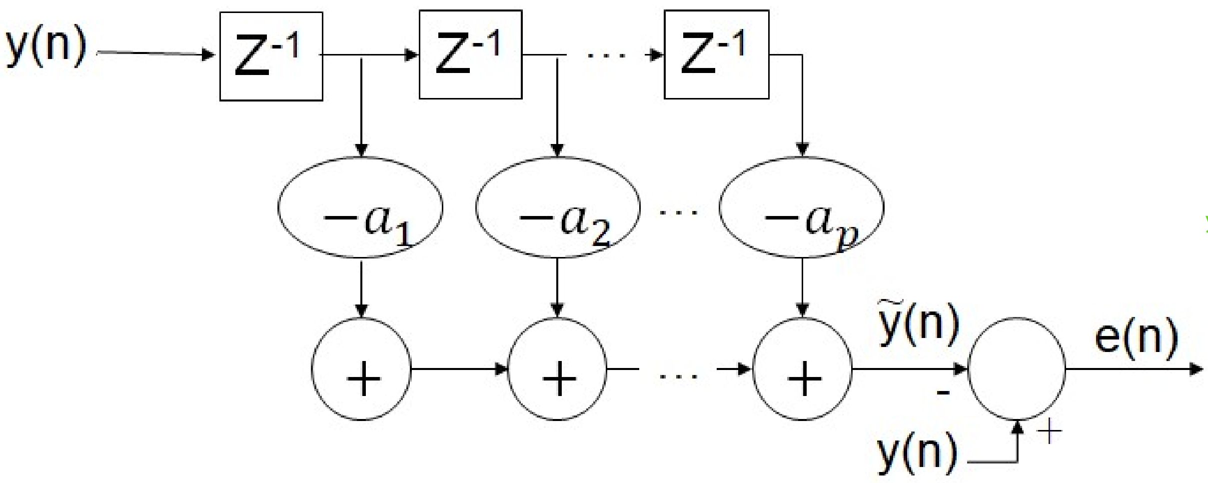
P = number of coefficients

Autoregresive (AR) or All-Pole Modeling

Solution (4/4):

FIR FILTER

- The optimization of the coefficient is performed by minimizing **E[e²(n)]**



y[n]= sum (k=1 a P) y[n-k]*a_k

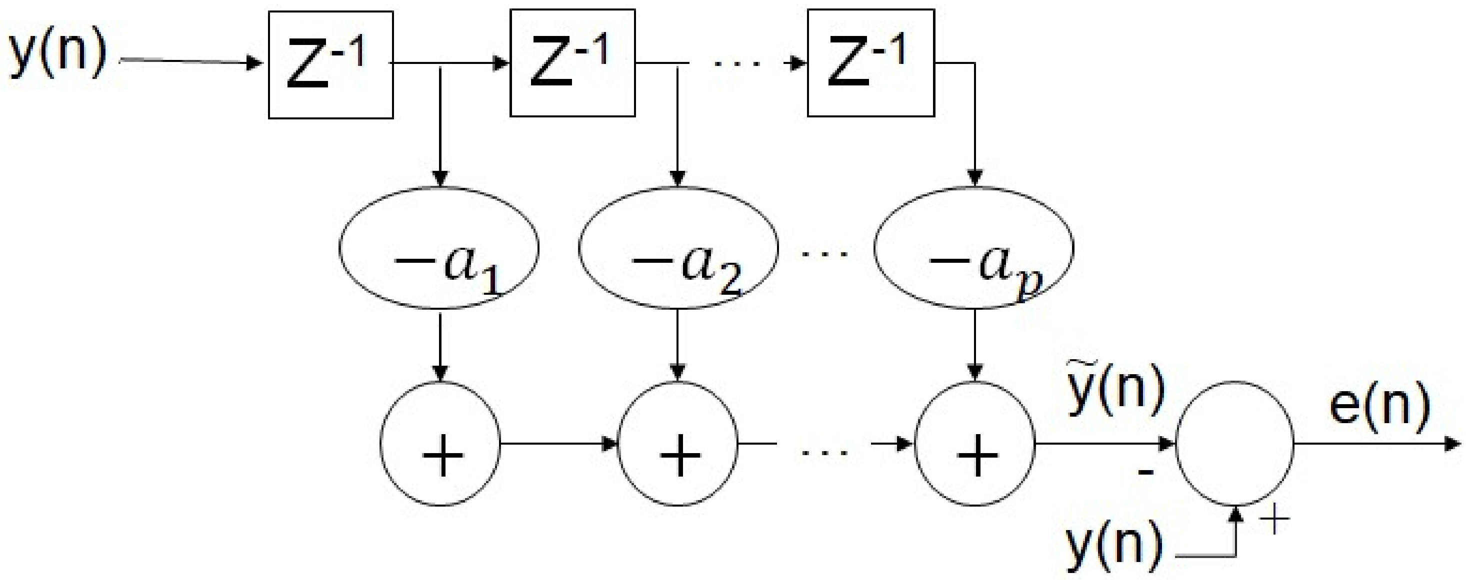
- The coefficients of the filter can be obtained following a similar approach to that of the Wiener filter leading to

