UNIVERSIDAD COMPLUTENSE
DE MADRID

Ejercicios del ALUMNO

APELLIDOS ....
NOMBRE D.N.I. n.º
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CURSO N.º DE MATRICULA FECHA

PROBLEMAS RESUELTOS (HOJA 12)

89) a) $A = M_g = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & -1 & 0 \\ 1 & -1 & 9 & -6 \\ 0 & 0 & -6 & 10 \end{pmatrix}$

¿simétrica? Sí ¿definida positiva? Sí

$1 > 0 \qquad \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 > 0 \qquad \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & -2 & 8 \end{vmatrix} = 4 > 0$

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & -1 & 0 \\ 1 & -1 & 9 & -6 \\ 0 & 0 & -6 & 10 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & -2 & 8 & -6 \\ 0 & 0 & -6 & 10 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 0 \\ -2 & 8 & -6 \\ 0 & -6 & 10 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 0 \\ 0 & 4 & -6 \\ 0 & -6 & 10 \end{vmatrix} = 40-36 = 4 > 0$$

Por el crit. de Sylvester, es def. pos.

Luego, A define un prod. escalar en $\mathbb{R}^4$

$\underline{u} = (4, -1, 0, -\frac{2}{3})$ ¿$\underline{u}_U$?

$\underline{u}_U = \alpha(2,1,0,0) + \beta(1,0,-1,0) = (2\alpha+\beta, \alpha, -\beta, 0)$

$(4, -1, 0, -\frac{2}{3}) - (2\alpha+\beta, \alpha, -\beta, 0) = (4-2\alpha-\beta, -1-\alpha, \beta, -\frac{2}{3})$

$(4-2\alpha-\beta, -1-\alpha, \beta, -\frac{2}{3}) \cdot (2,1,0,0) = 0 \Rightarrow$

$\Rightarrow (2\ 1\ 0\ 0) \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & -1 & 0 \\ 1 & -1 & 9 & -6 \\ 0 & 0 & -6 & 10 \end{pmatrix} \begin{pmatrix} 4-2\alpha-\beta \\ -1-\alpha \\ \beta \\ -\frac{2}{3} \end{pmatrix} = (3\ 4\ 1\ 0) \begin{pmatrix} 4-2\alpha-\beta \\ -1-\alpha \\ \beta \\ -2/3 \end{pmatrix} =$

$= 12 - 6\alpha - 3\beta - 4 - 4\alpha + \beta = 0 \Rightarrow -10\alpha - 2\beta + 8 = 0 \Rightarrow -5\alpha - \beta + 4 = 0$

$\boxed{5\alpha + \beta = 4}$

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$$\left(4-2\alpha-\beta, -1-\alpha, \beta, -\frac{2}{3}\right)\cdot(1,0,-1,0)=0 \Rightarrow$$

$$\Rightarrow (1\ \ 0\ \ -1\ \ 0)\begin{pmatrix}1&1&1&0\\1&2&-1&0\\1&-1&9&-6\\0&0&-6&10\end{pmatrix}\begin{pmatrix}4-2\alpha-\beta\\-1-\alpha\\\beta\\-\frac{2}{3}\end{pmatrix}=0 \Rightarrow$$

$$\Rightarrow (0\ \ 2\ \ -8\ \ 6)\begin{pmatrix}4-2\alpha-\beta\\-1-\alpha\\\beta\\-\frac{2}{3}\end{pmatrix}=0 \Rightarrow -2-2\alpha-8\beta-4=0$$

$$-2\alpha-8\beta-6=0$$

$$-\alpha-4\beta-3=0 \Rightarrow \boxed{\alpha+4\beta=-3}$$

$$\left.\begin{cases}5\alpha+\beta=4\\\alpha+4\beta=-3\end{cases}\right\}\ \beta=-1 \wedge \alpha=1$$

Luego, la proy. ortogonal de $\underline{u}$ sobre $\mathcal{U}$ es: $(1,1,1,0)$

b) ¿$\mathcal{U}^{\perp}$? $(x,y,z,t)\cdot(2,1,0,0)=0 \Rightarrow$

$$\Rightarrow (2\ 1\ 0\ 0)\begin{pmatrix}1&1&1&0\\1&2&-1&0\\1&-1&9&-6\\0&0&-6&10\end{pmatrix}\begin{pmatrix}x\\y\\z\\t\end{pmatrix}=0 \Rightarrow (3\ 4\ 1\ 0)\begin{pmatrix}x\\y\\z\\t\end{pmatrix}=0 \Rightarrow$$

