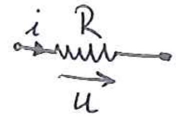
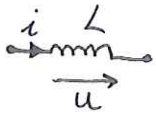
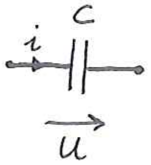
ELECTROTECNIA II



$u = Ri \Rightarrow i = G \cdot u$



$u = L \cdot i' \Rightarrow i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} u(t)\,dt$



$i = C \cdot u' \Rightarrow u(t) = u(t_0) + \frac{1}{C}\int_{t_0}^{t} i(t)\,dt$

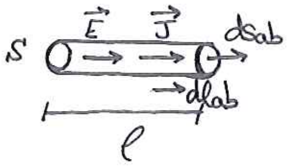
$\sum_{k=1}^{n} i_k = 0$

$\sum_{k=1}^{n} u_k = 0$

Leyes de Kirchoff

En bobinas acopladas: $u_k = \sum_{m=1}^{n} L_{km}\, i'_m$

En condensadores acoplados: $i_k = \sum_{m=1}^{n} C_{km}\, u'_m$

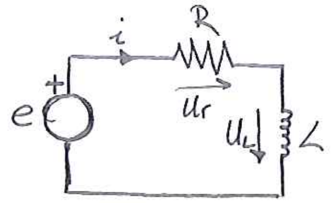


$\vec{E} = \rho \vec{J}$ $\quad i = \iint \vec{J} \cdot d\vec{S} \Rightarrow \begin{cases} i_{ab} = \iint \vec{J} \cdot d\vec{S}_{ab} \\ i_{ba} = \iint \vec{J}\, d\vec{S}_{ba} \end{cases}$

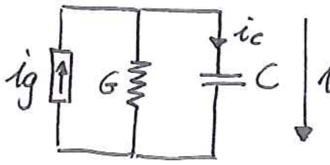
$u = \oint \vec{E} \cdot d\vec{l} \Rightarrow u_{ab} = \oint \vec{E} \cdot d\vec{l}_{ab}$

Si $\vec{E}$ y $\vec{J}$ uniformes en la sección $\Rightarrow \begin{cases} i = J \cdot S \\ u = E \cdot l \end{cases} \Rightarrow R = \frac{u}{i}$

Circuito de primer orden



¿i? $\quad e = R\cdot i + L\frac{di}{dt} \Rightarrow \boxed{i' + \frac{R}{L}i = \frac{e}{L}}$ ... $\frac{1}{\varepsilon}$



$i_g = G\cdot u + C u' \Rightarrow \boxed{u' + \frac{G}{C}u = \frac{i_g}{C}}$

De forma genérica, las ecuaciones: $\boxed{X'(t) + \sigma\cdot X(t) = g(t)}$

$r + \sigma = 0 \Rightarrow -\sigma = r \Rightarrow X(t) = K_t\,\varepsilon^{rt} + X_p(t) = K_t\,\varepsilon^{-\sigma t} + X_p(t)$

$X(0) = K_t + X_p(0) \Rightarrow K_t = X(0) - X_p(0) \Rightarrow \boxed{X(t) = \left[X(0) - X_p(0)\right]\cdot}$

$\boxed{X_p'(t) + \sigma X_p(t) = g(t)}$

- 1er caso: $g(t) = G_0$ (cc)
- 2º caso: $g(t) = \hat{G}\cdot \text{Sen}(\omega t + \theta) = \hat{G}\,\text{Cos}(\omega t + \theta')$

  $g(t) = I_m\{\underline{\hat{G}}\cdot\varepsilon^{j\omega t}\}$; $\underline{\hat{G}} = \hat{G}\,\varepsilon^{j\theta} = \hat{G}\,\text{Cos}\theta + j\hat{G}\,\text{Sen}\theta$
- 3er caso: $g(t) = I_m\left\{\sum_{k=0}^{n}\underline{\hat{G}_k}\,\varepsilon^{jk\omega_1 t}\right\}$

  $k=0 \Rightarrow g_0(t) = I_m\{\underline{\hat{G}_0}\} = I_m\{jG_0\}$; $\underline{\hat{G}_0} = jG_0$

Caso

Solución particular

1er caso: $X_p(t) = X_0 \longrightarrow \boxed{X_0 = \frac{G_0}{\sigma}}$

2º caso: $X_p(t) = I_m\{\underline{\hat{X}}\,\varepsilon^{j\omega t}\}$

3er caso: $X_p(t) = I_m\left\{\sum_{k=0}^{n}\underline{\hat{X}_k}\cdot\varepsilon^{jk\omega_1 t}\right\}$

$\underline{2^{\circ}\text{ caso}}$: $\mathrm{Im}\left\{ j\omega \hat{\underline{X}}\, \varepsilon^{j\omega t} \right\} + \sigma\, \mathrm{Im}\left\{ \hat{\underline{X}}\, \varepsilon^{j\omega t} \right\} = \mathrm{Im}\left\{ \hat{\underline{G}}\, \varepsilon^{\ldots}\right.$

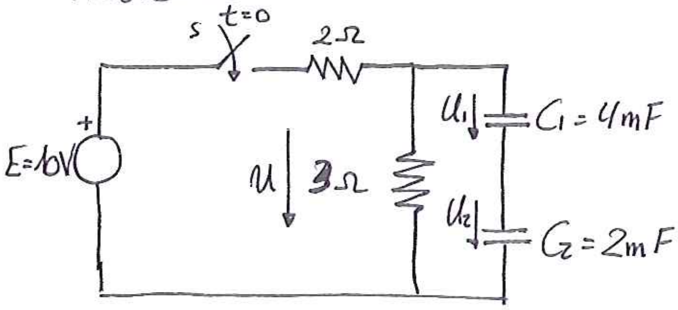
$$\Rightarrow \mathrm{Im}\left\{ \left[\hat{\underline{X}}(j\omega + \sigma) - \hat{\underline{G}} \right] \varepsilon^{j\omega t} \right\} = 0 \Leftrightarrow \hat{\underline{X}}(j\omega+\sigma) - \hat{\underline{G}} \ldots$$

$$\Rightarrow \boxed{\hat{\underline{X}} = \frac{\hat{\underline{G}}}{j\omega + \sigma}}$$

$\underline{3^{er}\text{ caso}}$: Como antes: $\mathrm{Im}\left\{ \sum_{k=0}^{n_c} \left[\hat{\underline{X}}_k (jk\omega_1 + \sigma) - \hat{\underline{G}}_k \right] \varepsilon^{j\ldots} \right. \Rightarrow$

$$\Leftrightarrow \boxed{\hat{\underline{X}}_k = \frac{\hat{\underline{G}}_k}{jk\omega_1 + \sigma}}$$

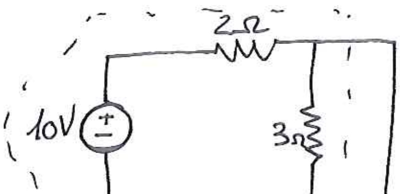
/// Problema



a) $u_1(t=0) = 0 = u_2(t=\ldots$

b) $u_1(0) = 3V \quad u_2(0) = \ldots$

a) Para $t = 0^+$



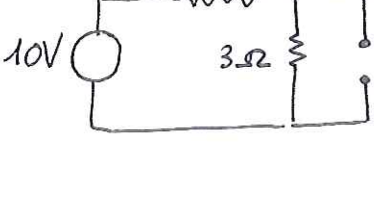
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{4}{3} mF$$

$$R_{eq} = \frac{2 \cdot 3}{2+3} = \frac{6}{5} \Omega$$

$$\sigma = \frac{1}{R_{eq} C_{eq}} = \ldots 625$$

$$\underline{u(0^+) = 0}$$

Para $t = \infty$



$$u(\infty) = 10 \cdot \frac{3}{3+2} = 6V$$

$$u_p(t) = 6V$$

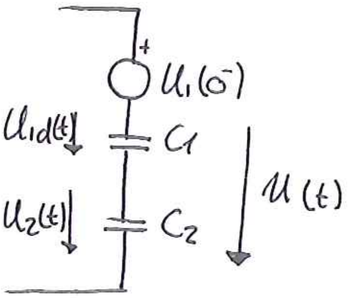
$$\Rightarrow \boxed{u(t) = [0 - 6]\ldots}$$