AUTOC

AUTOCORRELATION MATRIX

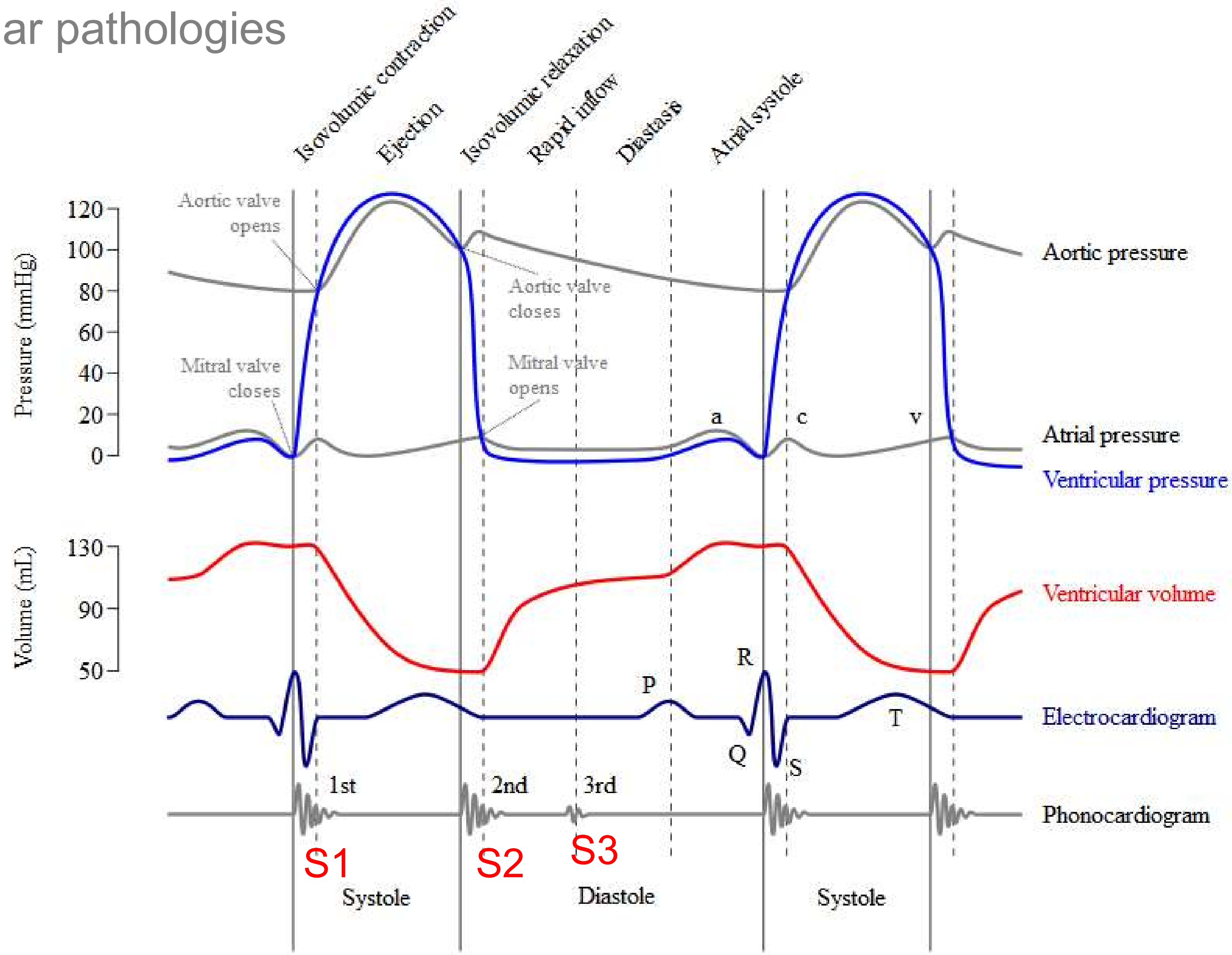
$$\begin{bmatrix} R_{YY}(0) & R_{YY}(1) & \dots & R_{YY}(P-1) \\ R_{YY}(1) & R_{YY}(0) & \dots & R_{YY}(P-2) \\ \vdots & \vdots & \ddots & \vdots \\ R_{YY}(P-1) & R_{YY}(P-2) & \dots & R_{YY}(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \end{bmatrix} = \begin{bmatrix} R_{YY}(1) \\ R_{YY}(2) \\ \vdots \\ R_{YY}(P) \end{bmatrix}$$

