

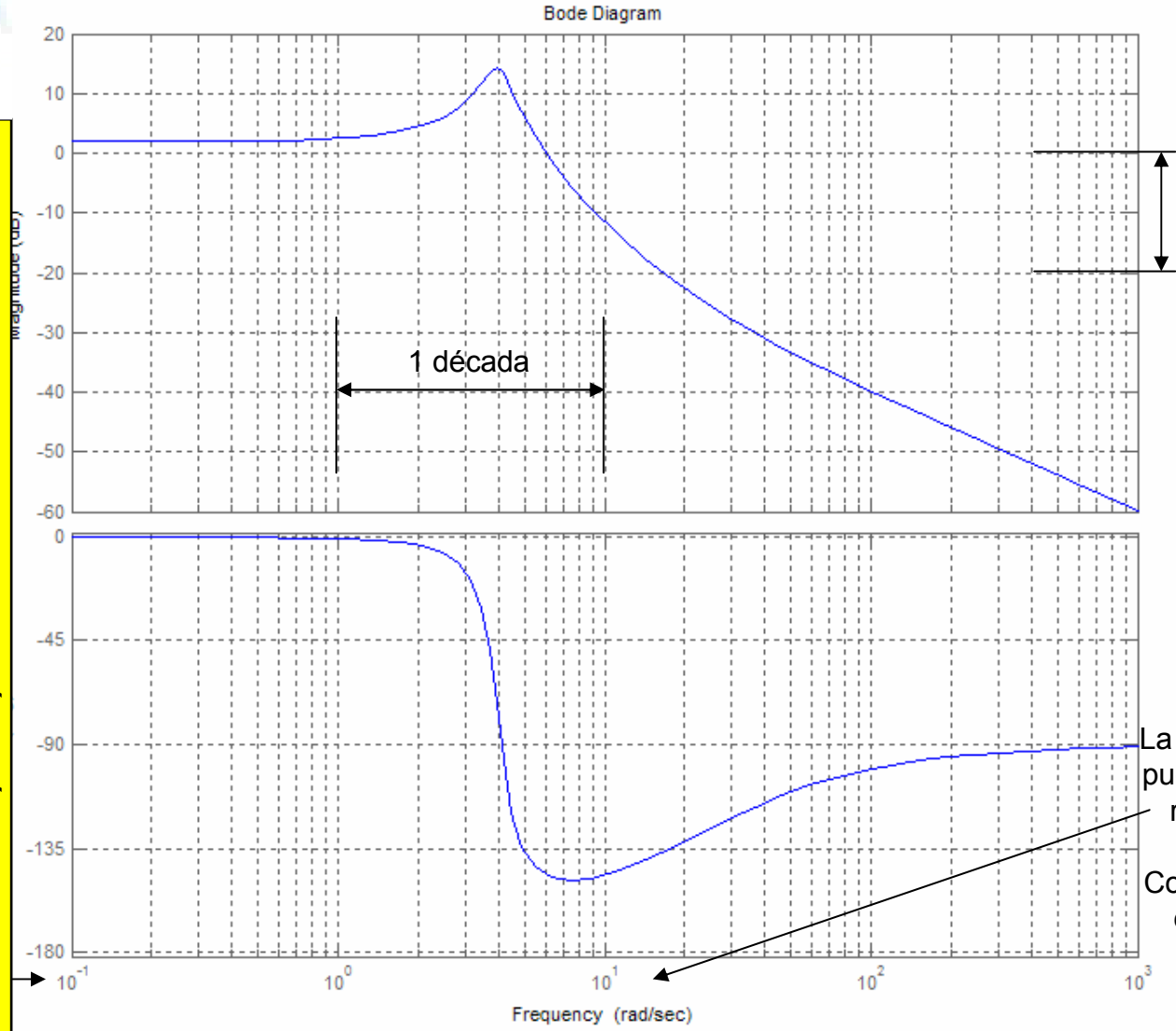
# Estado Asintótico de Diagramas de Bode

Análisis Dinámico de Sistemas  
2º curso Ingeniería de Telecomunicación

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The logo for Cartagena99 features the word "Cartagena99" in a stylized, green, cursive font. The text is set against a light blue background that resembles a stylized map of the Iberian Peninsula. A yellow and orange arrow-like shape points upwards from the bottom left towards the text.

# mía de un Diagrama de Bode



Bajar/subir 20 dB equivale a dividir/multiplicar por 10

La escala de frecuencias pueden venir en Hz o en rads/seg (pulsación).  
Como trabajamos con w emplearemos rads/s

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# Factorización de una función de transferencia

La esencial es factorizar la  $G(s)$  en fdt sencillas cuyos diagramas de Bode asintóticos conocemos.

$$G(s) = K \cdot s^{\pm N} \cdot \left[ \prod \frac{p_i}{s + p_i} \right] \cdot \left[ \prod \frac{s + c_i}{c_i} \right] \cdot \left[ \prod \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \right]$$

Polos/ceros En el origen
polos reales
ceros reales
Pares de polos complejos conjugados

En el diagrama de Bode logarítmico, el Bode del producto de fdt's es la suma de los Bodes de cada fdt por separado

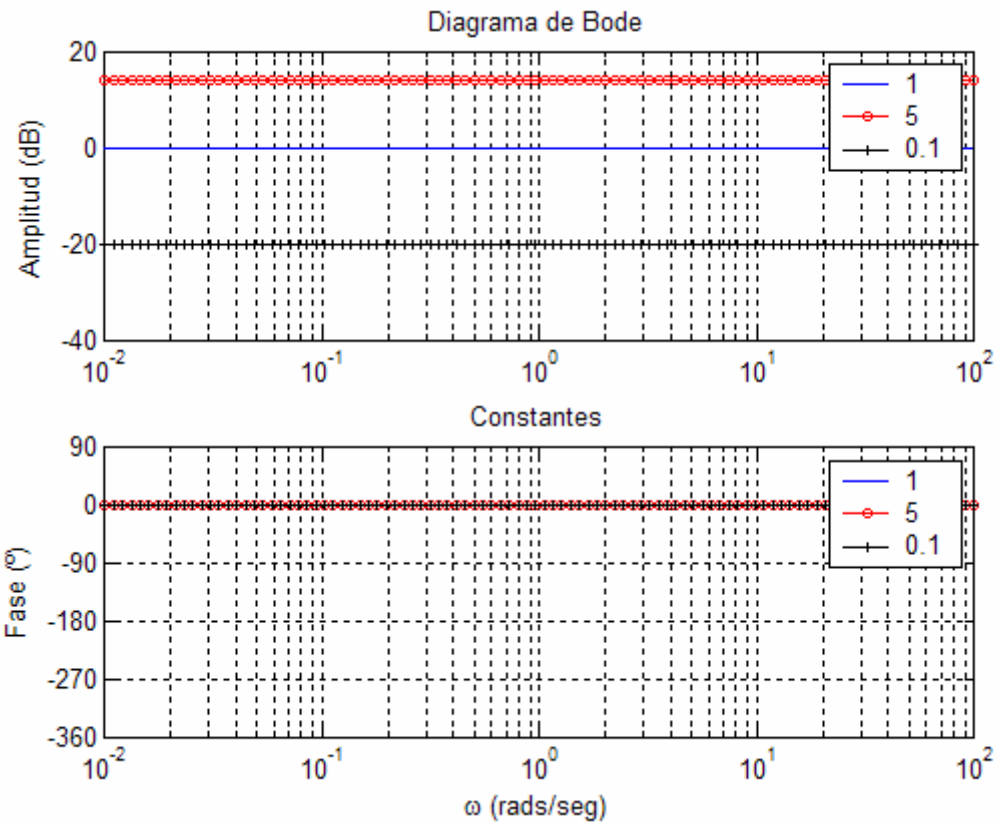
Por lo tanto, la fdt factorizada, el diagrama de Bode total es la suma de los diagramas de Bode sencillos

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# nos constantes: $G(s) = K$

Las curvas de magnitud son constantes

La fase es siempre  $0^\circ$  (o bien  $-180^\circ$  si la constante es negativa)

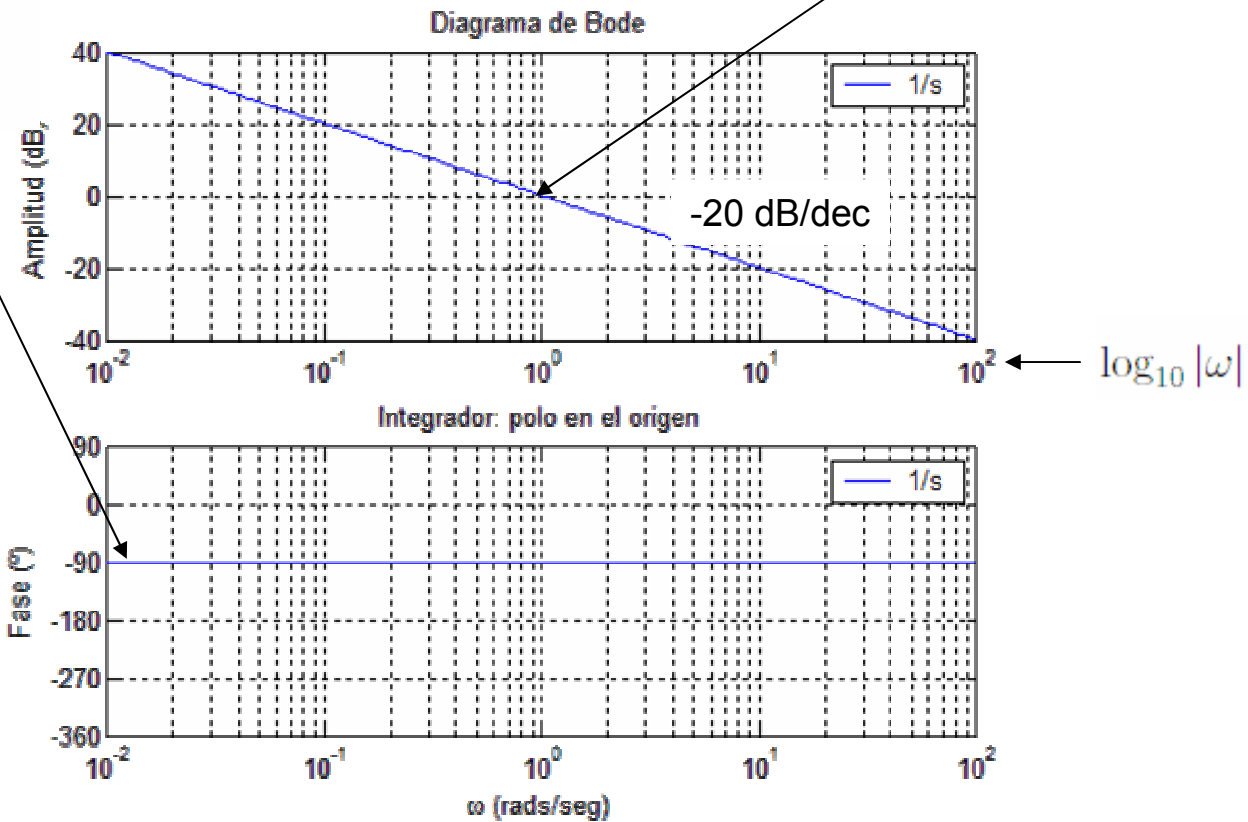


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# lo en el origen: $G(s) = 1/s$

$$= -20 \log_{10} |\omega|$$

Cruza en el punto  
( $\omega=1$  rad/s,  $A = 0$  dB)