$\Downarrow$

$$\begin{cases} u_1 = \dfrac{C_2}{C_1+C_2} u(t) \\ u_2 = \dfrac{C_1}{C_2+C_1} u(t) \end{cases}$$

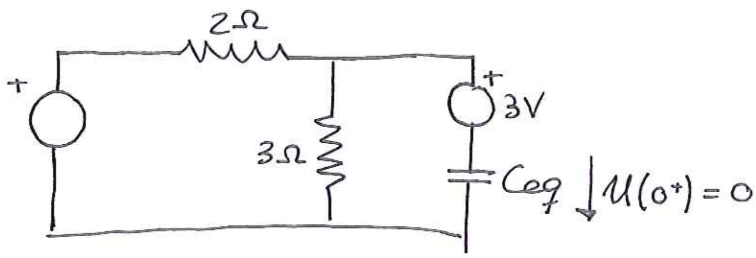
b) $u(t)=u(t_0)+\frac{1}{C}\int_{t_0}^{t} i(\varepsilon)\,d\varepsilon$

$u_1(t)=u_1(0^-)+\frac{1}{C_1}\int_{0^-}^{t} i(\varepsilon)\,d\varepsilon$; $u_1(0^+)=u(0^-)+\frac{1}{C_1}\int_{0^-}^{0^+} i(\varepsilon)\,d$ … $=u_1(0^-)$

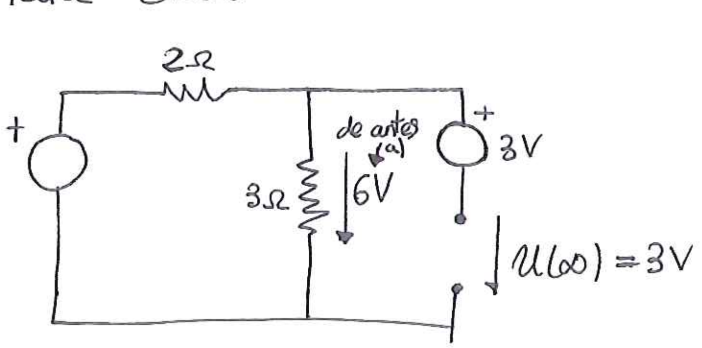


$u(t)=u_{1d}(t)+u_2(t)$

Para $t=0^+$



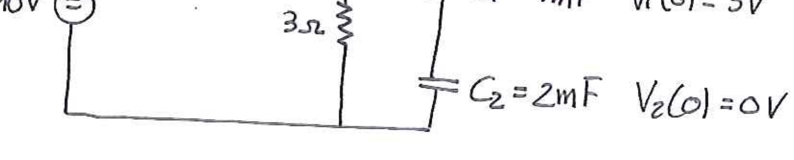
Para $t=\infty$



$\Rightarrow u(t)=[0-3]\,e^{-625t}+$ …

$\Rightarrow \left| \begin{array}{l} u_{1d}=\frac{1}{3}u(t) \Rightarrow u \ldots \\ u_2=\frac{2}{3}u(t) \end{array} \right.$

Ejercicio.



Laplace:

$\hat{X}(s)=\int_{0^-}^{\infty} x(t)\,e^{-st}$ …

$e(t)=E_0 \Rightarrow \hat{e}(s)=\int_0^{\infty} E_0\,e^{-st}\,dt=\frac{E_0}{s}$

RESISTENCIAS

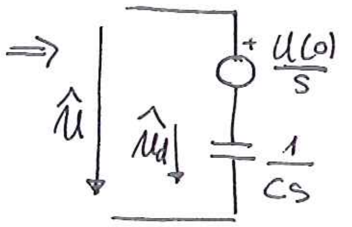
$$u(t) = R \cdot i(t) \Rightarrow \underbrace{\int_0^{\infty} u(t)\, e^{-st}\, dt}_{\hat{u}(s)} = R \underbrace{\int_0^{\infty} i(t)\, e^{-st}\, dt}_{\hat{i}(s)} \Rightarrow \boxed{\hat{u}(s) = R \cdot \hat{i}(s}$$

CONDENSADORES

$$i(t) = C \frac{du(t)}{dt} = C\dot{u}(t) \qquad \hat{x}'(s) = s\hat{x}(s) - x(0) \Rightarrow$$

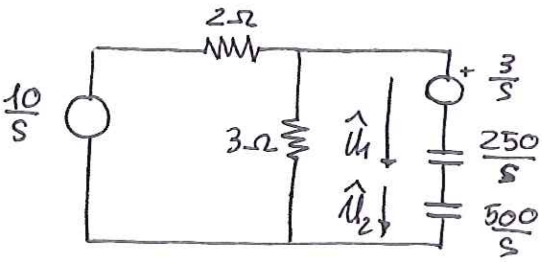
$$\Rightarrow \boxed{\hat{i}(s) = C\left[s\hat{u}(s) - u(0)\right]} \qquad \hat{i} = Cs\hat{u} - Cu(0) \Rightarrow \boxed{\hat{u}}$$

$\Rightarrow$

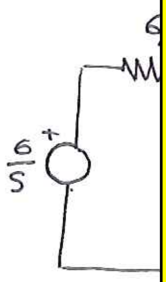


Volvemos al problema...

$$\frac{1}{C_1 s} = \frac{250}{s} \qquad \frac{1}{C_2 s} = \frac{500}{s}$$



$$\frac{3}{3+2} \times \frac{10}{s} = \frac{6}{s}$$



$$\hat{u}_2 = \frac{500/s}{\frac{6}{5} + \frac{750}{s}} \left(\frac{6}{s} - \frac{3}{s}\right) = \frac{500}{750 + \frac{6}{5}s} \cdot \frac{3}{5} = \frac{1250}{s+625} \cdot \frac{1}{s}$$

$$\left\{ \begin{array}{l} x(t) = \dots \\ \Rightarrow \hat{x}(\dots \end{array} \right.$$

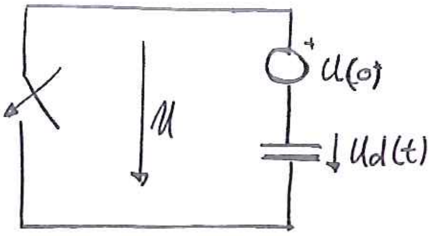
$$\hat{u}_2 = \frac{\lambda_1}{s+625} + \frac{\lambda_2}{s} \Rightarrow \lambda_1 = \hat{u}_2 (s+625)\Big|_{s=-625} = 1250 \times \frac{1}{-625} = -2$$

$$\lambda_2 = \hat{u}_2 \cdot s\Big|_{s=0} = \frac{1250}{0+625} = 2$$

$$\boxed{\begin{array}{l} u_2(t) = -2\, \dots \\ u_1(t) = -\, e^{-6\dots} \end{array}}$$

$$u_{1d}(t) = \frac{1}{2} u_2(t) = -e^{-625t} + 1\,V \qquad u_1(t) = u_{1d}(t) + 3V$$

Impulsos

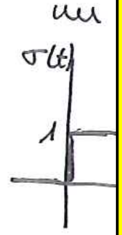


$t=0^-\quad u(0^-)\neq 0$

$t=0^+\quad u(0^+)=0=u(0^-)+ud(0$

$\Rightarrow ud(0^+)=-u(0^-)\Rightarrow ud($

$ud(t)=-u(0^-)\sigma(t)$



$$i=C\,ud'(t)=C\frac{d}{dt}\left[u(0^-)\sigma(t)\right]=$$

$$=-C\,u(0^-)\frac{d\sigma(t)}{dt}=\underline{\underline{-C\,u(0^-)\delta(t)}}$$

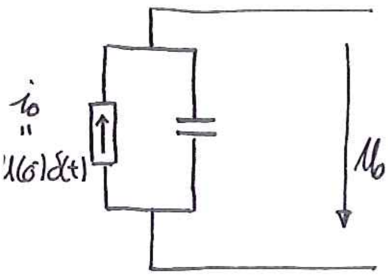
$q(t)=C\,u(t)\qquad i=$

$$\int_{0^-}^{0^+} i(t)dt=-C\,u(0^-)\int_{0^-}^{0^+}\delta(t)dt=-C\,u(0^-)=q(0^+)-q(0^-)$$