$$\Rightarrow \boxed{3x+4y+z=0}$$

$$(1,0,-1,0)\cdot(x,y,z,t)=0 \Rightarrow (1\ 0\ -1\ 0)\begin{pmatrix}1&1&1&0\\1&2&-1&0\\1&-1&9&-6\\0&0&-6&10\end{pmatrix}\begin{pmatrix}x\\y\\z\\t\end{pmatrix}=0 \Rightarrow$$

$$\Rightarrow (0\ \ 2\ \ -8\ \ 6)\begin{pmatrix}x\\y\\z\\t\end{pmatrix}=0 \Rightarrow 2y-8z+6t=0 \Rightarrow \boxed{y-4z+3t=0}$$

$$\begin{cases}3x+4y+z=0 \Longrightarrow 3x=-4(4z-3t)-z \Rightarrow 3x=-17z+12t\\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad x=-\frac{17}{3}z+4t\\ y-4z+3t=0 \Rightarrow y=4z-3t\end{cases}$$

$$\mathcal{U}^{\perp}=\left\{\left(-\frac{17}{3}z+4t, 4z-3t, z, t\right) / z,t\in\mathbb{R}\right\}=L\left(\left\{\left(-\frac{17}{3},4,1,0\right),(4,-3,0,1)\right\}\right)=$$

$$=L(\{(-17,12,3,0),(4,-3,0,1)\})$$

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¿$\underline{u}_{u^\perp}$? $\underline{u}_{u^\perp} = (-17\alpha+4\beta,\ 12\alpha-3\beta,\ 3\alpha,\ \beta)$

$(4,-1,0,-\frac{2}{3}) - (-17\alpha+4\beta,\ 12\alpha-3\beta,\ 3\alpha,\ \beta) =$

$= (4+17\alpha-4\beta,\ -1-12\alpha+3\beta,\ -3\alpha,\ -\frac{2}{3}-\beta)$

$(-17\ \ 12\ \ 3\ \ 0)\begin{pmatrix}1&1&1&0\\1&2&-1&0\\1&-1&9&-6\\0&0&-6&10\end{pmatrix}\begin{pmatrix}4+17\alpha-4\beta\\-1-12\alpha+3\beta\\-3\alpha\\-\frac{2}{3}-\beta\end{pmatrix} = 0 \Rightarrow 38\beta-76\alpha=0$

$(4\ \ -3\ \ 0\ \ 1)\begin{pmatrix}\ \end{pmatrix}\begin{pmatrix}\ \end{pmatrix} = 0 \Rightarrow 38\alpha-20\beta=\frac{2}{3}$

$\left.\begin{array}{l}19\alpha-10\beta=\frac{1}{3}\\-2\alpha+\beta=0\end{array}\right\} \sim \left\{\begin{array}{l}19\alpha-10\beta=\frac{1}{3}\\-20\alpha+10\beta=0\end{array}\right.$

$-\alpha=\frac{1}{3} \Rightarrow \alpha=\frac{-1}{3} \wedge \beta=\frac{-2}{3}$

$$\boxed{\underline{u}_{u^\perp} = \left(\frac{17}{3}-\frac{8}{3},\ \frac{-12}{3}+\frac{6}{3},\ -1,\ \frac{-2}{3}\right) = (3,-2,-1,-2)}$$

(90) $\mathbb{R}^3$, $B=\{\underline{v}_1,\underline{v}_2,\underline{v}_3\}$

$\underline{v}_1\cdot\underline{v}_2=0 \quad \underline{v}_1\cdot\underline{v}_1=1 \quad \underline{v}_2\cdot\underline{v}_2=1 \quad \underline{v}_3\cdot\underline{v}_3=2$

$(2\underline{v}_2-\underline{v}_3)\perp v_1 \quad (2\underline{v}_2-\underline{v}_3)\perp v_3$

$(2\underline{v}_2-\underline{v}_3)\cdot\underline{v}_1=0 \Rightarrow 2\underline{v}_2\cdot\underline{v}_1-\underline{v}_3\cdot\underline{v}_1=0 \Rightarrow v_3\cdot v_1=0$

a) $G=\begin{pmatrix}1&0&\\0&1&\\&&2\end{pmatrix}$

$(2\underline{v}_2-\underline{v}_3)\cdot\underline{v}_3=0 \Rightarrow 2\underline{v}_2\cdot\underline{v}_3-\underline{v}_3\cdot\underline{v}_3=0$

$2\underline{v}_2\cdot\underline{v}_3=2 \Rightarrow v_2\cdot v_3=1$

Así: $\boxed{G=\begin{pmatrix}1&0&0\\0&1&1\\0&1&2\end{pmatrix}}$

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b) ¿ $B' = \{\underline{w}_1, \underline{w}_2, \underline{w}_3\}$ base ortonormal a partir de $B = \{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$?