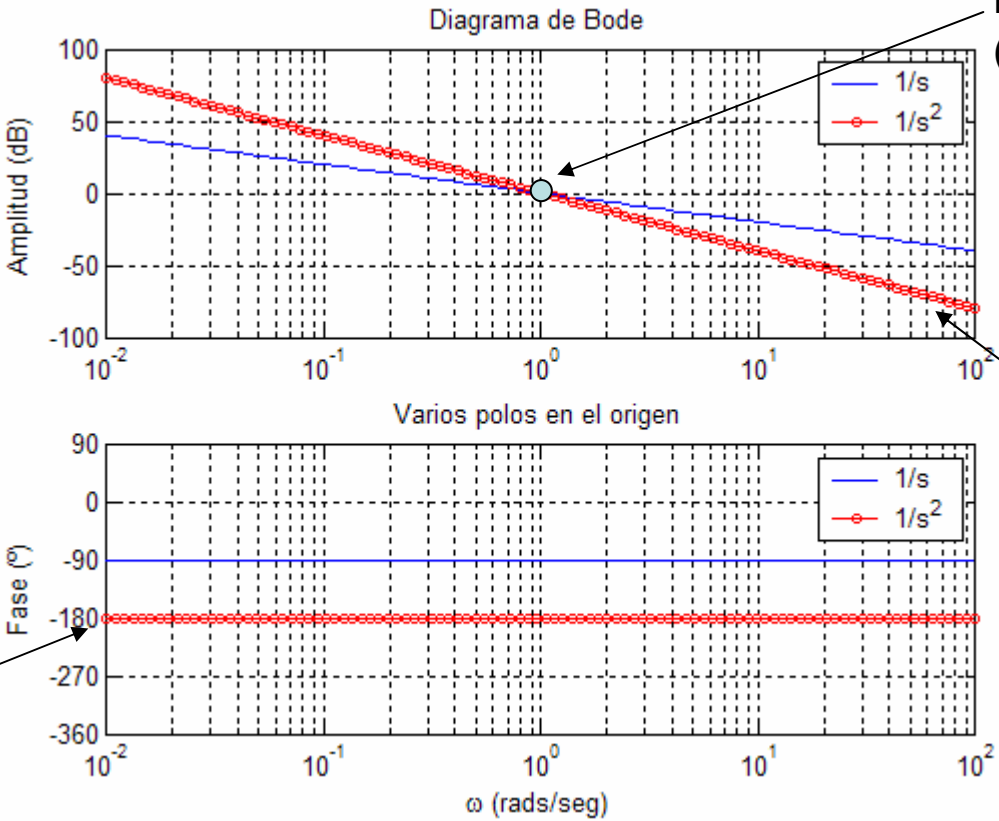
$$= -90^\circ$$

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# Varios polos en el origen: $G(s) = 1/s^N$



$$20 \log_{10} \left| \frac{1}{(j\omega)^2} \right| = -20 \log_{10} |\omega^2| = -20 \times 2 \log_{10} |\omega| = -40 \log_{10} |\omega|$$

$$\arg \left\{ \frac{1}{(j\omega)^2} \right\} = -180^\circ$$

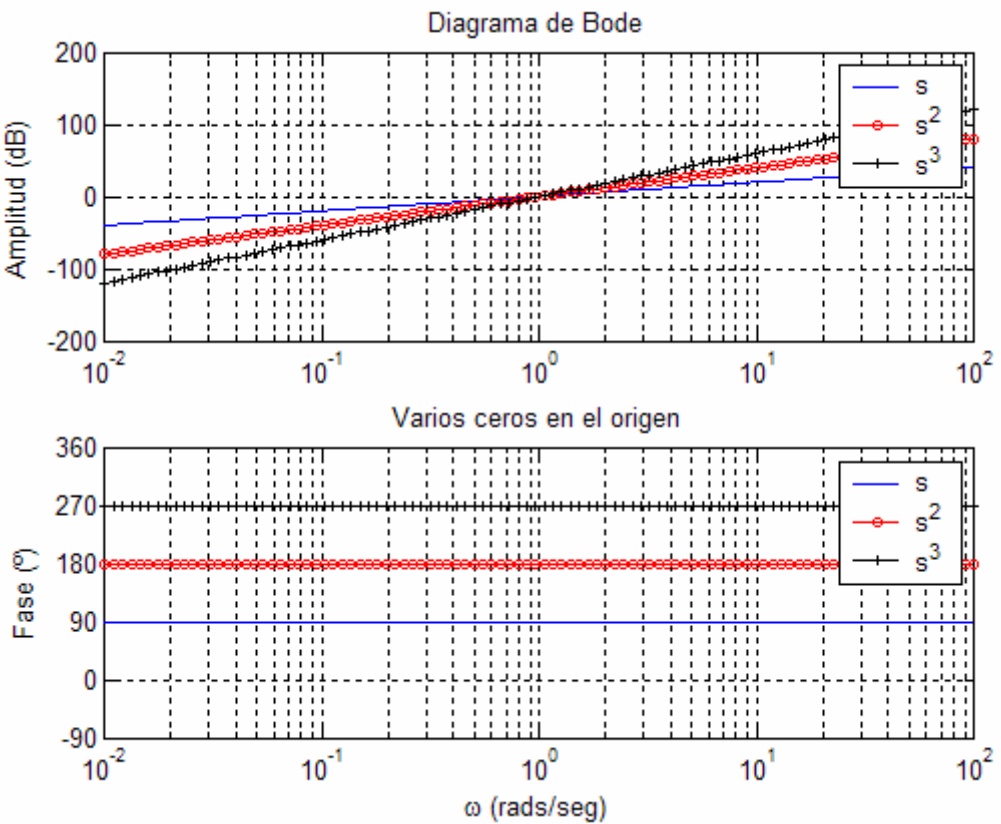
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## ceros en el origen

$$|N| = 20N \log_{10} |\omega|$$

$$\angle N = +90N^\circ$$



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real



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Frecuencias bajas ( $\omega \approx 0$ ):

$$20 \log_{10} \left| \frac{p_i}{j\omega + p_i} \right| \approx 20 \log_{10} \left| \frac{p_i}{p_i} \right| = 0$$

$$\arg \left\{ \frac{p_i}{j\omega + p_i} \right\} \approx \arg \left\{ \frac{p_i}{p_i} \right\} = 0$$

Frecuencias medias ( $\omega \approx p_i$ ):

$$20 \log_{10} \left| \frac{p_i}{jp_i + p_i} \right| \approx 20 \log_{10} \left| \frac{1}{j + 1} \right| = -3\text{dB}$$

$$\arg \left\{ \frac{p_i}{j\omega + p_i} \right\} \approx \arg \left\{ \frac{1}{j + 1} \right\} = -45^\circ$$

Pendiente  
-20 dB/dec

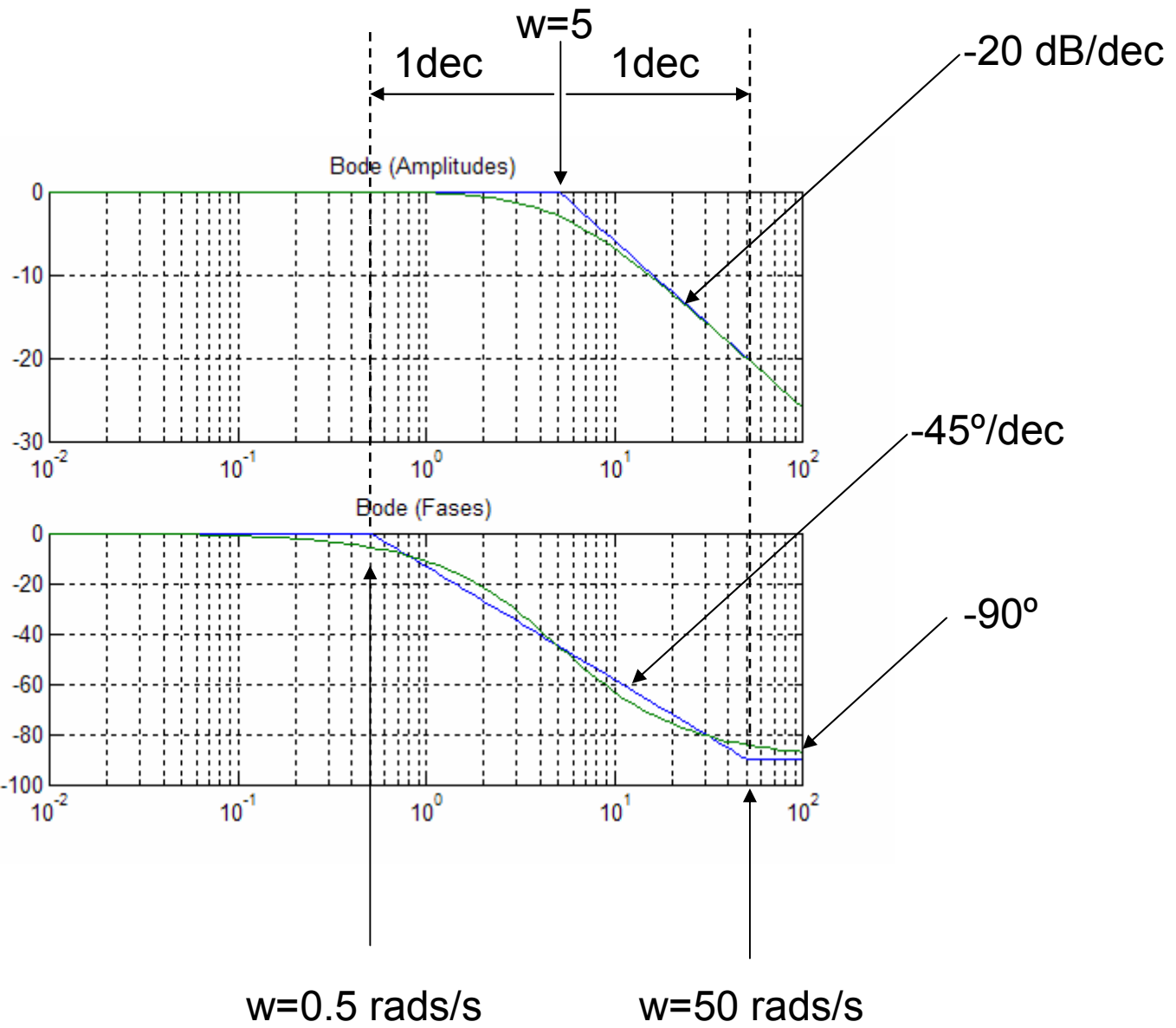
Frecuencias altas ( $\omega \approx \infty$ ):

$$20 \log_{10} \left| \frac{p_i}{j\omega + p_i} \right| \approx 20 \log_{10} \left| \frac{p_i}{j\omega} \right| = 20 \log_{10} |p_i| - 20 \log_{10} |j\omega|$$

$$\arg \left\{ \frac{p_i}{j\omega + p_i} \right\} \approx \arg \left\{ \frac{1}{j\omega} \right\} = -90^\circ$$







real

5  
+  
5

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Real

The logo for Cartagena99 features the word "Cartagena99" in a stylized, green, cursive font. The text is set against a background of a blue and orange gradient that resembles a stylized sun or a wave. The "99" is slightly larger and more prominent than the "Cartagena" part.

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Frecuencias bajas ( $\omega \approx 0$ ):

$$20 \log_{10} \left| \frac{j\omega + c_i}{c_i} \right| \approx 20 \log_{10} \left| \frac{c_i}{c_i} \right| = 0$$

$$\arg \left\{ \frac{j\omega + c_i}{c_i} \right\} \approx \arg \left\{ \frac{c_i}{c_i} \right\} = 0$$

Frecuencias medias ( $\omega \approx c_i$ ):

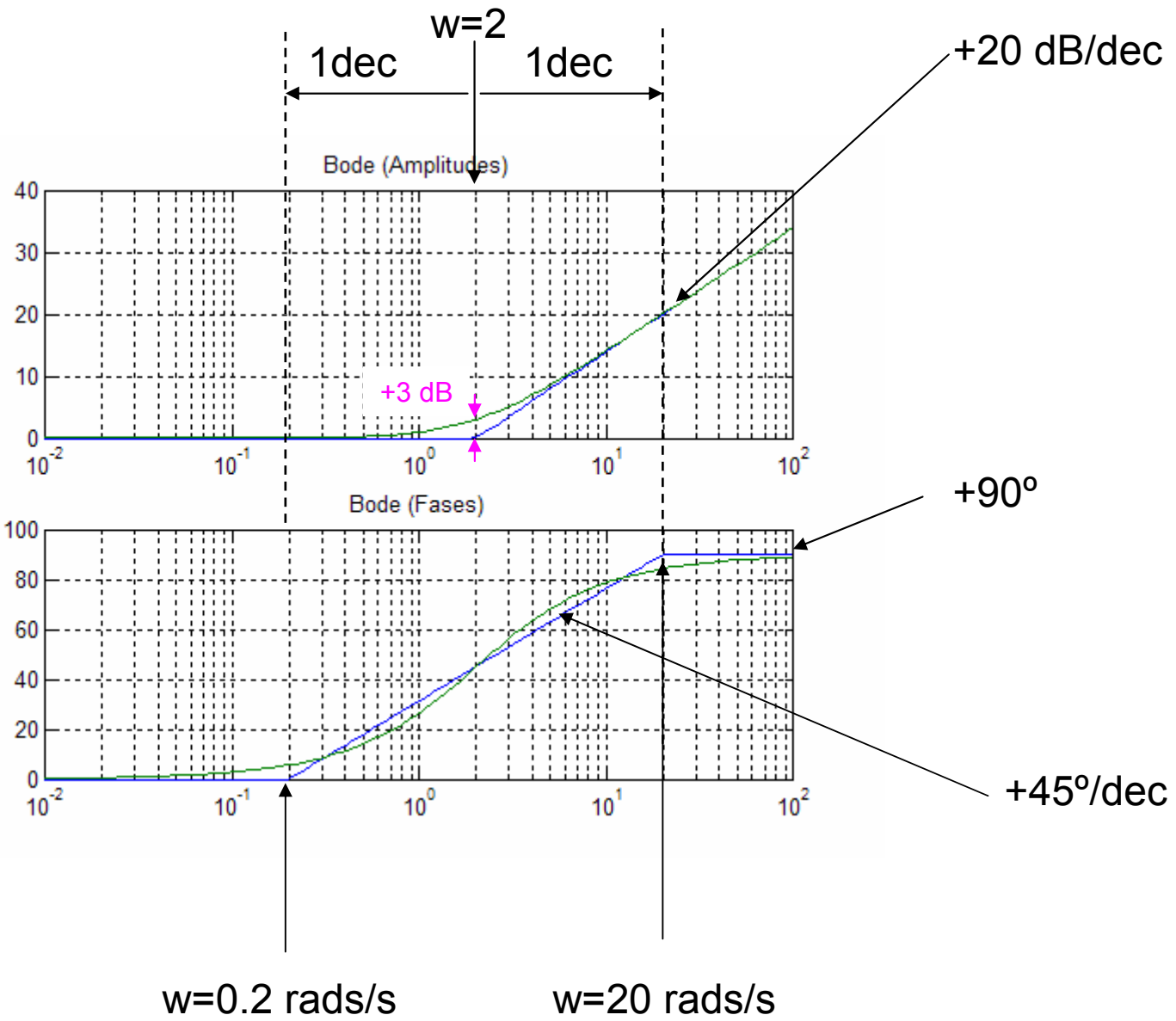
$$20 \log_{10} \left| \frac{j c_i + c_i}{c_i} \right| \approx 20 \log_{10} \left| \frac{j + 1}{1} \right| = +3\text{dB}$$

$$\arg \left\{ \frac{j\omega + c_i}{c_i} \right\} \approx \arg \left\{ \frac{j + 1}{1} \right\} = +45^\circ$$