Autoregresive (AR) or All-Pole Modeling

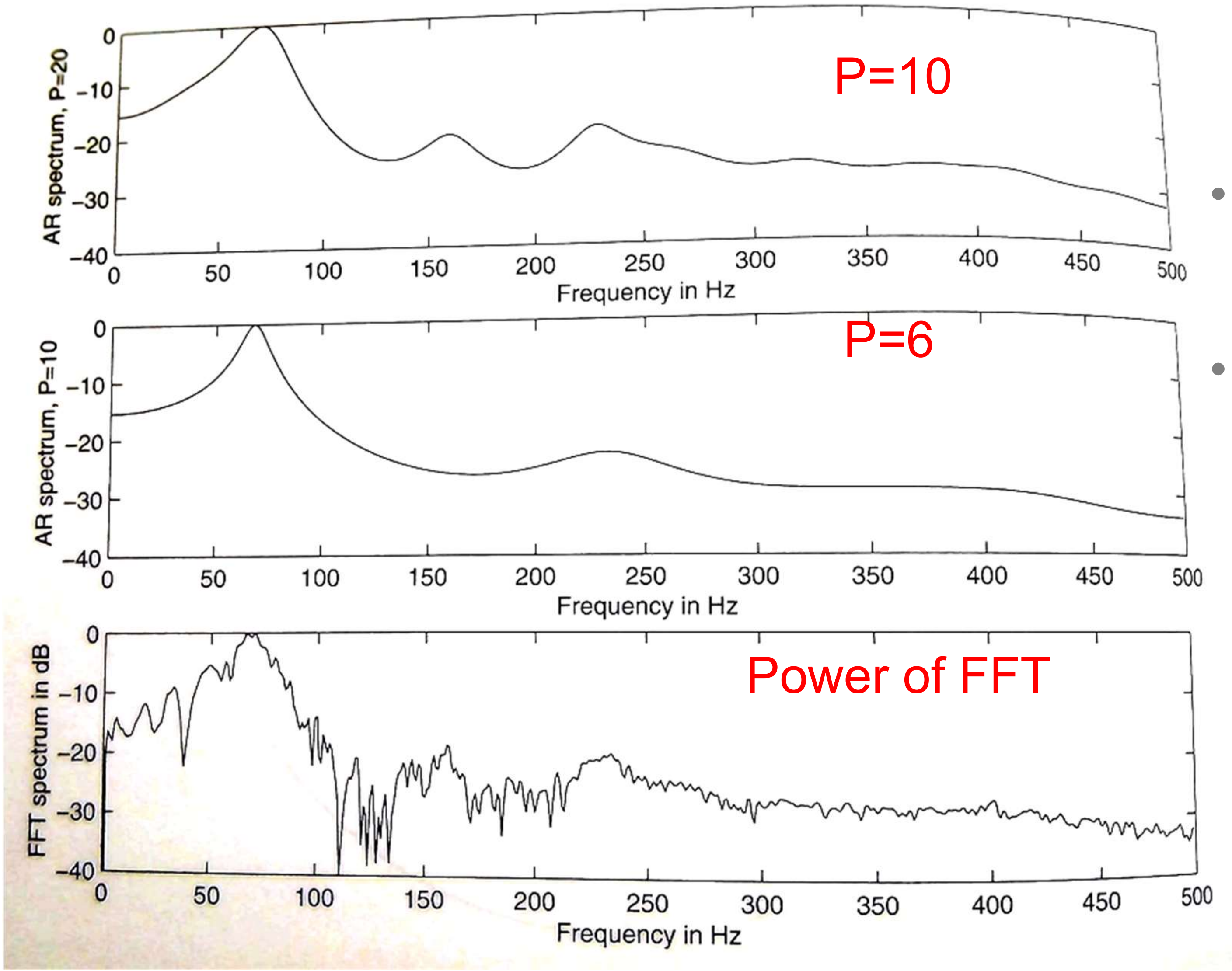


Example: PCG signals

- The frequency analysis of a phonocardiogram signal yields intereting insight about cardiovascular pathologies