$q($

$q($

$$i=C\,u'=CDu$$



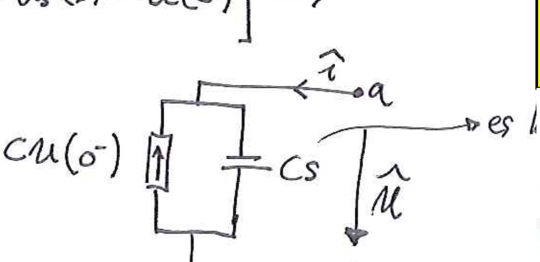
$$u_0=\frac{1}{CD}C\,u(0^-)\delta(t)$$

$$u_0(t)=\int_0^t u(0^-)\delta(t)dt=u(0^-)\int_{0^-}^t\delta(t)dt=u_0($$

Mejor por Laplace:

$$i(t)=C\frac{du(t)}{dt}\Rightarrow \hat{i}(s)=C\left[s\hat{u}_s(s)-u(0^-)\right]\Rightarrow$$

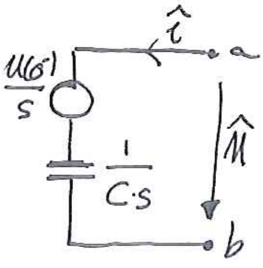
$$\Rightarrow \boxed{\hat{i}=Cs\hat{u}-\underbrace{C\,u(0^-)}_{i_g}}\longrightarrow \text{eq. Norton}$$



$\hat{\delta}(s)=1$

despejando $\hat{u}$

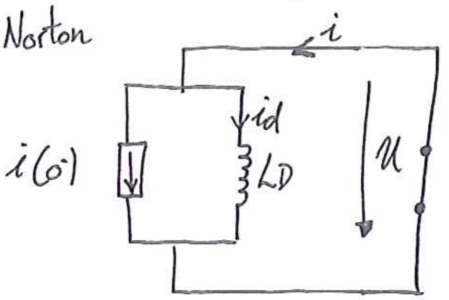
$$\boxed{\hat{u}=\frac{1}{Cs}\hat{i}+\frac{u(0^-)}{s}}\longrightarrow$$



BOBINA IDEAL.

$$i(t)=i(0^-)+\frac{1}{L}\int_{0^-}^{t}u(t)\,dt \qquad u=L\frac{di}{dt}=Li'=LDi$$

Norton



$u=0$

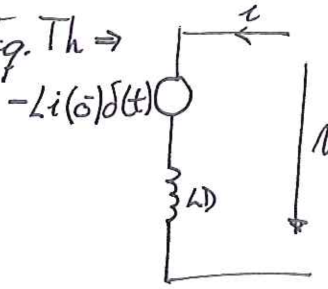
$id(0^-)=0$

$$t=0^+ \Rightarrow i=0=i(0^-)+id(t) \qquad i(0^+)=0=i(0^-)+id(0^+) \Rightarrow id$$

$$id(t)=-i(0^-)\,\sigma(t)$$

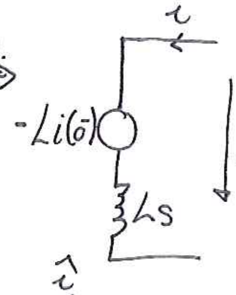
$$u=LD\,id=L\,id'=L\frac{d\,id}{dt}=L\frac{d}{dt}\left[-i(0^-)\sigma(t)\right]=\underline{-L\,i(0^-)\,\delta(t)}$$

Eq. Th ⇒

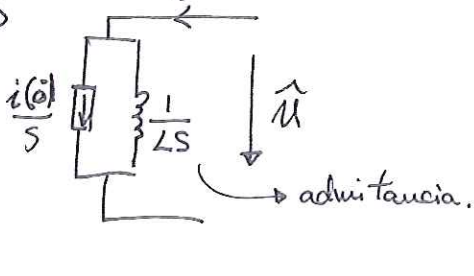


Con Laplace:

$$\hat{u}=L\left[s\hat{i}-i(0^-)\right]=Ls\hat{i}-Li(0^-) \quad \xRightarrow{Th.}$$

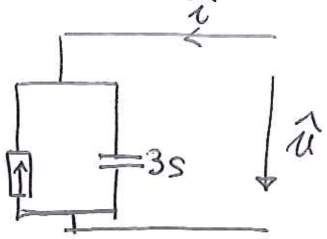


$$\text{Despejando } \hat{i}=\frac{1}{Ls}\hat{u}+\frac{i(0^-)}{s} \quad \xRightarrow{Norton}$$



Problema examen Junio 2012

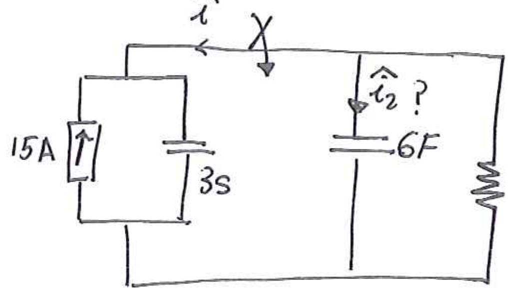
$$\begin{cases} C_1 = 3F \\ U_1(0^-) = 5V \end{cases}$$



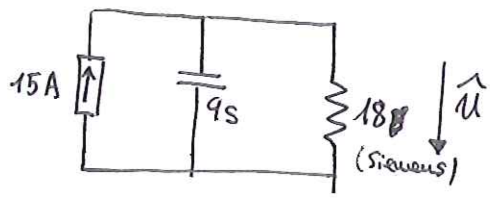
$3 \times 5 = 15A = C_1 U(0^-)$

$C_1 \cdot s = 3s$
admitancia

$$\begin{cases} G = 18 \, S \\ C_2 = 6F \\ U_2(0^-) = 0\,V \end{cases}$$



$\hat{i}_2$?
$i_2(t)$?
$U_r(t)$?



$$\hat{i}_2 = \frac{6s}{3s + 6s + 18} \cdot 15 = \frac{90s}{9s + 18} = \frac{10s}{s+2}$$

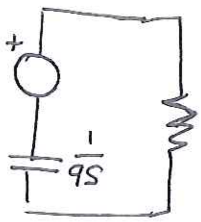
$$= 10 \, \frac{s+2-2}{s+2} = 10 - \frac{20}{s+2} \Rightarrow$$

$$\Rightarrow i_2(t) = 10\delta(t) - 20e^{-2t} \, A$$

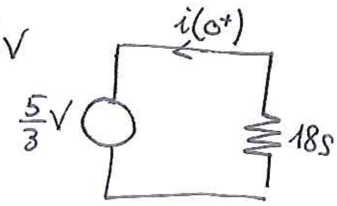
$$15 = (9s + 18)\hat{U}_r \Rightarrow \hat{U}_r = \frac{5}{3} \frac{1}{s+2} \Rightarrow U_r(t) = \frac{5}{3} e^{-2t}$$

$f_n = 2Hz \quad \tau = \frac{1}{f_n} = 0'5 \; s$

$$u(\tau) = u(0^+)e^{-1} = \frac{u(0^+)}{2'73} \qquad u(2\tau) = \frac{u(0^+)}{2'73^2} \qquad u(3\tau) = \frac{u(0^+)}{2'73^3}$$

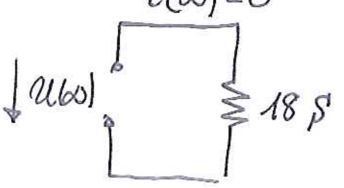


$U(0^+) = \frac{5}{3} V$



$i(0^+) = -18 \times \frac{15}{3} = -$

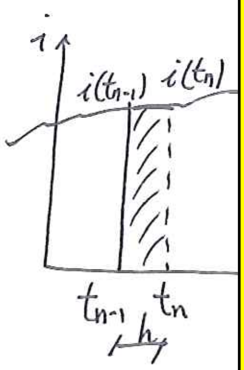
$i(\infty) = 0$



IIIb) Integración numérica en condensadores.