Frecuencias altas ( $\omega \approx \infty$ ):

$$20 \log_{10} \left| \frac{j\omega + c_i}{c_i} \right| \approx 20 \log_{10} \left| \frac{j\omega}{c_i} \right| = -20 \log_{10} |c_i| + 20 \log_{10} |j\omega|$$

$$\arg \left\{ \frac{j\omega + c_i}{c_i} \right\} \approx \arg \{j\omega\} = +90^\circ$$



real

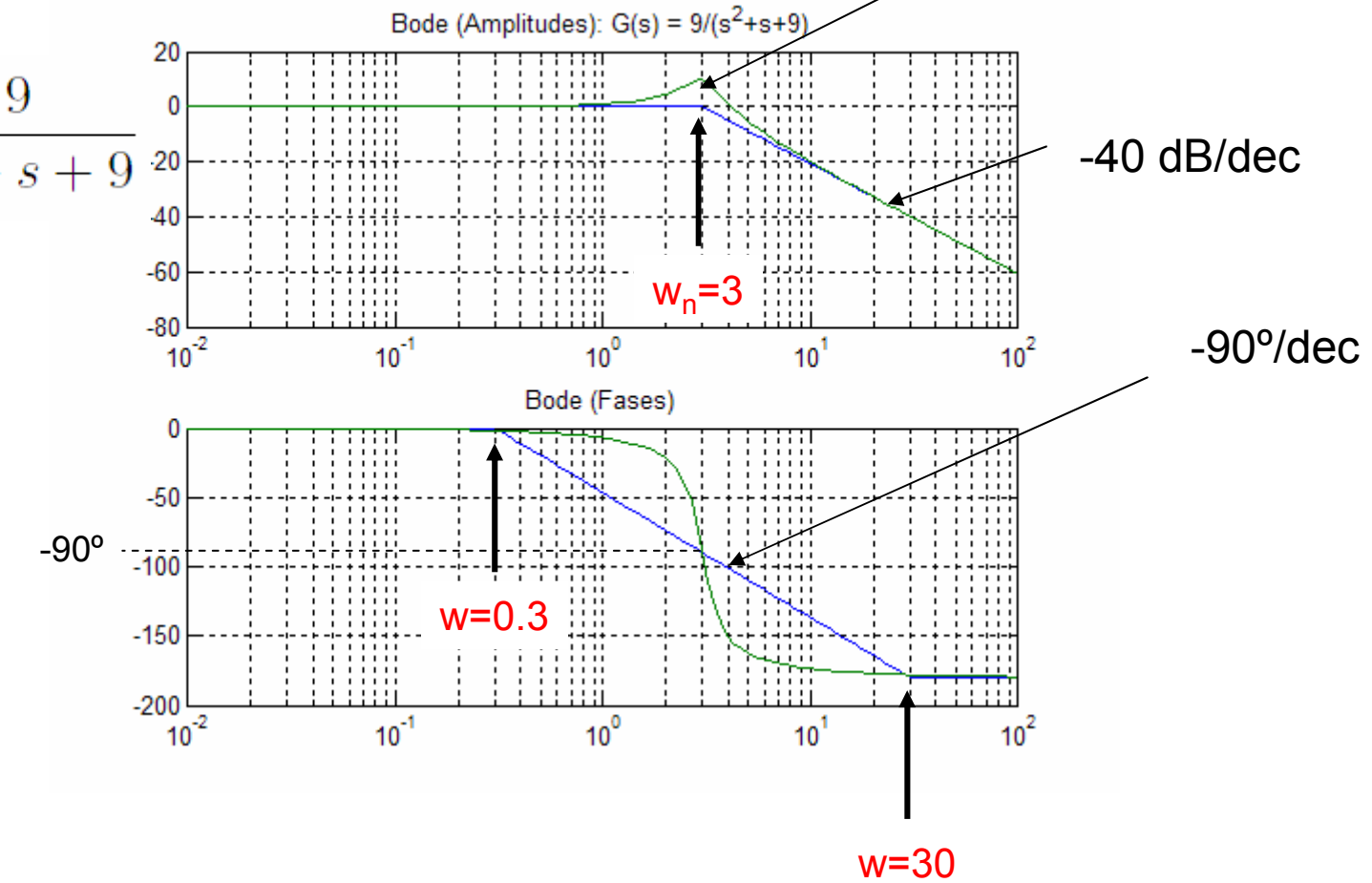
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## complejos conjugados

La resonancia depende del factor de amortiguamiento  $\xi$  pequeño  $\rightarrow$  resonancia grande (ver tablas graficas Puente)

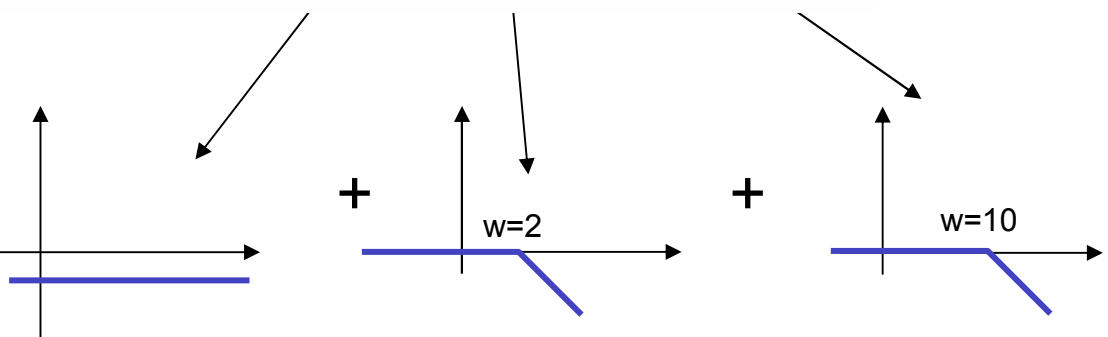


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$$G(s) = \frac{10}{(s + 2)(s + 10)}$$

tercero: factorizamos en bloques básicos (de Bodes conocidos)

$$G(s) = \frac{10}{(s + 2)(s + 10)} = 0,5 \cdot \frac{2}{s + 2} \cdot \frac{10}{s + 10}$$



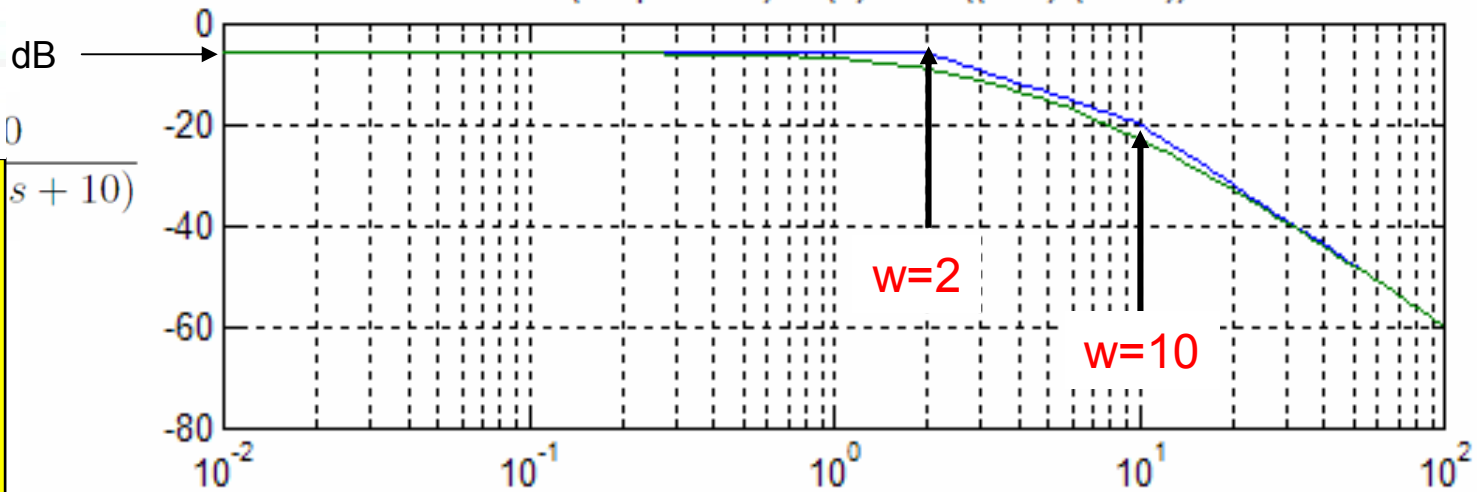
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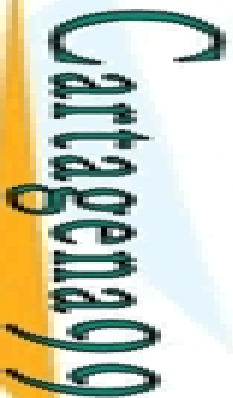
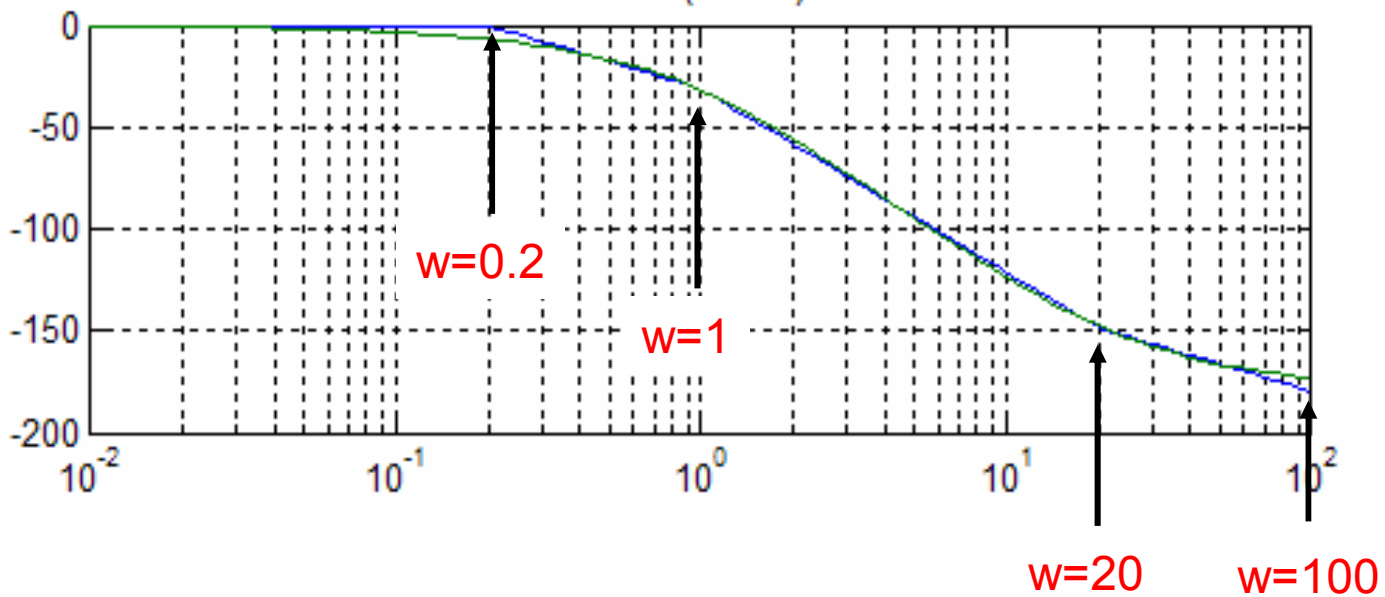
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# Modelo (dos polos reales y term. constante)