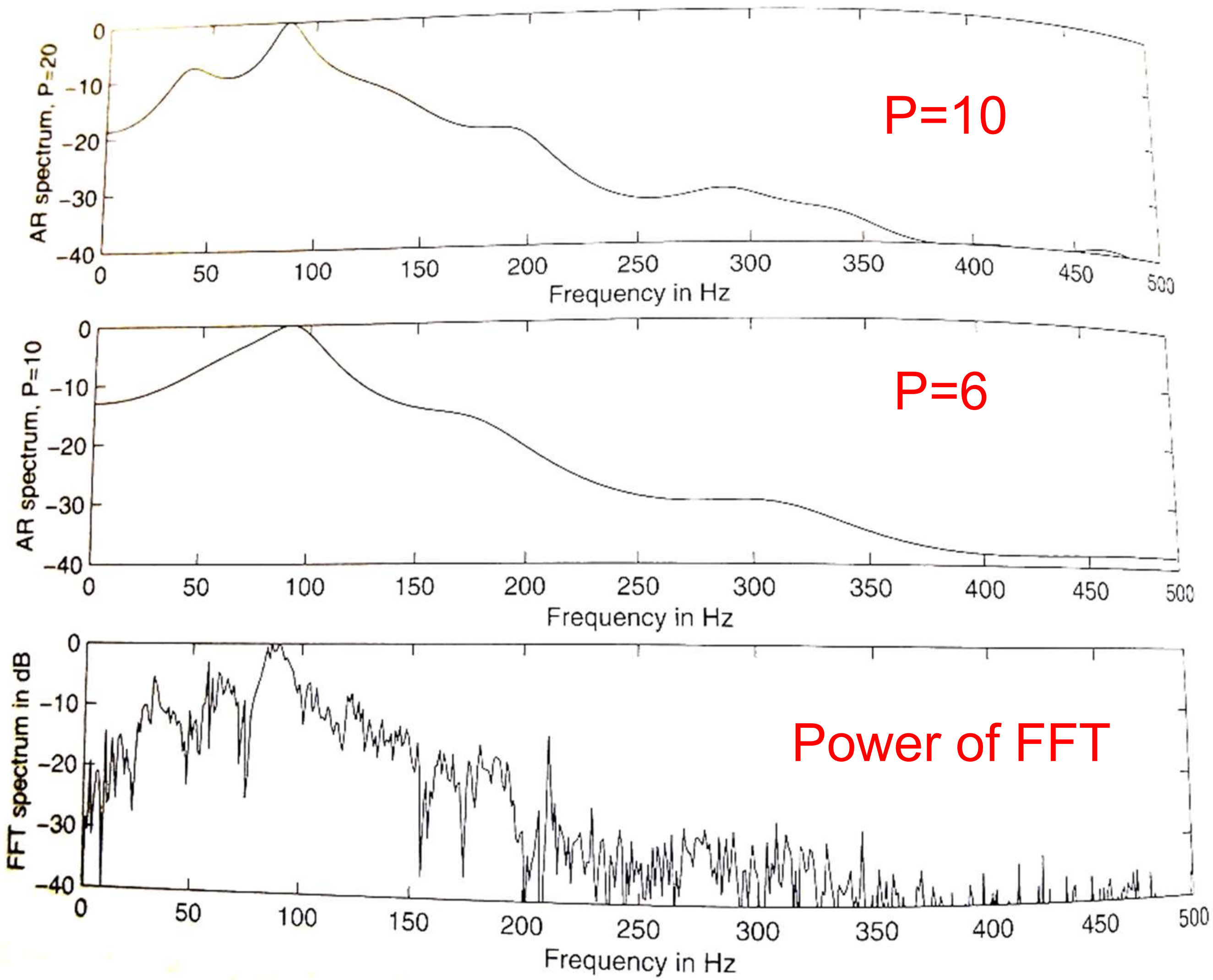
Example: PCG signals



- Normal subject

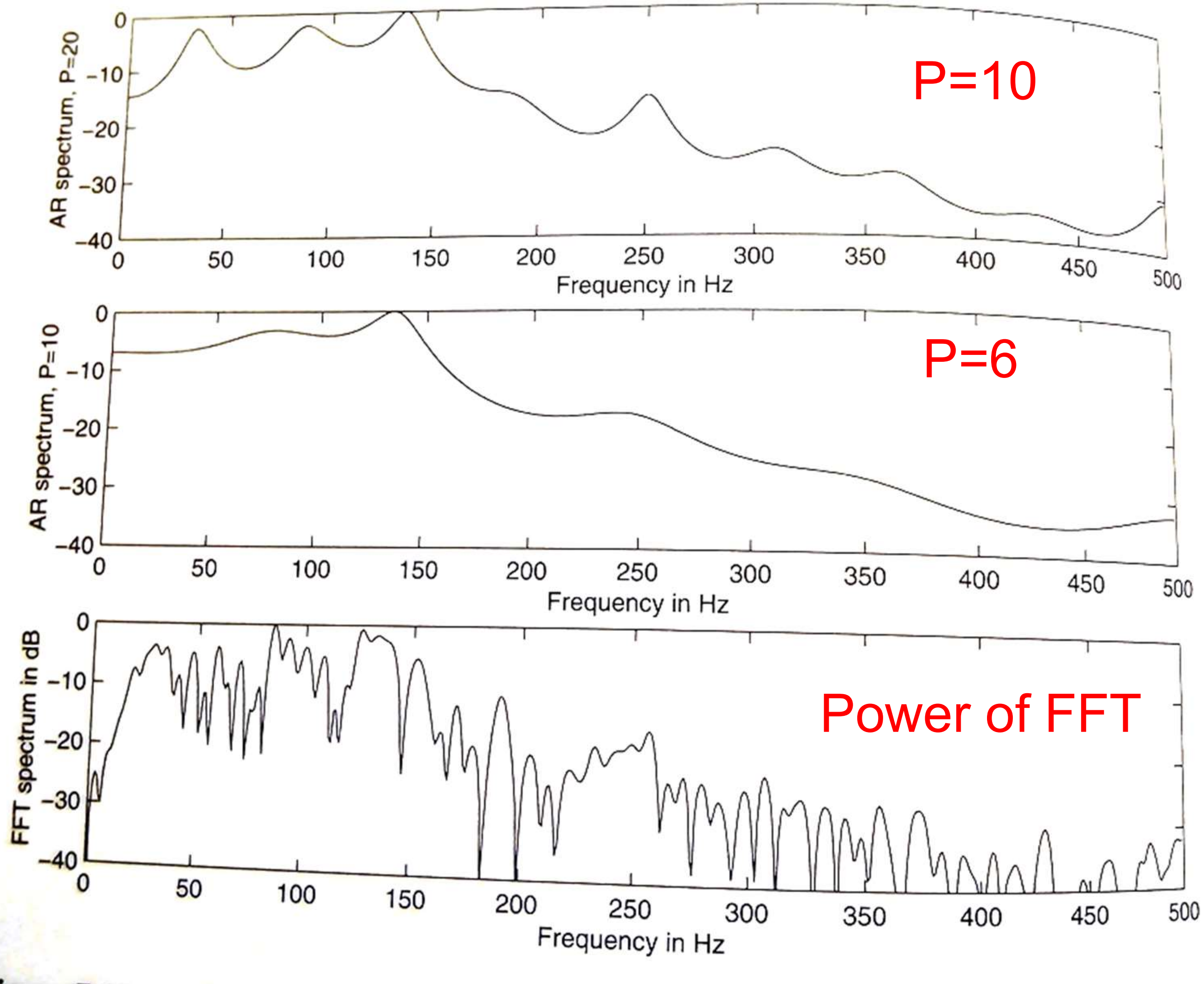
  (male, 23 years)
- S1 from systolic portion
- Easier interpretation than that of the periodogram (power of FFT)