$U(t) \quad t = 0'1\,s$

$$i = C\frac{dU}{dt} \Rightarrow U(t) = U(t_0) + \frac{1}{C}\int_{t_0}^{t} i(t)\,dt$$



$t_n = t_{n-1} + h$ → paso de integración (intervalo entre instantes)

$$u(t_n) = u(t_{n-1}) + \frac{1}{C}\underbrace{\frac{i(t_n) - i(t_{n-1})}{2}h}_{A_{TRAPECIO}}$$

$U(t_n) = U_n$

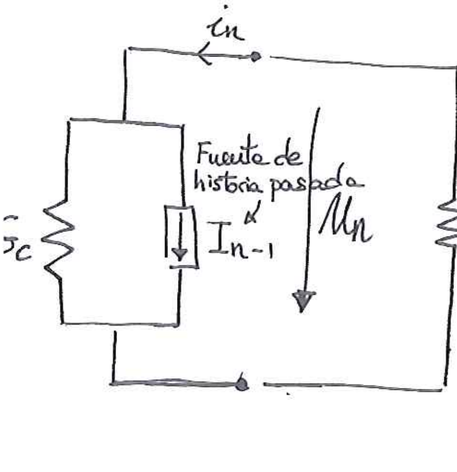
$u(t_{n-1}) = U_{n-1}$ e igual

Por tanto:

$$U_n = U_{n-1} + \frac{h}{2C}\left[i_n + i_{n-1}\right] \Rightarrow i_n + i_{n-1} = \frac{2C}{h}(U_n - U_{n-1})$$

en siemens ⇒

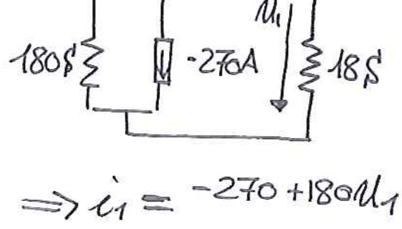
$\boxed{i_n = G_c \cdot U_n + I_{n-1}}$ $\boxed{I_{n-1} = -i_{n-1} - G_c U_{n-1}}$



$G_c = \frac{2\cdot 9}{0'1} = 180\,S$

$t=0$
$\begin{cases} i_0 = -30A \\ U_0 = \frac{5}{3}V \end{cases} \Rightarrow I_0 = -G_c \cdot \frac{5}{3} \ldots$

$= -2\ldots$



$n = 1 \Rightarrow t_1 = 0'1s$

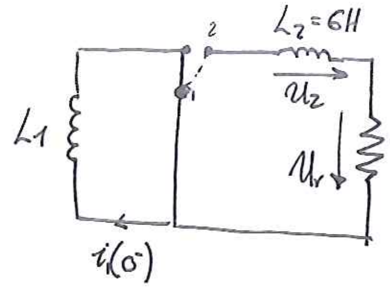
$(18 + 180)\cdot U_1 = -(-270) \Rightarrow \underline{U_1 = 1'36\,V}$

$\Rightarrow i_1 = -270 + 180U_1 = \underline{-24'54A} \Rightarrow I_1 = -G_c U_1 - i_1 = -220'$

$(18+180)U_2 = -(-220'909) \Rightarrow \underline{U_2 = 1'117V} \quad i_2 = -\underline{18 \cdot 1'1170A}$

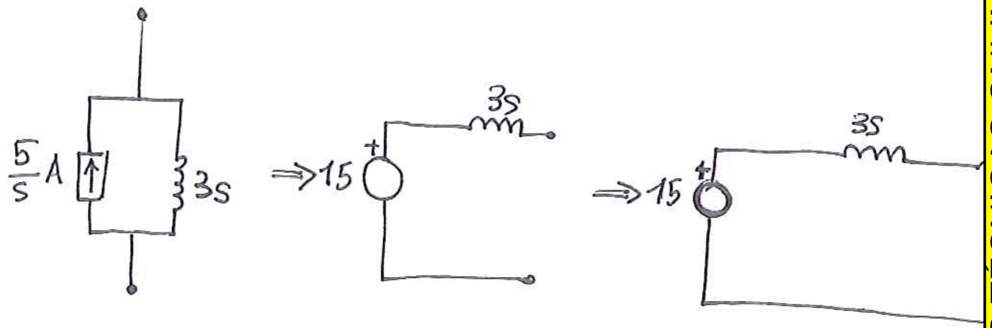
y así sucesivamente. (Sale bien porque $h < \varepsilon$)

Problema examen. Julio 2013.



Va a haber un impulso de tens ... a bruscamente la i_2

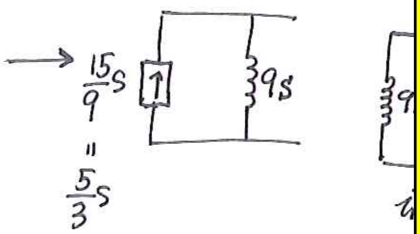
$L_1 = 3H$
$i_1(0^-) = 5A$



$$\hat{U}_2 = \frac{6s \cdot 15}{18 + 9s} = \frac{90s}{9(s+2)} = 10\frac{s}{s+2} = 10 - \frac{20}{s+2} \Rightarrow$$

$$\Rightarrow u_2(t) = 10\delta(t) - 20e^{-2t} \quad (V)$$

$$\hat{U}_r = \frac{18}{9(s+2)} \times 15 = \frac{30}{s+2} \Rightarrow u_r(t) = 30e^{-2t}$$



b) Hallar $u_r(0'1s)$ por integración numérica.

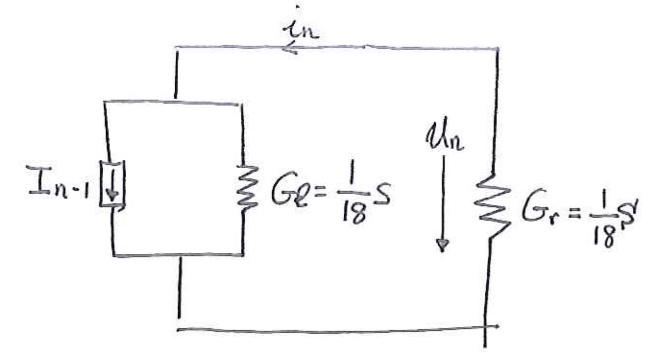
$h = 0'1s$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} u(t)\,dt \qquad h = t_n - t_{n-1} \Rightarrow$$

$$\Rightarrow i_n = i_{n-1} + \frac{1}{L}\int_{t_{n-1}}^{t_n} u(t)\,dt \approx i_{n-1} + \frac{1}{L}\frac{u_n - u_{n-1}}{2} \cdot h \Rightarrow$$

$$\Rightarrow i_n = i_{n-1} + \underbrace{\left(\frac{h}{2L}\right)}_{G_e}(u_n - u_{n-1}) \quad \begin{cases} i_n = G_e u_n + I_{n-1} \\ I_{n-1} = i_{n-1} + G_e u_{n-1} \end{cases}$$

$$\boxed{G_e = \frac{h}{2L}}$$

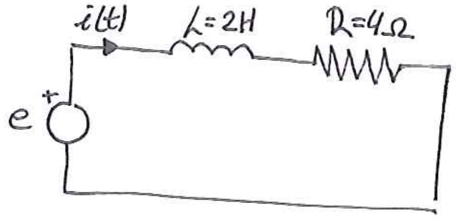


$u_0 = 30V$
$i_0 = -\frac{5}{3}A$ | $I_0 = i_0 + G_\ell u_0 = -1$

$[G_\ell + G_r] u_1 = -I_0 \Rightarrow u_1 =$ … $4'55 V$

$i_1 = G_\ell u_1 + I_0$

// Problema



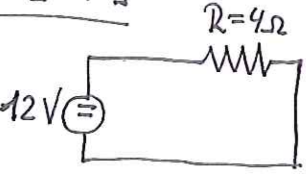
$\mathcal{L}\left[\text{Sen}(\omega t+\varphi)\right] = \frac{s \cdot S}{}$

$\text{Sen}\,\theta = \text{Cos}(\theta -$

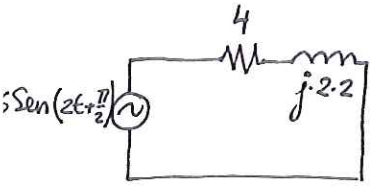
$\text{Cos}\,\theta = \text{Sen}(\theta +$

$$e(t) = \begin{cases} 12 + 16\,\text{Sen}\left(2t + \frac{\pi}{2}\right) & \forall t \in [-\infty, 0] \\ 6 + 8\,\text{Sen}\left(2t + \frac{\pi}{2}\right) & \forall t \in [0, \infty] \end{cases}$$