Bode (Amplitudes):  $G(s) = 10/((s+2)*(s+10))$



Bode (Fases)



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el Bode asintótico de

$$G(s) = \frac{s + 5}{(s + 0,1)(s + 3)}$$

zación en Bodes Básicos

$$= \frac{s + 5}{(s + 0,1)(s + 3)} = \frac{5}{0,1 \cdot 3} \cdot \frac{0,1}{s + 0,1} \frac{s + 5}{5} \frac{3}{s + 3}$$

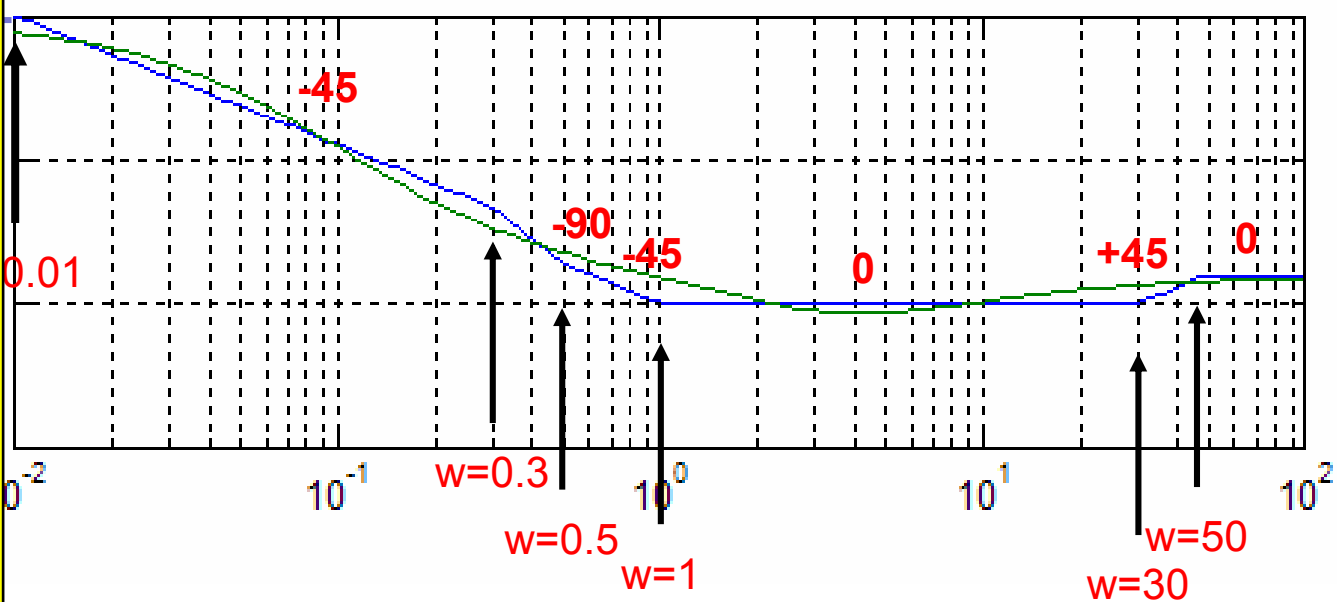
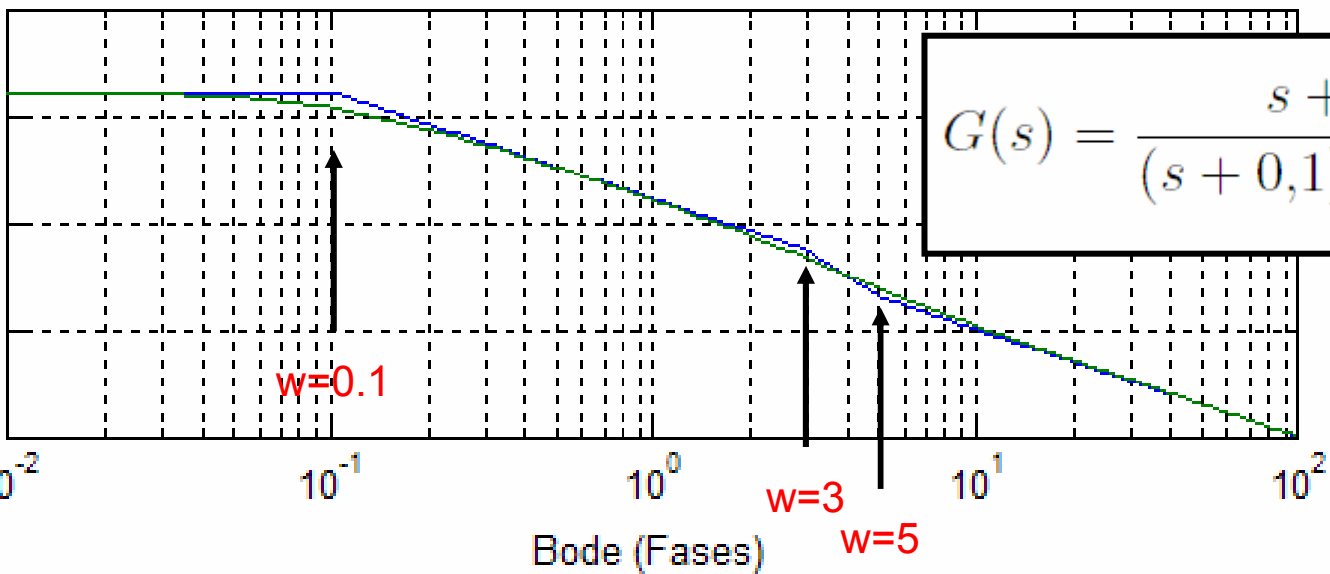
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## lo

Bode (Amplitudes):  $G(s) = (s+5)/((s+0.1)*(s+3))$

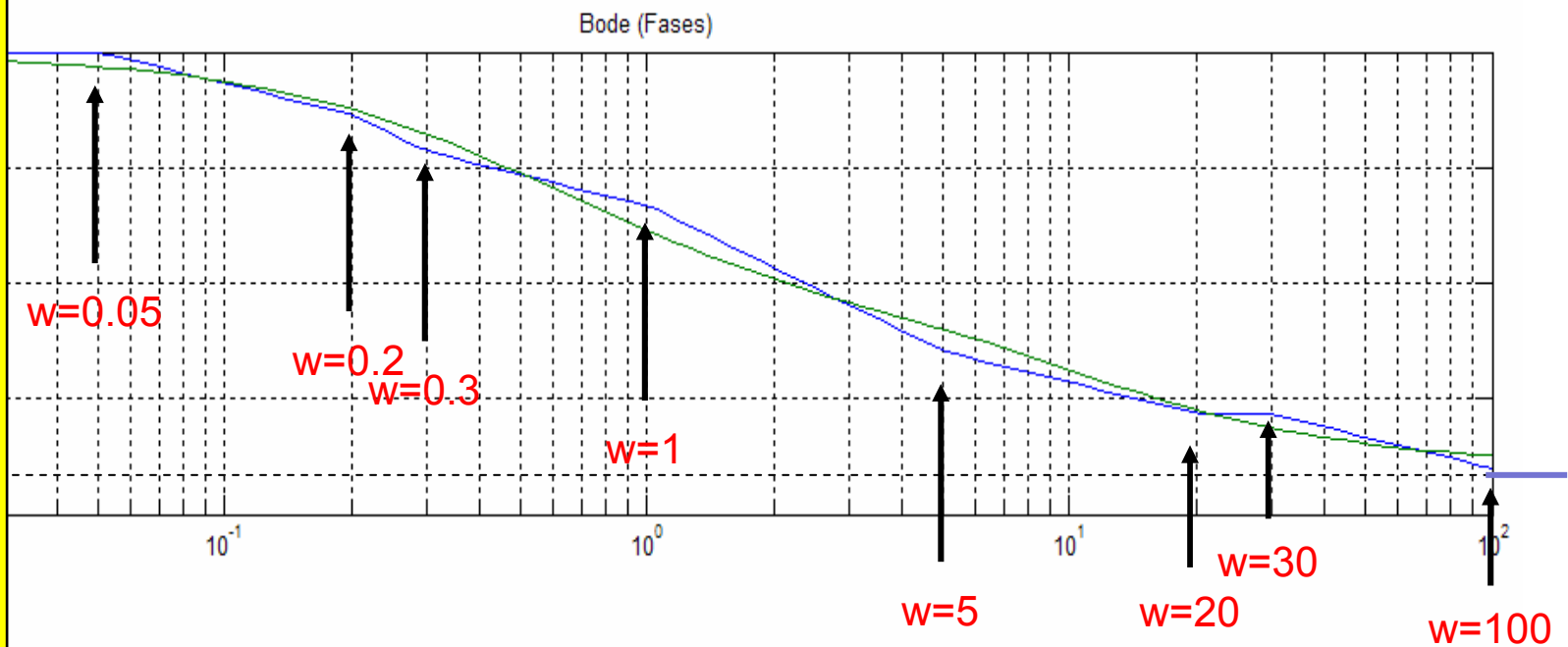
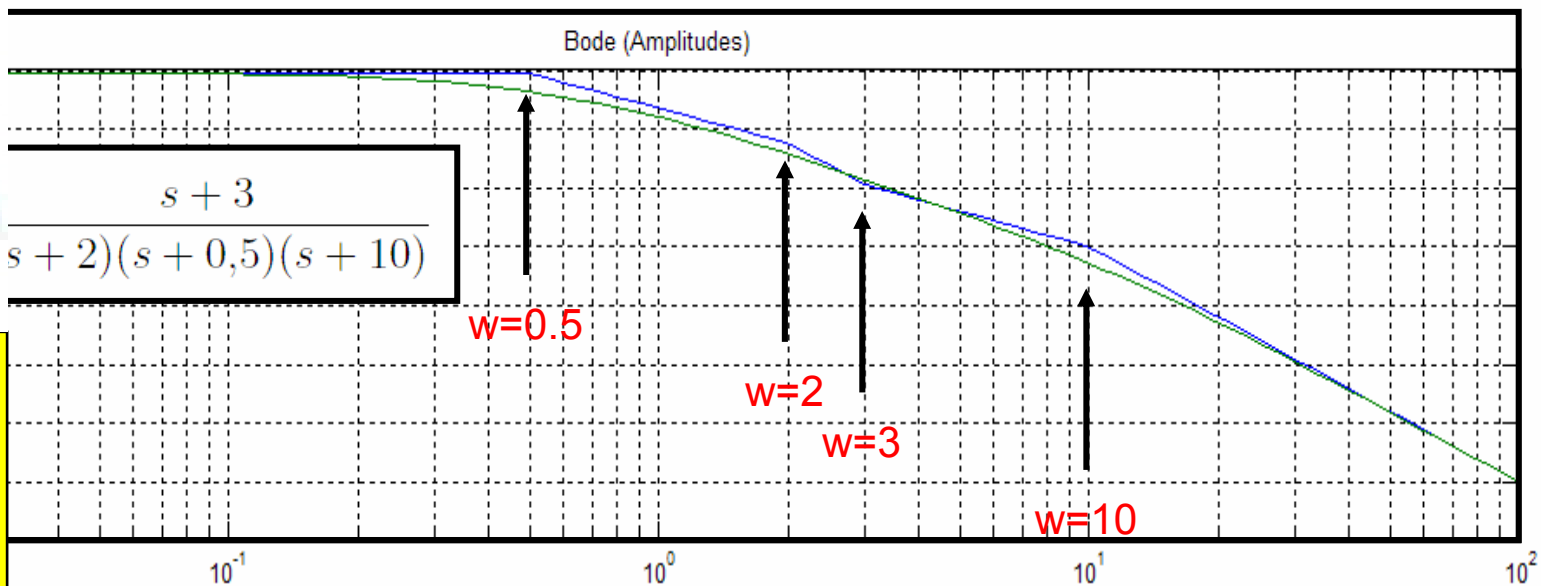
$$G(s) = \frac{s + 5}{(s + 0,1)(s + 3)}$$