Example: PCG signals



- Normal subject (male, 23 years)
- S2 from diastole portion
- Easier interpretation than that of the periodogram (power of FFT)

Example: PCG signals

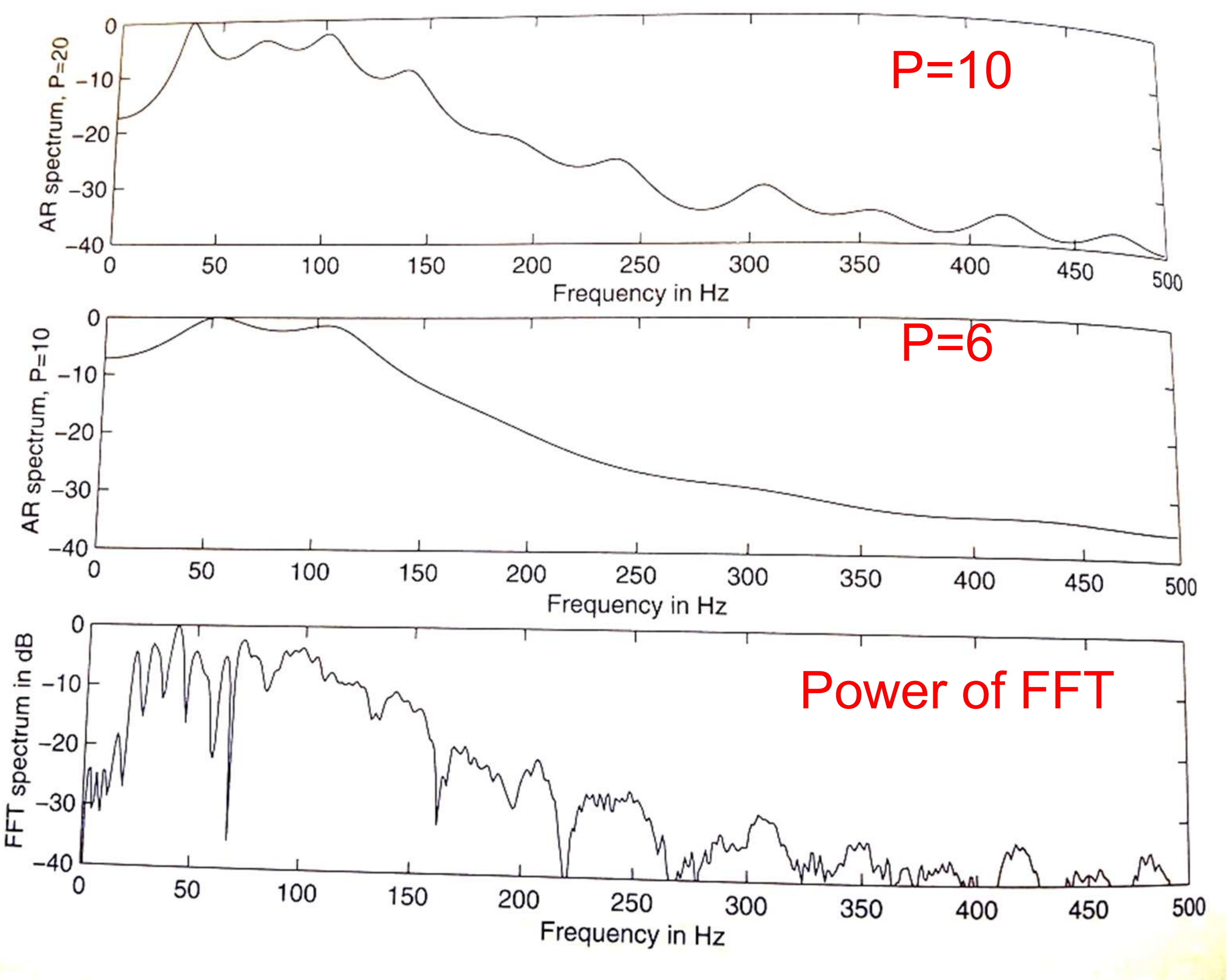


- Patient with systolic murmur

  (female, 14 months)

- S1 from systolic portion
- More medium frequency components

Example: PCG signals



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SUMMARY

- Linear prediction
- Autoregressive filter
- PCG example