$t \in [-\infty, 0]$



$i_1(t) = I_1 = \frac{12}{4} = 3A$



$i_2(t) = \text{Im}\left\{\hat{I}_2 e^{j2t}\right\}$

$\hat{I}_2 = \frac{16j}{4+j4} = \frac{4j}{1+j} = j\frac{4(1-j)}{1^2+1^2} = 2(1+j) = 2$

$i(t) = i_1(t) + i_2(t) = 3 + \text{Im}\left\{2\sqrt{2}e^{j\frac{\pi}{4}}e^{j2t}\right\} \Rightarrow i(0) = 5A$

$t > 0$

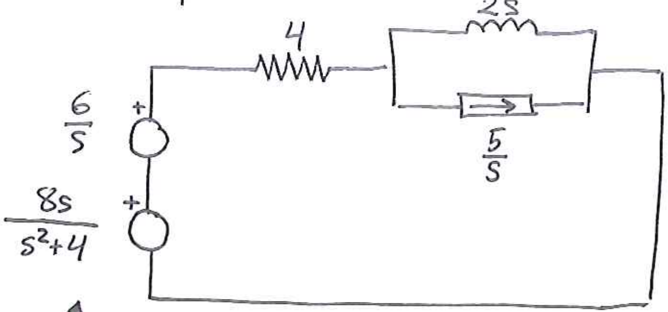
$i(t) = [i(0) - i_p(0)]e^{-\sigma t} + i_p(t)$ donde $i_p(t) = 1'5 + \text{Im}\left\{\sqrt{2}e\right.$

porque no cambia φ ni f.

$\Rightarrow i_p(0) = 2'5A$

$\sigma = \frac{R}{L} = 2\,Hz \Rightarrow i(t) = [5 - 2'5]e^{-2t} + 1'5 + \text{Im}\left\{\sqrt{2}e^{j\frac{\pi}{4}}e^{j2t}\right\}$

Por Laplace:



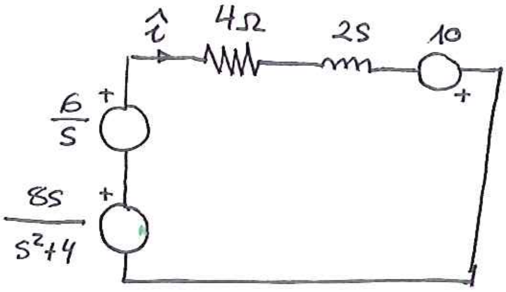
$$v(t) = \mathrm{Im}\left\{ \underline{\hat{V}}\, e^{j\omega t} \right\}$$

$$\underline{\hat{V}} = \hat{v}\, e^{j\theta} = \hat{V}_r + j\hat{V}_x \Rightarrow$$

$$\Rightarrow \hat{V}(s) = \int_0^{\infty} \mathrm{Im}\left\{ \underline{\hat{V}}\, e^{j\omega t} \right\} e^{-st}\, dt$$

$$= \int_0^{\infty} \mathrm{Im}\left\{ \underline{\hat{V}}\, e^{(j\omega - s)t} \right\} dt = \mathrm{Im}\left[\underline{\hat{V}}\, \frac{e^{(j\omega - s)t}}{j\omega - s} \right]_0^{\infty} = \mathrm{Im}\left\{ \underline{\hat{V}}\, \frac{s + j\omega}{s^2 + \omega^2} \right\} =$$

$$= \mathrm{Im}\left\{ \frac{(\hat{V}_r + j\hat{V}_x)(s + j\omega)}{s^2 + \omega^2} \right\} = \boxed{\frac{\omega \hat{V}_r + \hat{V}_x \cdot s}{s^2 + \omega^2}}$$



Superposición (no hace falta)

$$\hat{i} = \hat{i}_1 + \hat{i}_2 + \hat{i}_3 = \frac{1'5}{s} + \frac{2'5}{s+2} + \frac{s+2}{s^2+4}$$

$$\hat{i}_1 = \frac{6/s}{4+2s} = \frac{1'5}{s+2}$$

$$\hat{i}_2 = \frac{8s/(s^2+4)}{4+2s} = \frac{s+2}{s^2+4} - \frac{1}{s+2}$$

$$\hat{i}_3 = \frac{10}{4+2s} = \frac{5}{s+2}$$

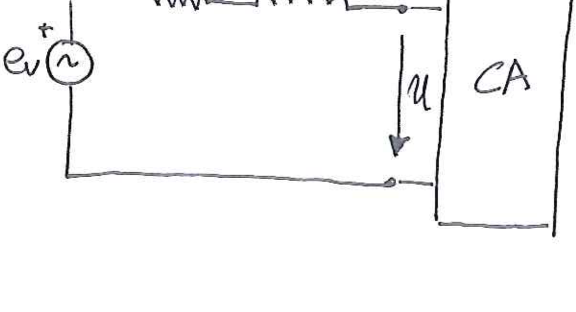
$$V(s) = \hat{V}_r \ldots$$

$$i(t) = 1'5 + 2'5\, e^{-2t} + \mathrm{Im}\left\{ \sqrt{2}\, e^{j\frac{\pi}{4}}\, e^{j2t} \right\}$$

$$\omega = \ldots$$

$$\underline{\hat{V}} = 1 + j \ldots$$

TRANSITORIO DE CORTOCIRCUITO. pág 164



$$e = Ri + Li' + u \begin{cases} e = \mathrm{Im}\{\underline{\hat{E}}\, e^{j\omega t}\}; \ \underline{\hat{E}} = \hat{E} \ldots \\ u = \mathrm{Im}\{\underline{\hat{U}}\, e^{j\omega t}\}; \ \underline{\hat{U}} = \ldots \\ i = \mathrm{Im}\{\underline{\hat{I}}\, e^{j\omega t}\}; \ \underline{\hat{I}} = \ldots \end{cases}$$

$\underline{Z} = R + j\omega L = R + jX = Z e^{j\varphi} \quad \sigma = \frac{R}{L}$

$\underline{S} = P + jQ$

$\underline{\hat{E}} = \underline{\nu}\,\underline{\hat{U}} \qquad \underline{\nu} = \nu e^{j\delta}$ • Demostrar que: $\underline{\nu} = 1 + \frac{2(R+jX}{\hat{U}^2}$

- - - - - - -

$$\left.\begin{array}{l}\underline{\hat{E}} = \underline{Z}\,\underline{\hat{I}} + \underline{\hat{U}} \\ \underline{S} = \underline{U}\,\underline{I}^* \\ \Downarrow \\ \underline{S} = \frac{1}{2}\underline{\hat{U}}\,\underline{\hat{I}}^*\end{array}\right\} \underline{\hat{E}} = \underline{Z}\left(\frac{2\underline{S}}{\underline{\hat{U}}}\right)^* + \underline{\hat{U}} = \underline{Z}\,\frac{2\underline{\hat{S}}}{\underline{U}^*} + \underline{\hat{U}} = \underline{Z}\,\frac{2\cdot\underline{S}^*}{\hat{U}^2}\underline{\hat{U}} + $$

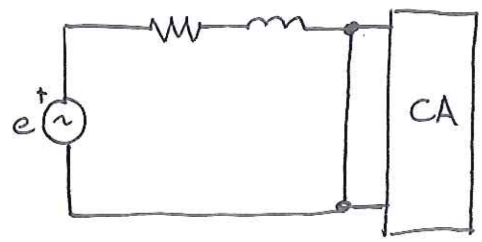
$$= \hat{U}\underbrace{\left(1 + \frac{2\underline{S}^*}{\hat{U}^2}\underline{Z}\right)}_{\underline{\nu}} \Rightarrow$$

$$\Rightarrow \boxed{\underline{\nu} = 1 + \frac{2(P - jQ)(R + jX)}{\hat{U}^2}}$$

En vacío $P = Q = 0 \Rightarrow \underline{\nu}$

Datos del problema: $R, L, \omega, \hat{U}, \beta, P, Q$

$$\left.\begin{array}{l}\underline{\hat{U}} = \hat{U} e^{j\beta} \\ \underline{\nu}\end{array}\right\} \underline{\hat{E}} = \underline{\nu}\,\underline{\hat{U}} \left\{\begin{array}{l}\boxed{\hat{E} = \nu\hat{U}} \\ \boxed{\theta_e = \delta + \beta}\end{array}\right.$$