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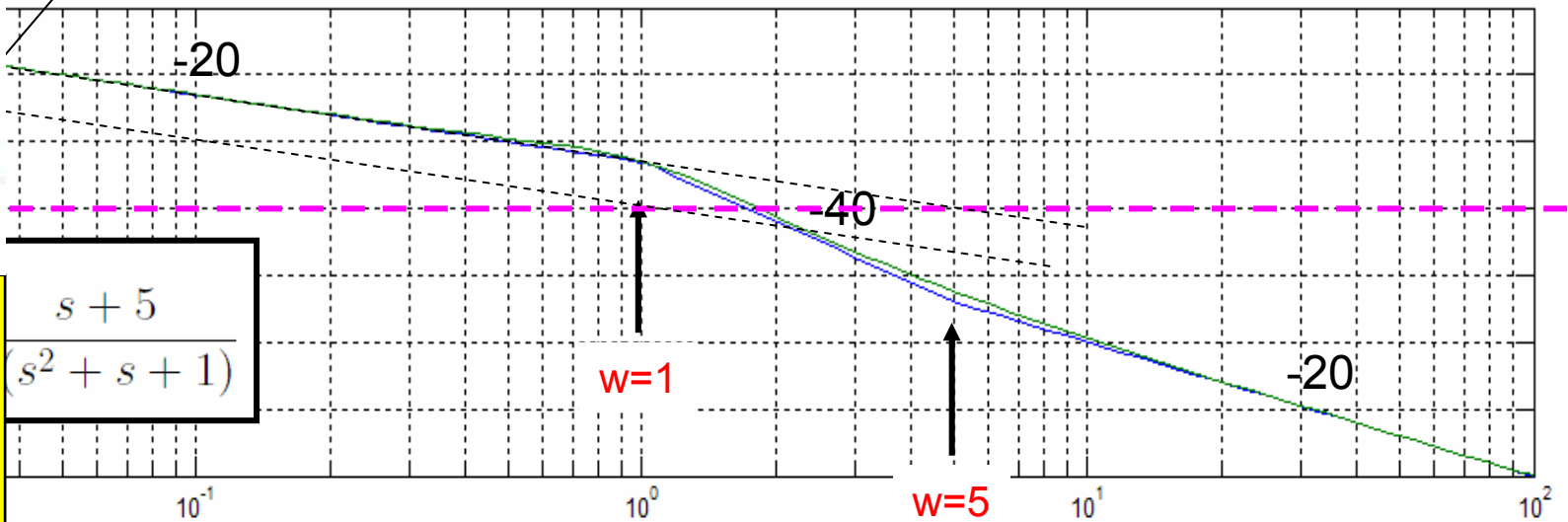
$$20 \cdot \log_{10} |3 / (2 \cdot 5 \cdot 10)|$$



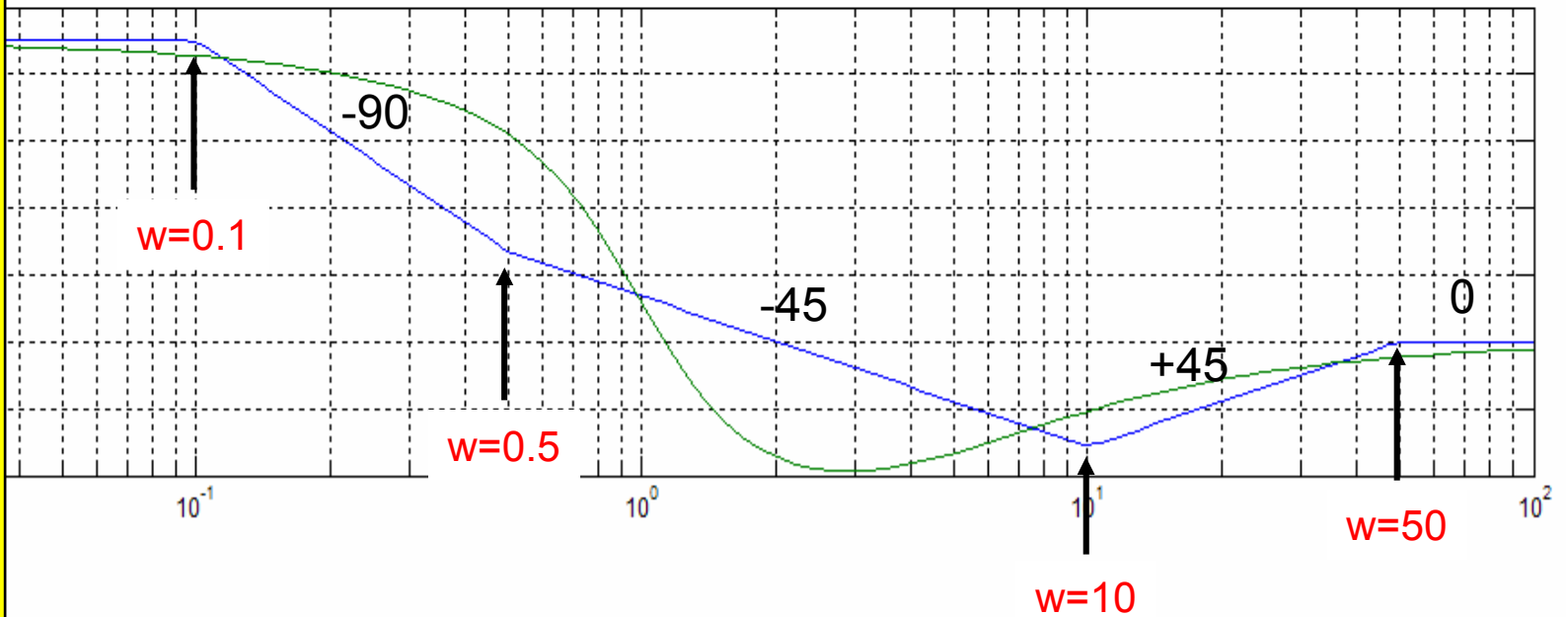
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$$20 \cdot \log_{10}|5| = 13.97 \text{dB}$$

Bode (Amplitudes):  $G(s) = (s+5)/(s^2+s+1)$



Bode (Fases)



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**plos: sistemas de fase mínima**

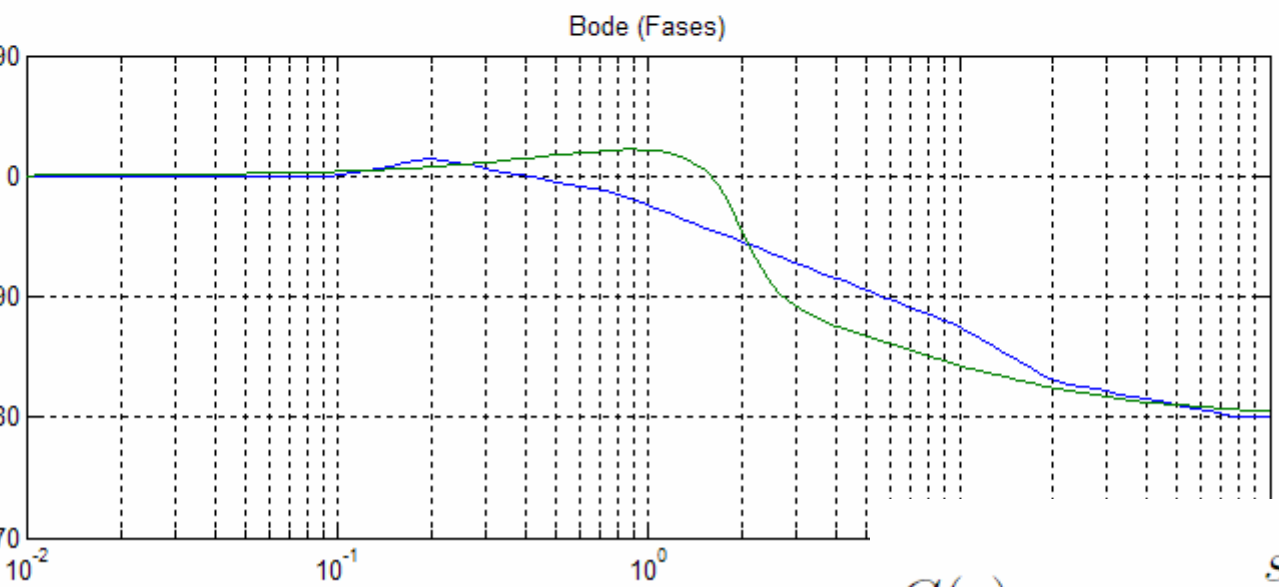
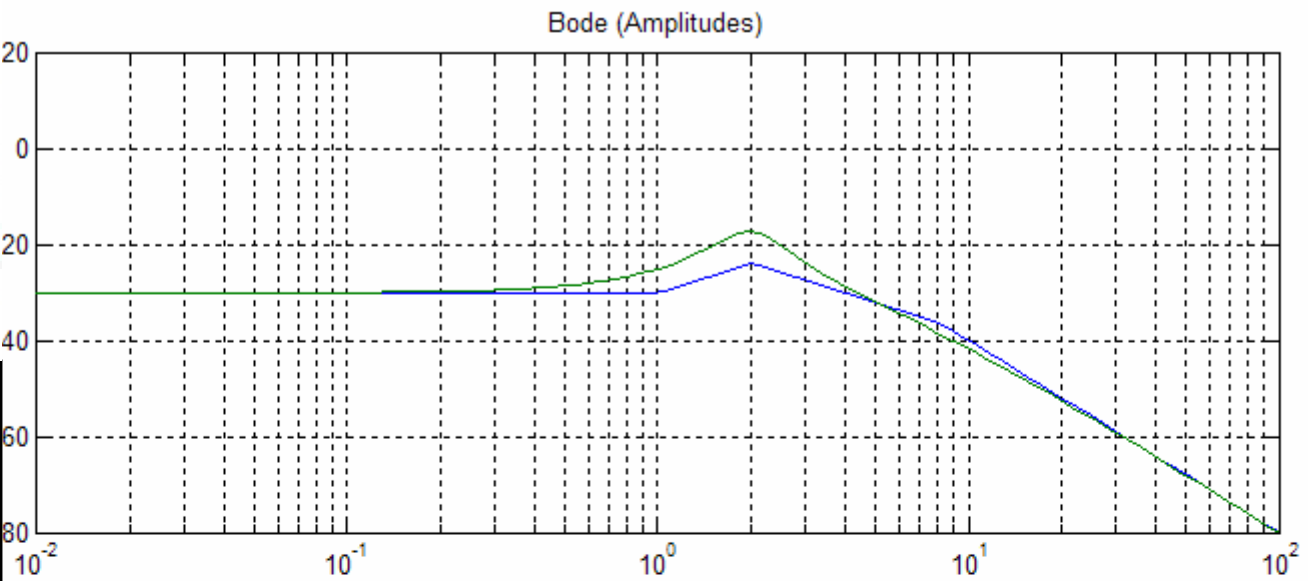
The logo for Cartagena99 features the word "Cartagena99" in a stylized, green, cursive font. The text is set against a light blue background that resembles a stylized map of the city of Cartagena. The logo is partially framed by a yellow and orange graphic element on the left side.