$$\boxed{\underline{w}_1 = \underline{v}_1}$$

$$\underline{w}_2 = \underline{v}_2 + \alpha_{21}\underline{w}_1 \Rightarrow \underbrace{\underline{w}_1\cdot\underline{w}_2}_{0} = \underbrace{\underline{w}_1\cdot\underline{v}_2}_{0} + \alpha_{21}\underbrace{\underline{w}_1\cdot\underline{w}_1}_{\neq 0} \Rightarrow \alpha_{21} = 0$$

Así $\boxed{\underline{w}_2 = \underline{v}_2}$

$$\underline{w}_3 = \underline{v}_3 + \alpha_{32}\underline{w}_2 + \alpha_{31}\underline{w}_1$$

$$0 = \underline{w}_3\cdot\underline{w}_2 = \underbrace{\underline{w}_2\cdot\underline{v}_3}_{1} + \alpha_{32}\underbrace{\underline{w}_2\cdot\underline{w}_2}_{1} + \alpha_{31}\overbrace{\underline{w}_1\cdot\underline{w}_2}^{=0} \Rightarrow \alpha_{32} = -1$$

$$0 = \underline{w}_3\cdot\underline{w}_1 = \underbrace{\underline{v}_3\cdot\underline{w}_1}_{0} + \alpha_{32}\underbrace{\underline{w}_2\cdot\underline{w}_1}_{0} + \alpha_{31}\underbrace{\underline{w}_1\cdot\underline{w}_1}_{1} \Rightarrow \alpha_{31} = 0$$

luego, $\boxed{\underline{w}_3 = \underline{v}_3 - \underline{v}_2}$

$$\left.\begin{array}{l} \|\underline{w}_1\| = \sqrt{(1\,0\,0)\begin{pmatrix}1&0&0\\0&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}} = 1 \\ \|\underline{w}_2\| = \sqrt{(0\,1\,0)\begin{pmatrix}1&0&0\\0&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}} = 1 \\ \|\underline{w}_3\| = \sqrt{(0\,{-1}\,1)\begin{pmatrix}1&0&0\\0&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}0\\-1\\1\end{pmatrix}} = \sqrt{(0\,0\,1)\begin{pmatrix}0\\-1\\1\end{pmatrix}} = 1 \end{array}\right\}\text{ SON UNITARIOS}$$

Por tanto, $B' = \{\underline{v}_1, \underline{v}_2, \underline{v}_3 - \underline{v}_2\}$ es base ORTONORMAL

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La matriz asociada, en esta base, para este prod. escalar, es la matriz identidad: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

c) $\underline{w} = (1,3,-1)_B = 1\underline{v}_1 + 3\underline{v}_2 - 1\underline{v}_3 = -1\underline{w}_1 + 3\underline{w}_2 - 1(\underline{w}_2 + \underline{w}_3) =$

$= -\underline{w}_1 + 2\underline{w}_2 - \underline{w}_3$

$$\boxed{\underline{w} = (-1, 2, -1)_{B'}}$$

$$\|\underline{w}_B\| = \sqrt{(1\ 3\ -1)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}} = \sqrt{(1\ 2\ 1)\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}} = \sqrt{6}$$

$$\|\underline{w}_{B'}\| = \sqrt{(-1\ 2\ -1)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}} = \sqrt{6}$$

- La norma es la misma, pues es INVARIANTE para matrices congruentes.

(91) $\left\{\begin{array}{cccc|c} 1 & 3 & -1 & 2 & 0 \\ 2 & -1 & 3 & -1 & 0 \end{array}\right\} \sim \left\{\begin{array}{cccc|c} 1 & 3 & -1 & 2 & 0 \\ 0 & -7 & 5 & -5 & 0 \end{array}\right\} \Rightarrow x_2 = \frac{5}{7}x_3 - \frac{5}{7}x_4$

$x_1 = \frac{-8}{7}x_3 + \frac{1}{7}x_4$

$\mathcal{U} = L(\{(\frac{-8}{7}, \frac{5}{7}, 1, 0), (\frac{1}{7}, -\frac{5}{7}, 0, 1)\}) = L(\{(-8,5,7,0), (1,-5,0,7)\})$

$\left.\begin{array}{l} (a,b,c,d)\cdot(-8,5,7,0) = 0 \\ (\qquad)\cdot(1,-5,0,7) = 0 \end{array}\right\} \left\{\begin{array}{l} -8a + 5b + 7c = 0 \\ a - 5b + 7d = 0 \end{array}\right\} \Rightarrow a = c + d \wedge b = \frac{1}{5}c + \frac{8}{5}d$

Luego, $\mathcal{U}^{\perp} = L(\{(1, \frac{1}{5}, 1, 0), (1, \frac{8}{5}, 0, 1)\})$