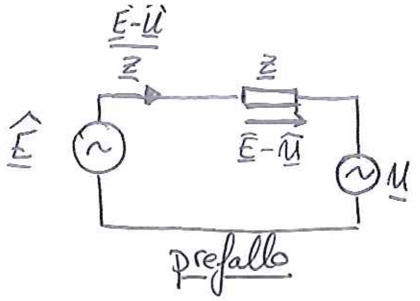


$$\left\{\begin{array}{l} e = Ri + Li' + 0 \\ i(0)\end{array}\right\} \Rightarrow i(t) = [i(0) - i \quad t)$$

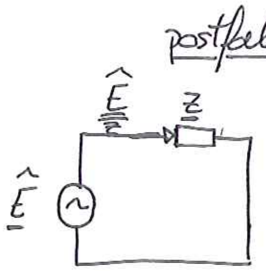
Prefallo (antes del corto):

$e = Ri + Li' + u \quad \rightarrow \mathrm{Im}\{\underline{\hat{E}} e^{j\omega t}\}$

$i(t) = \mathrm{Im}\left\{\left(\frac{\underline{\hat{E}} - \underline{\hat{U}}}{\underline{Z}}\right) e^{j\omega t}\right\}$ $\quad \boxed{i(0) = \mathrm{Im}\left\{\left(\frac{\underline{\hat{E}} - \underline{\hat{U}}}{\underline{Z}}\right)\right.}$



$$i(t) = [i(0) - i_p(0)]e^{-\gamma t} + i_p(t)$$



$$i_p(t) = \mathrm{Im}\left\{\frac{\hat{E}}{Z}e^{j\omega t}\right\} \qquad i_p(0) = \mathrm{Im}\left\{\frac{\hat{E}}{Z}\right\} \Rightarrow$$

$$\Rightarrow i(0) - i_p(0) = \left\{-\frac{\hat{U}}{Z}\right\} \Rightarrow$$

$$\Rightarrow \boxed{i(t) = \mathrm{Im}\left\{-\frac{\hat{U}}{Z}e^{-\frac{R}{X}\omega t}\right\} + \mathrm{Im}\left\{\frac{\hat{E}}{Z}e^{j\omega t}\right\}}$$

Ahora con senos

$$i(t) = -\underbrace{\frac{\hat{U}}{Z}}_{\frac{\hat{I}_s}{\nu}}\mathrm{Sen}(\beta-\varphi)e^{-\frac{R}{X}\omega t} + \underbrace{\frac{\hat{E}}{Z}}_{\hat{I}_s = \frac{\hat{E}}{Z}}\mathrm{Sen}(\omega t + \theta_e - \varphi)$$

$$\boxed{\frac{i(t)}{\hat{I}_s} = -\frac{1}{\nu}\mathrm{Sen}(\beta-\varphi)e^{-\frac{R}{X}\omega t} + \mathrm{Sen}(\omega t + \delta + \beta - \varphi)}$$

$$i(t) = \underbrace{i_t(t)}_{\text{transitorio}} + i_p(t)$$

$$\left.\begin{array}{l}\beta-\varphi=0\\ \beta-\varphi=\pi\end{array}\right\} \Rightarrow \boxed{i_t(0)=0}$$

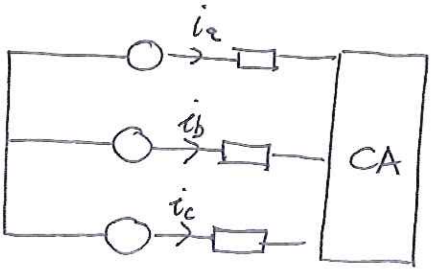
$$\varphi = \frac{\pi}{2} \rightarrow \beta = \pm\frac{\pi}{2} \Rightarrow i_t(0) = 0$$

$$u(t) = \hat{U}\,\mathrm{Sen}(\omega t + \beta)$$

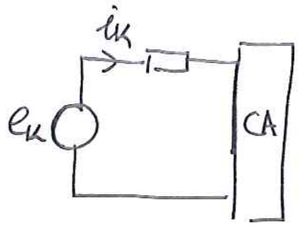
El caso más desfavorable (I_{max})

$i_t(0) = +.$

* Ahora un cortocircuito trifásico

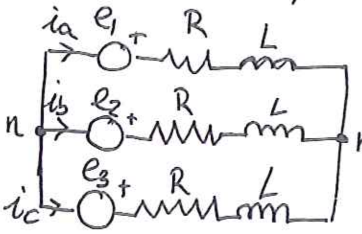


Prefallo en carga monofásico



$k = 1, 2, 3$
a, b, c

Cortocircuito trifásico



1^{er} orden

Vamos a llegar a la ec. dif

$$\begin{cases} e_a = R i_a + L i_a' + u_{n'n} \\ e_b = R i_b + L i_b' + u_{n'n} \\ e_c = R i_c + L i_c' + u_{n'n} \end{cases}$$

Sumando: $e_a + e_b + e_c = 0 = R(i_a + i_b \ldots$

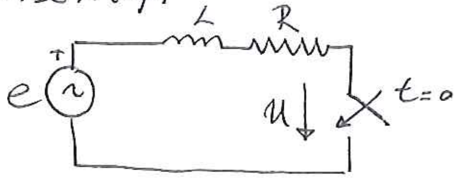
$+ 3 u_{n'n} \Rightarrow u_{n'n} = 0$

$$\Rightarrow \begin{cases} e_a = R i_a + L i_a' \\ e_b = R i_b + L i_b' \\ e_c = R i_c + L i_c' \end{cases} \rightarrow 3$$

* En la pág 167, las ecs 429 y 430 muestran las soluciones.

El problema monofásico, más sencillo: Prefallo en vacío.

FASE A (trafo)



$i(0) = 0$

$e(t) = \hat{E} \operatorname{Sen}(\omega t + \beta)$

$\omega = 100\pi \ \frac{rad}{s}$

$\hat{E} = 230 \cdot \sqrt{2}\ V$

$Z = 8 m\Omega = \sqrt{R^2 + X^2}$ $\quad X = \omega L$

$t < 0 \Rightarrow$

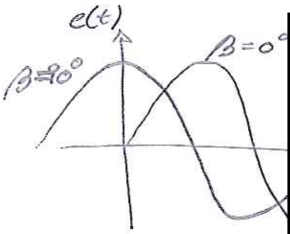
Vamos a comprobar lo de Z: $u_{cc} = Z_{cc} I_{2n} = 0'05 \cdot 230 V \Rightarrow I_{2n} = \frac{0'05 \cdot 230}{8 \cdot 10^{-3}}$

$S_n = 3 \cdot 230 \cdot 1'4375 = 991'875\ kVA \approx 1 MVA$ (como queríamos)

$tg\,\varphi = \frac{X}{R}$

Caso a: $\frac{R}{X} = 0 \Rightarrow \varphi = 90^\circ$

Caso b: $\frac{R}{X} = 0'2 \Rightarrow \varphi = 78'69^\circ$



$$i(t) = \left[i(0) - i_p(0)\right] e^{-\sigma t} + i_p(t) \qquad \overset{\text{simétrica}}{i_p(t)} = i_s(t) \Rightarrow$$

$$\Rightarrow i(t) = \underbrace{\left[\underset{0}{i(0)} - i_s(0)\right] e^{-\sigma t}}_{\substack{i_t(t) \\ \text{(transitoria)}}} + i_s(t) \qquad i(t) = i_s(t) + i_t(t) \Rightarrow i(0) = 0 =$$

$$\Rightarrow i_t(0) = -i_s(0)$$

$$\sigma = \frac{R}{L} = \frac{R}{\omega L}\omega = \frac{R}{X}\omega \Rightarrow i(t) = -i_s(0)\, e^{-\frac{R}{X}\omega t} + i_s(t)$$

$$i_s(t) = \hat{I}_s \operatorname{Sen}(\omega t + \phi) = \operatorname{Im}\left\{\underline{\hat{I}_s}\, e^{j\omega t}\right\}; \ \underline{\hat{I}_s} = \hat{I}_s e^{j\phi}$$

$$e(t) = \hat{E} \operatorname{Sen}(\omega t + \beta) = \operatorname{Im}\left\{\underline{\hat{E}}\, e^{j\omega t}\right\}; \ \underline{\hat{E}} = \hat{E} e^{j\beta} \quad \Rightarrow \underline{\hat{I}_s} = \frac{\hat{E}}{R + jX}$$

$$\Rightarrow \hat{I}_s = \frac{\hat{E}}{Z} \ \text{ y } \ \phi = \beta - \varphi \qquad \hat{I}_s = \frac{230\sqrt{2}\,V}{8\,m V} = 40'6586\ kA.$$

$$i_s(t) = \hat{I}_s \operatorname{Sen}(\omega t + \beta - \varphi) = \frac{\hat{E}}{Z} \operatorname{Sen}(\omega t + \beta - \varphi) \Rightarrow$$

$$\Rightarrow i(t) = \underbrace{-\hat{I}_s \operatorname{Sen}(\beta - \underset{-\frac{2\pi}{3}\ b}{\varphi})\, e^{-\frac{R}{X}\omega t}}_{i_t(t)} + \underbrace{\hat{I}_s \operatorname{Sen}(\omega t + \underset{-2\pi/3 \text{ para } b}{\beta} - \varphi)}_{i_s(t)}$$