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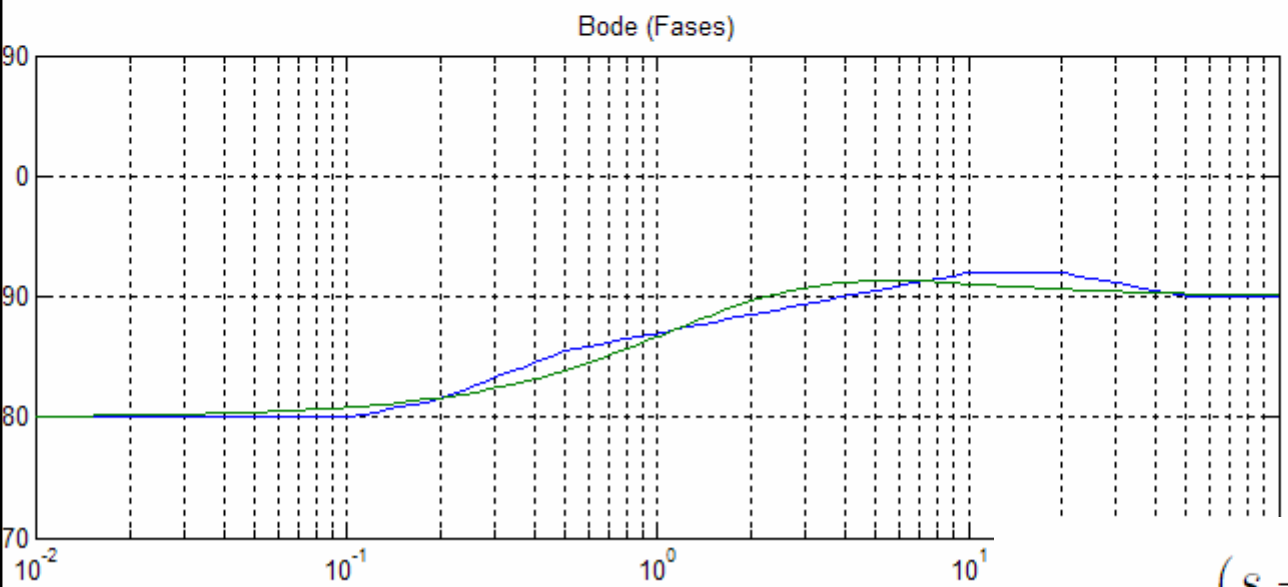
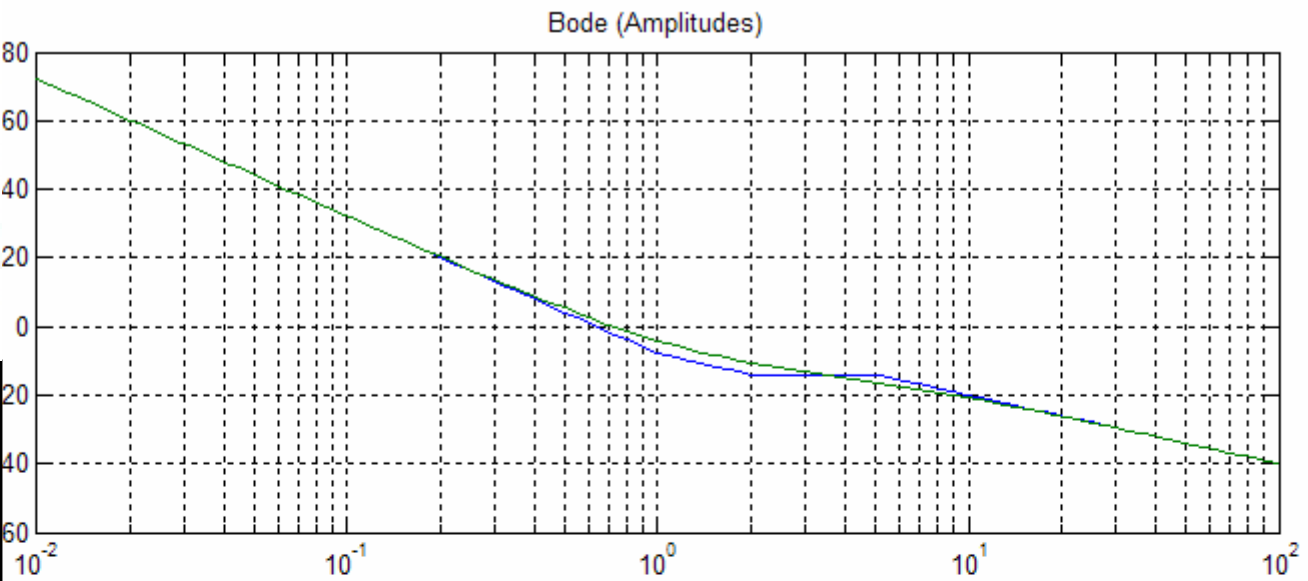
$$G(s) = \frac{s + 1}{(s + 8)(s^2 + s + 4)}$$



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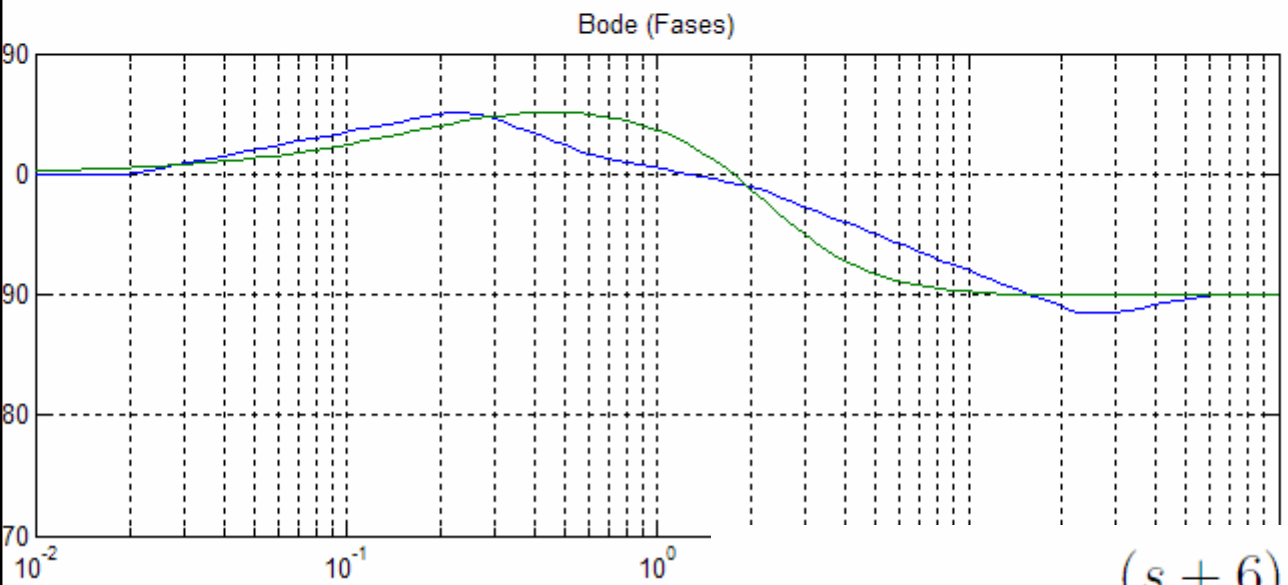
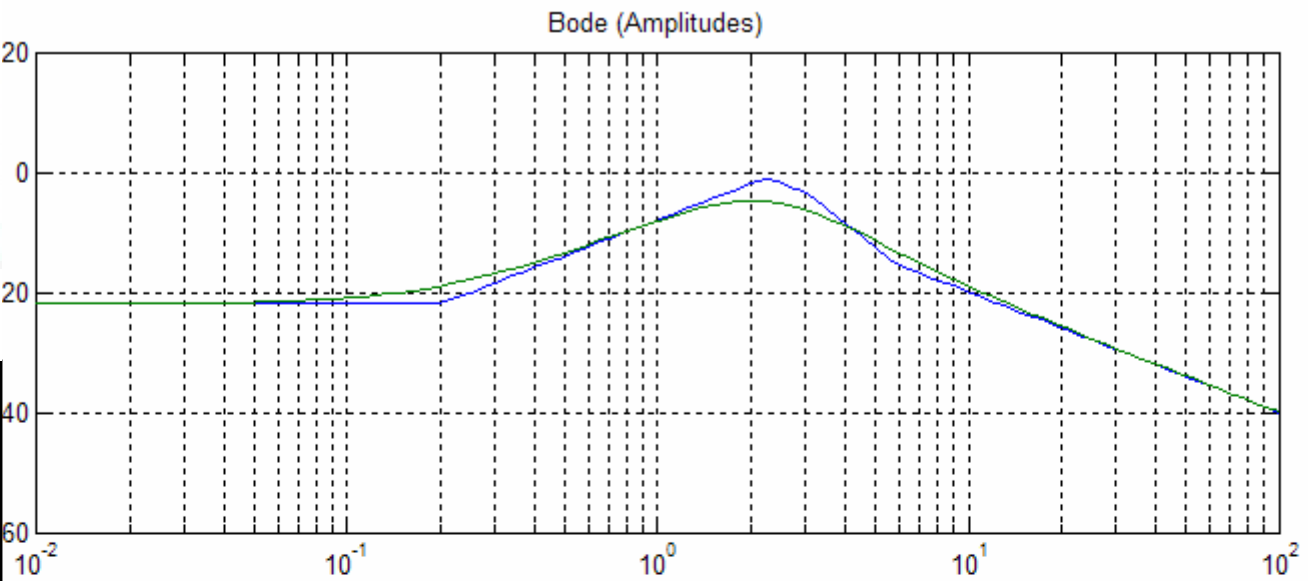
$$G(s) = \frac{(s + 2)(s + 1)}{(s + 5)s^2}$$



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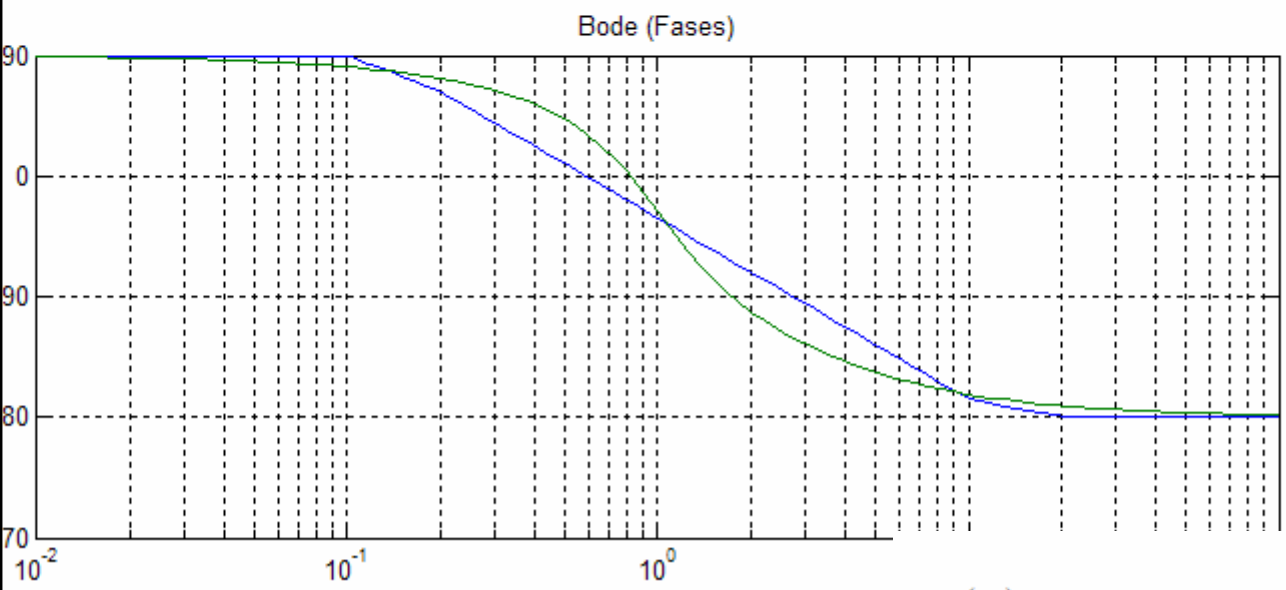
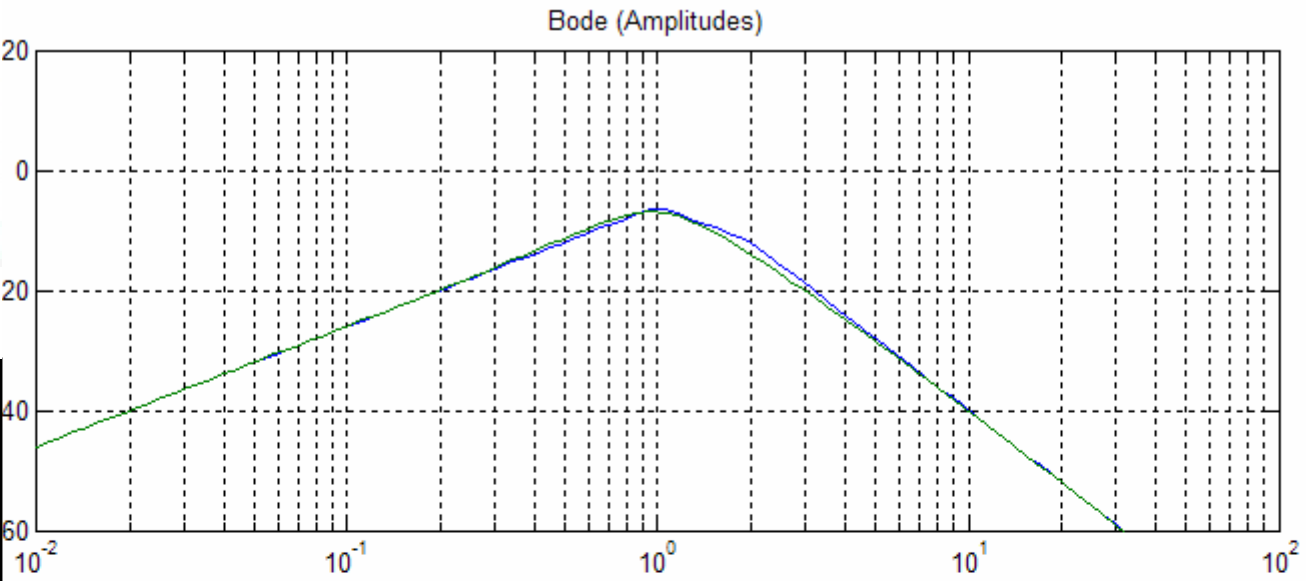
$$G(s) = 1/5 \frac{(s + 6)(5s + 1)}{(s + 3)(s^2 + 3s + 5)}$$