Si $\begin{cases} \beta - \varphi = 0 \\ \beta - \varphi = \pm\pi \end{cases} \Rightarrow i_t(t) = 0 \ \forall t \Rightarrow$ Entra directamente en rég permane

Caso a: $\varphi = \frac{\pi}{2}$ $\Rightarrow \beta = \frac{\pi}{2} \Rightarrow i(t) = \hat{I}_s \operatorname{Sen}\omega t$ Situación más favora

$\Rightarrow \beta = 0 \Rightarrow i(t) = \hat{I}_s\left[1 + \operatorname{Sen}\left(\omega t - \frac{\pi}{2}\right)\right] = \hat{I}_s(1 - \operatorname{Cos}\omega t)$ S

$i\left(t = \frac{\pi}{\omega}\right) = 2\hat{I}_s \rightarrow$ Valor de cresta: $2\hat{I}_s$

Caso b: $\varphi \neq 90^\circ$

$$i_t(0) = -\hat{I}_s \operatorname{Sen}(\beta - \varphi) \quad \beta - \varphi = \pm\pi/2 \Rightarrow i_t(0) = \mp\hat{I}_s \Rightarrow$$

$$\rightarrow i(t) = \mp\hat{I}_s\, e^{-\frac{R}{X}\omega t} + \hat{I}_s \operatorname{Sen}\left(\omega t \pm \frac{\pi}{2}\right)$$

Calcular la cresta ya es más complicado. De forma aproximada:

$$t_p \approx \frac{\pi}{\omega} \Rightarrow i(t_p) = \hat{I}_s e^{-\frac{R}{X}\pi} + \hat{I}_s \operatorname{Sen}\left(\pi - \frac{\pi}{2}\right) = \hat{I}_s \left[1 + e^{-\frac{R}{X}\pi}\right] \approx V$$ ... pico

aprox

$$I_p = \hat{I}_s = \left(1 + e^{-d2\pi}\right) = 1'5335$$

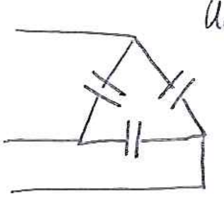
Las condiciones de máximo se dan para $\beta = 0$ (Demostrado en el ...

$$\Rightarrow i(t) = \hat{I}_s \operatorname{Sen}\varphi\, e^{-\frac{R}{X}\omega t} + \hat{I}_s \operatorname{Sen}(\omega t - \varphi) \qquad t_p \Rightarrow \left.\frac{di(t)}{dt}\right|_{t=t_p} = ...$$

Está en la pág 166. (TABLE 1).

Sistemas de segundo orden u orden superior.

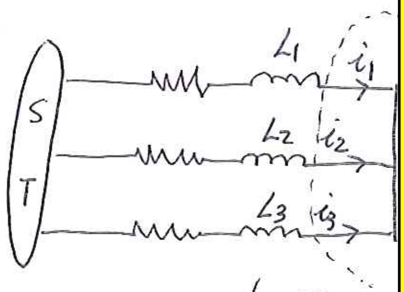
Cuando hay 2 elementos almacenadores de energía q... un lazo capacitivo ni conjunto de corte inductivo



$U_1 + U_2 + U_3 = 0$

Hay una combinación lineal

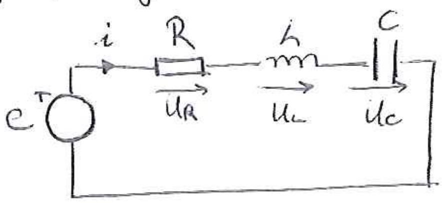
2º orden



2º orden

n Condensadores en serie o paralelo que se puedan agrup... equivalente son primer orden.

- Ejemplo segundo orden:



$$e = Ri + Li' + U_c \qquad i = ...$$

$$\Rightarrow e' = Ri' + Li'' + U_c' = Ri' + ...$$

$$\Rightarrow i'' + \frac{R}{L} i + \frac{1}{LC} i = \frac{e'}{L} \Rightarrow \boxed{X'' + 2\xi\omega_0 X' + \omega_0^2 X = g(t)}$$

$X = X(t)$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

↳ pulsación natural

$$2\xi\omega_0 = \frac{R}{L} \Rightarrow \xi = \frac{R}{L}\cdot\frac{1}{2\omega_0} = \frac{1}{2}R\sqrt{\frac{C}{L}} = \xi$$

↳ c... ...miento.

$$X(t) = X_t(t) + X_p(t) \qquad X_t(t) = \begin{cases} K_1 e^{r_1 t} + K_2 e^{r_2 t} & (\xi > 1) \\ (K_1 + K_2 t)e^{rt} & (\xi = 1) \\ \underline{K} e^{\underline{r}t} + \underline{K}^* e^{\underline{r}^* t} & (\xi < 1) \end{cases}$$

Sob...

$$r^2 + 2\xi\omega_0 r + \omega_0^2 = 0 \Rightarrow r = \frac{-2\xi\omega_0 \pm \sqrt{(2\xi\omega_0)^2 - 4\omega_0^2}}{2} = -\xi\omega_0 \pm$$

$$\xi > 1 \Rightarrow \begin{cases} r_1 = -\xi\omega_0 + \omega_0\sqrt{\xi^2 - 1} \\ r_2 = -\xi\omega_0 - \omega_0\sqrt{\xi^2 - 1} \end{cases}$$

$$\xi < 1 \Rightarrow \underline{r} = -\xi\omega_0 + j\omega_0\sqrt{1-\xi^2} = \alpha + j\beta \quad \text{con} \begin{cases} \alpha = -\xi\omega_0 \\ \beta = \omega_0\sqrt{1-\xi^2} \end{cases}$$

$$X(t) = X_t(t) + X_p(t) \Rightarrow X(0) = X_t(0) + X_p(0)$$

$$X'(t) = X_t'(t) + X_p'(t) \Rightarrow X'(0) = X_t'(0) + X_p'(0)$$

$$X_t(0) = X(0) - X_p(0) = K_1 + K_2 \Rightarrow r_2 K_1 + r_2 K_2 = r_2\left[X(0) - X_p(0)\right]$$

$$X_t'(0) = X'(0) - X_p'(0) = r_1 K_1 + r_2 K_2 = \left(X'(0) - X_p'(0)\right)$$

$$X_t' = r_1 K_1 e^{r_1 t} + r_2 K_2 e^{r_2 t} \quad (\xi > 0)$$

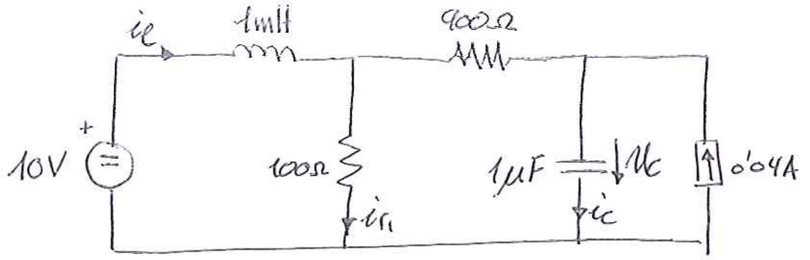
Por tanto $\boxed{K_1 = \frac{r_2\left[X(0) - X_p(0)\right] - \left[X'(0) - X_p'(0)\right]}{r_2 - r_1}}$ pág 15