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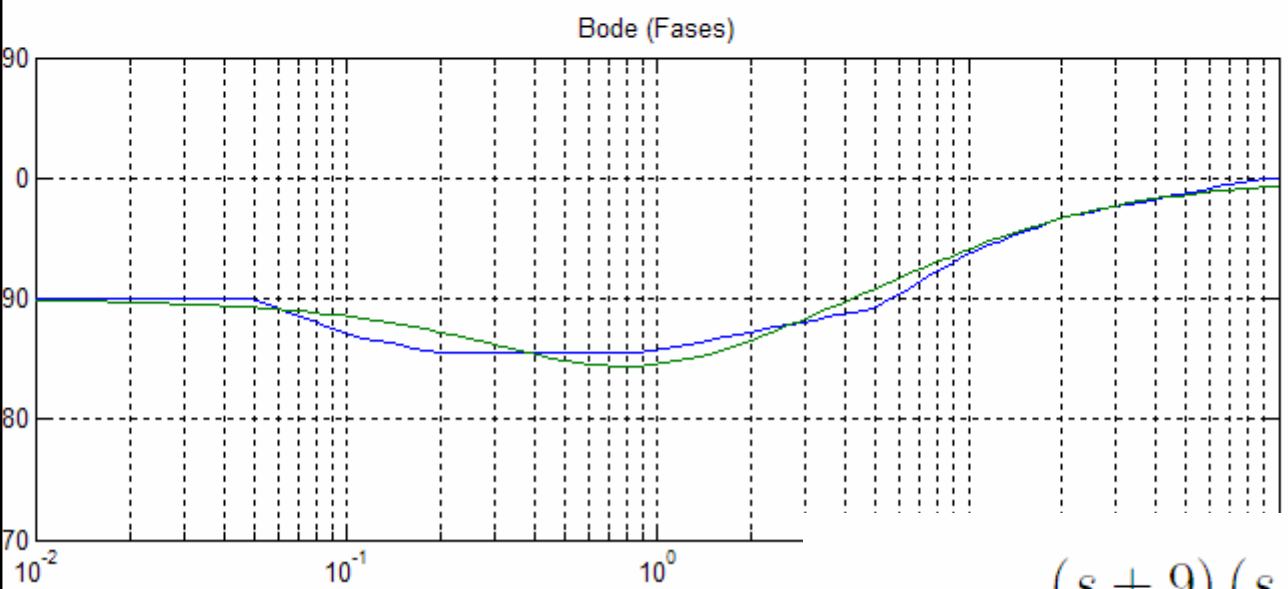
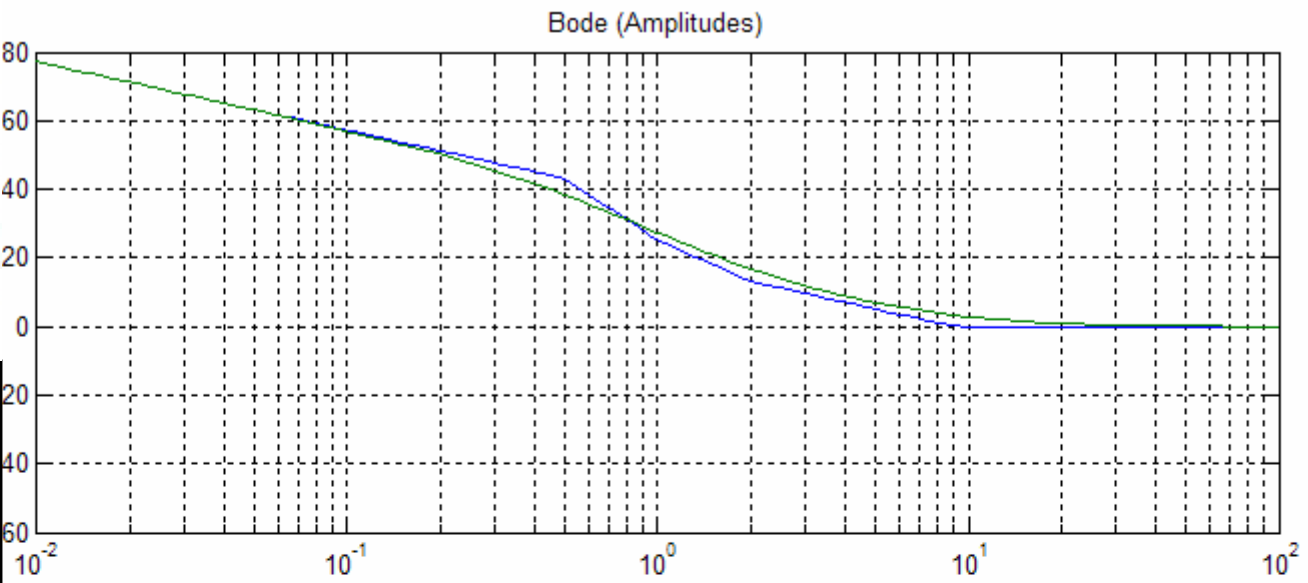
$$G(s) = \frac{s}{(s + 2)(s^2 + s + 1)}$$



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$$G(s) = 4 \frac{(s + 9)(s + 2)(s + 1)}{s(2s + 1)^2}$$



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## Clases de fase no mínima

Los sistemas que tienen polos o ceros en el semiplano positivo

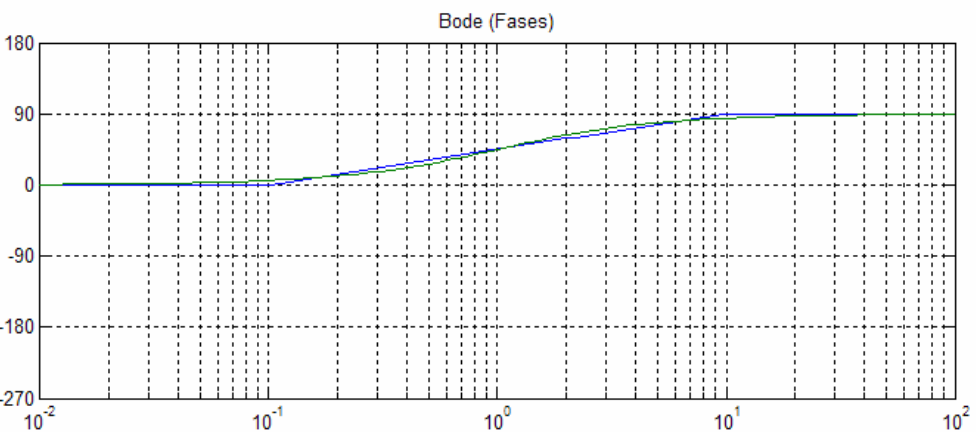
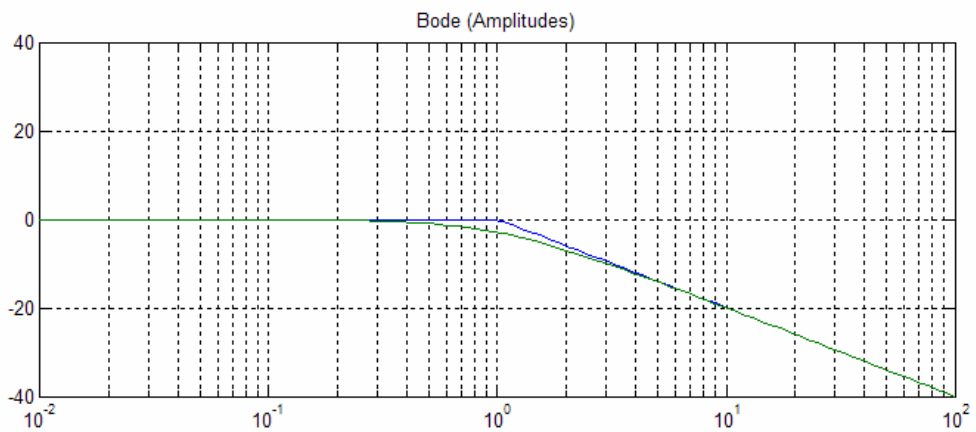
El programa de módulos es idéntico al de sus homólogos de fase mínima

Los sistemas, sin embargo son distintas

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# le fase no mínima



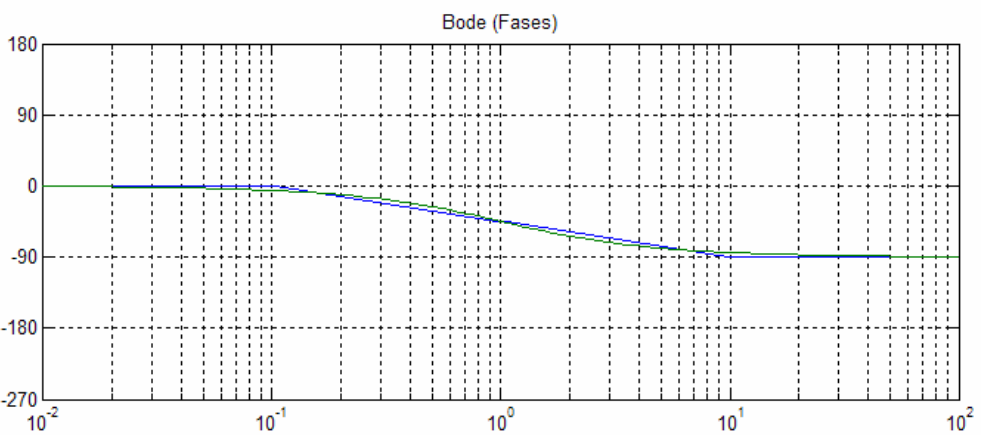
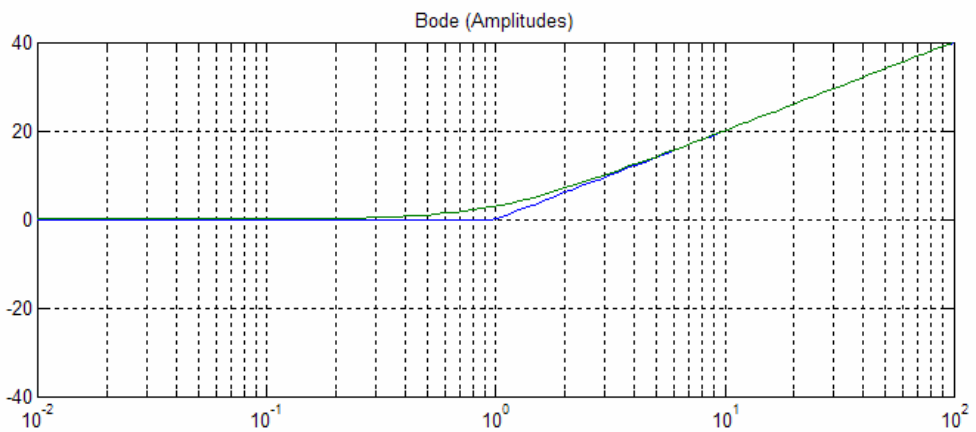
$$\frac{-1}{-1}$$

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# le fase no mínima



$\frac{1}{s}$

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**pos: sistemas de fase no mínima**

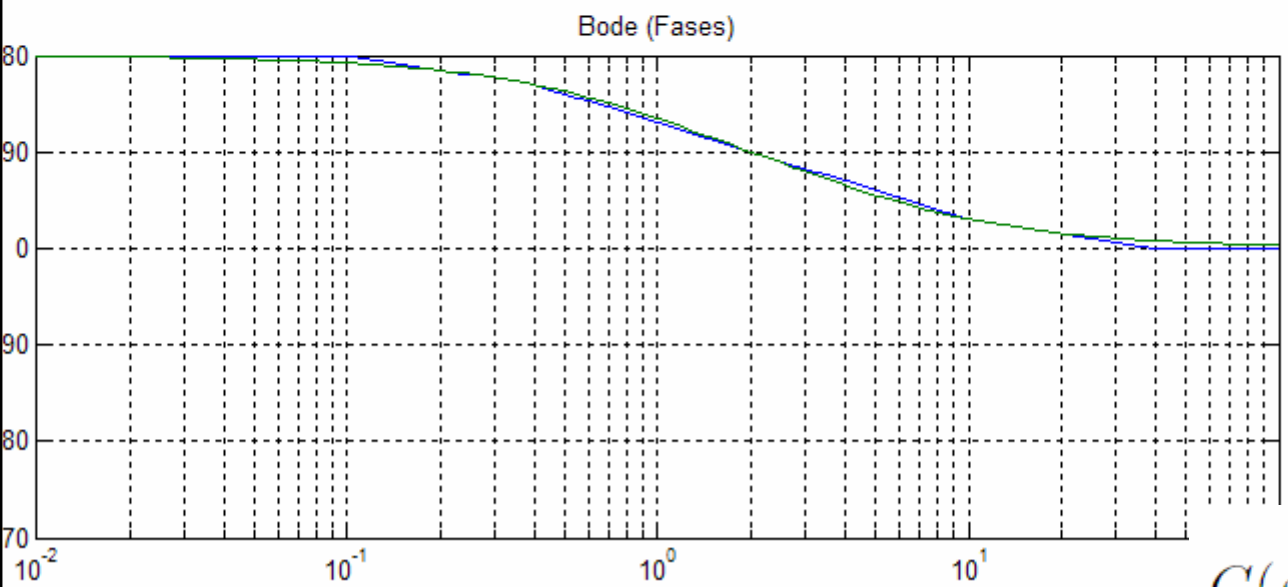
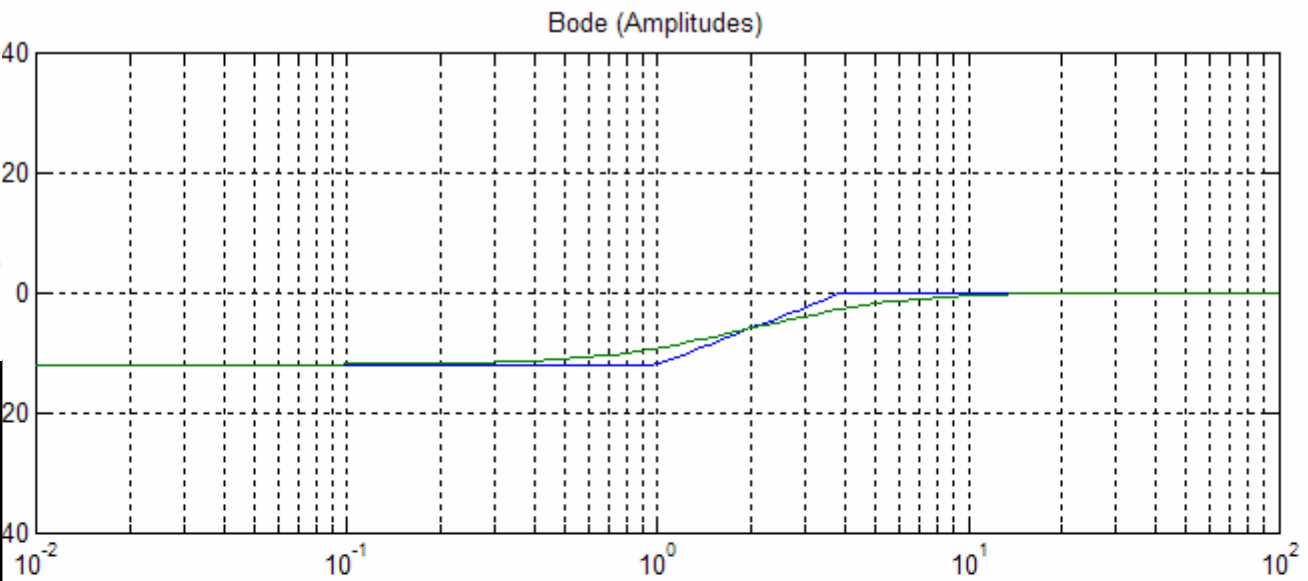
The logo for Cartagena99 features the word "Cartagena99" in a stylized, green, cursive font. The text is set against a light blue background that resembles a stylized map of the city of Cartagena. Below the text, there is a yellow and orange graphic element that looks like a stylized arrow or a banner.

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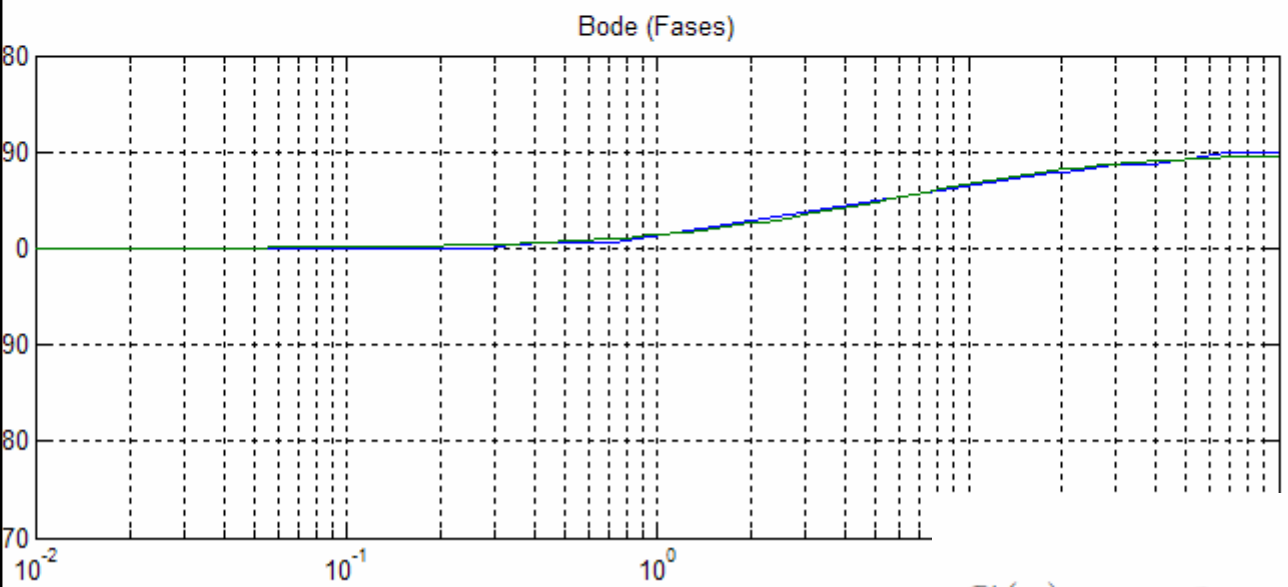
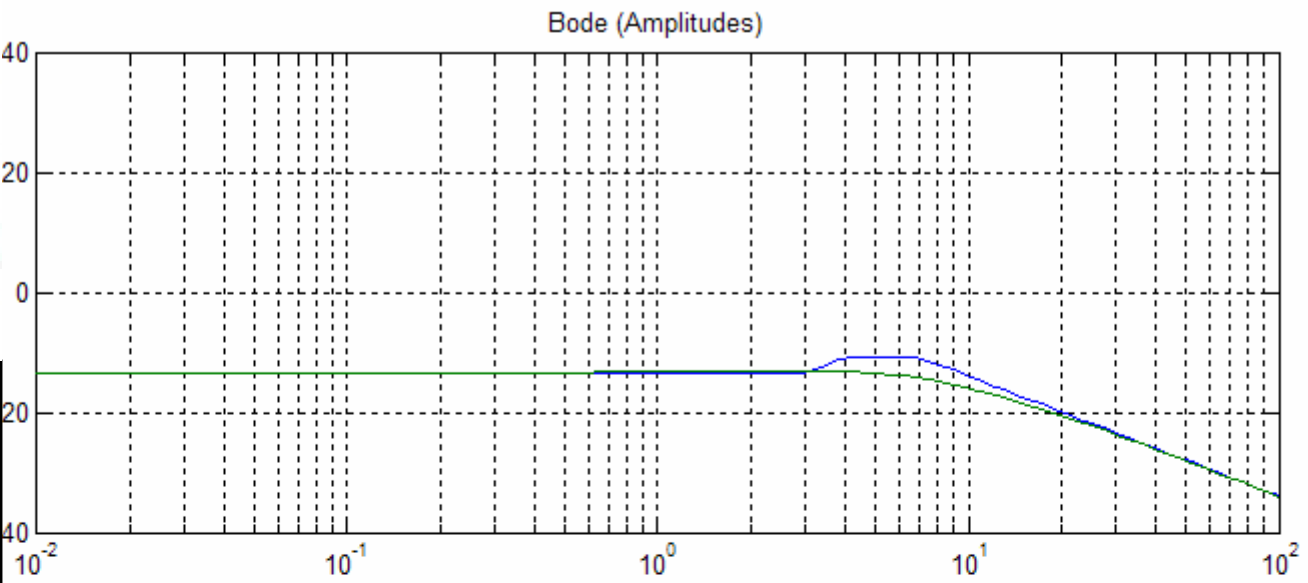
$$G(s) = \frac{s - 1}{s + 4}$$



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$$G(s) = -2 \frac{s + 3}{(s + 4)(s - 7)}$$



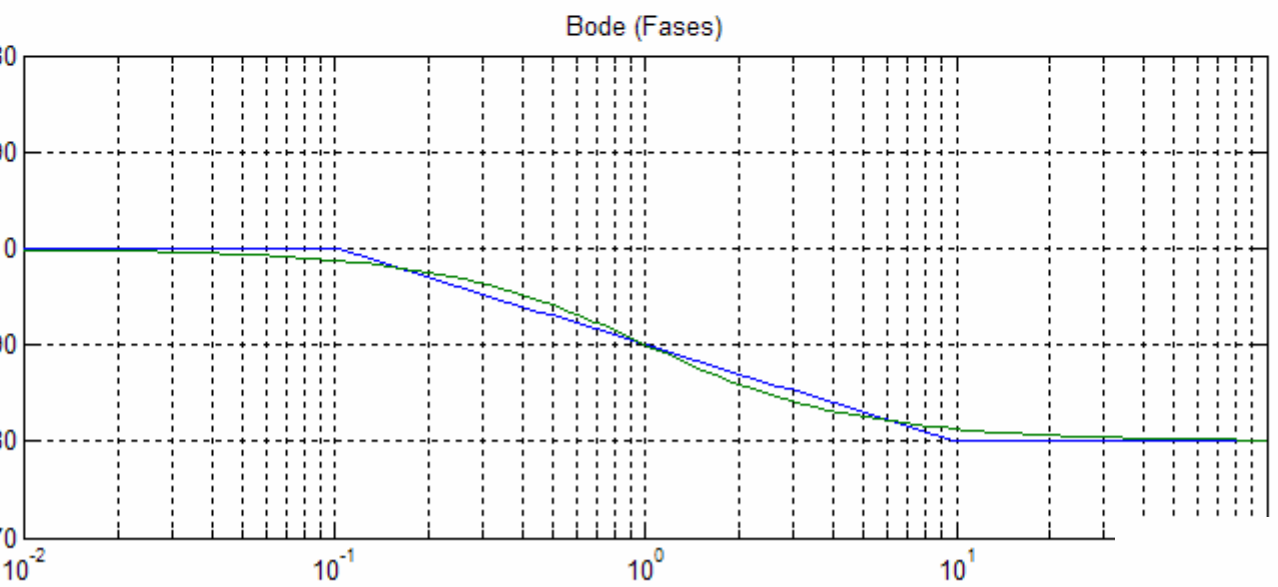
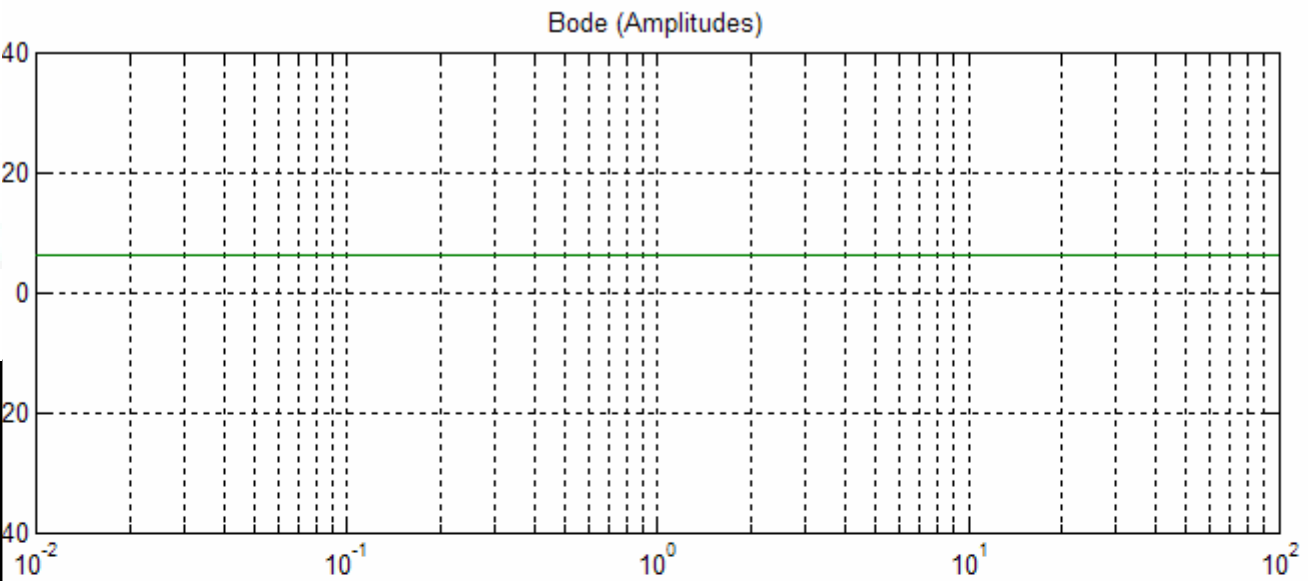
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$$G(s) = \frac{-2s + 2}{s + 1}$$



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