$\xi > 1$

y $\boxed{K_2 = \frac{r_1\left[X_p(0) - X(0)\right] + X'(0) - X_p'(0)}{r_2 - r_1}}$

$$\boxed{X(t) = K_1 e^{r_1 t} + K_2 e^{r_2 t} + X_p(t)}$$

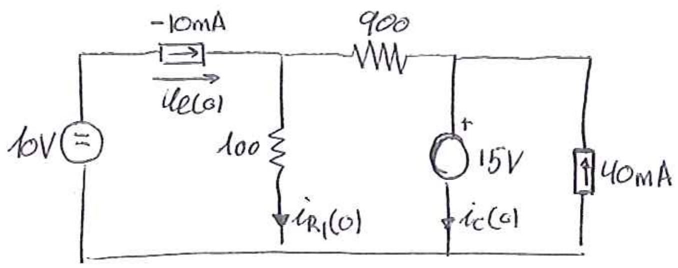
/// Problema

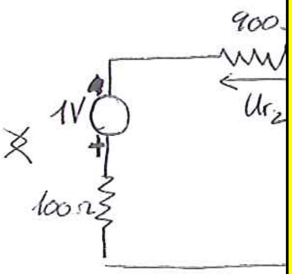


$i_\ell(0) = -$ ...

$u_c(0) = 15$ ...

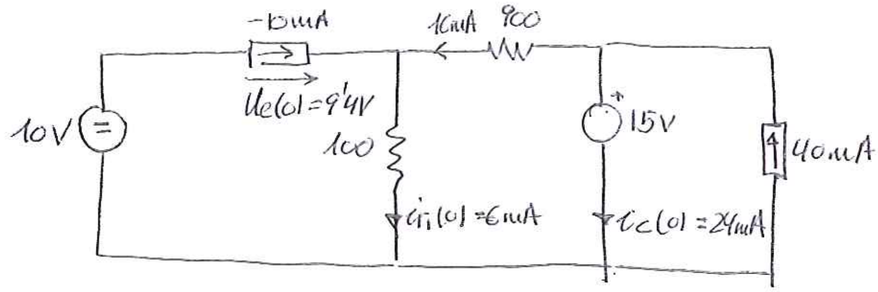
$t=0)$



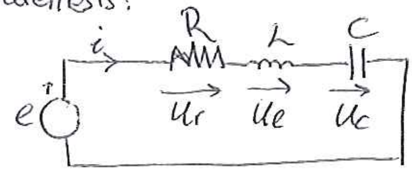


$$u_{r_2} = \frac{900}{100+900}(15+1) = \frac{9}{10}\cdot 16 = 14'4V$$

Nos queda:



* Paréntesis:



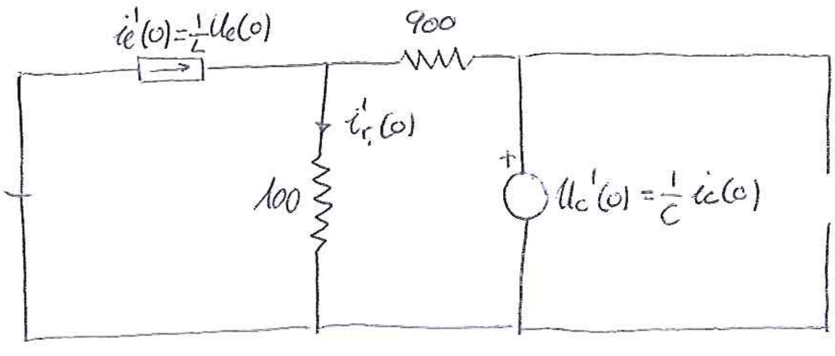
$$e = u_r + u_\ell + u_c = Ri + Li' + u_c$$

$$e' = u_r' + u_\ell' + u_c' = Ri' + Li'' + u_c'$$

$u_r' = Ri'$

$$\left\{\begin{array}{l} u_\ell = Li' \rightarrow u_\ell(0) = Li'(0) \\ i = Cu_c' \rightarrow i(0) = Cu_c'(0) \end{array}\right. \Rightarrow \left\{\begin{array}{l} i'(0) = \frac{1}{L}u_\ell(0) \\ u_c'(0) = \frac{1}{C}i(0) \end{array}\right.$$

* Fin paréntesis



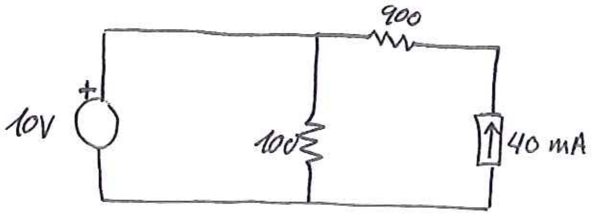
Circuito ...

$$i_L'(0) = \frac{1}{10^{-3}}\, 9'4 = 9400 \text{ A/s} \qquad u_c'(0) = \frac{1}{10^{-6}}\, 24\cdot 10^{-3} = 24000\ \frac{V}{s}$$

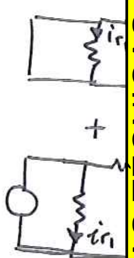
Aplicamos superposición

$$i_{r_1}'(0) = \frac{900}{100+900}\, 9400 + \frac{24000}{900+100} = \underline{\underline{8484 \text{ A/s}}}$$

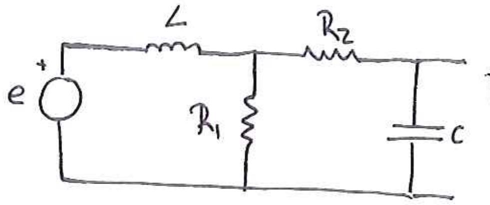
$t = \infty$)



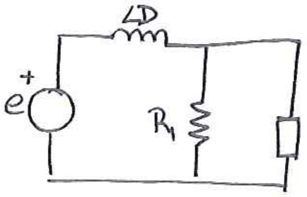
Superposición:



$$i_{r_1}(\infty) = 0 + 0'1\,A = \underline{\underline{0'1\,A}}$$



↳ la quita porque quiere.
(le da igual por superposición)



$$Z_2 = R_2 + \frac{1}{CD} = \frac{1+R_2CD}{CD} \qquad e = Zi$$

$$Z = LD + \frac{R_1Z_2}{R_1+Z_2} = LD + \frac{R_1\,\frac{1+R_2CD}{CD}}{R_1 + \frac{1+R_2CD}{CD}} = LD + \frac{R_1(1+R_2CD)}{1+(R_1+R_2)CD}$$

$$= \frac{LD\left[1+(R_1+R_2)CD\right] + R_1(1+R_2CD)}{1+(R_1+R_2)CD} = \frac{(R_1+R_2)LCD^2 + \left[L+R_1R_2C\right]D + R_1}{1+(R_1+R_2)CD}$$

$$\Rightarrow e\cdot d = n\cdot i \Rightarrow \underbrace{\left[1+(R_1+R_2)CD\right]}_{g(t)} e = \left[\underbrace{(R_1+R_2)LD^2}_{a_2} + \underbrace{(L+R_1R_2C)}_{a_1}D + \underbrace{R_1}_{a_0}\right]\cdot$$

Solución: $a_2 r^2 + a_1 r + a_0 \Rightarrow D^2 + \dfrac{(L+R_1R_2C)}{(R_1+R_2)LC} D + \dfrac{R_1}{(R_1+R_2)LC} \Rightarrow \; \omega_0^2 =$

$$\Rightarrow \omega_0^2 = \frac{R_1}{(R_1+R_2)LC} = 10^8 \Rightarrow \underline{\underline{\omega_0 = 10^4}} \ \frac{rad}{s}$$

$$2\xi\omega_0 = \frac{(L+R_1R_2C)}{(R_1+R_2)LC} \Rightarrow \underline{\underline{\xi = 4'55}} > 1 \Rightarrow \text{Sobreamortiguad}$$

Por tanto:

$$\boxed{i_{r_1}(t) = K_1 e^{r_1 t} + K_2 e^{r_2 t} + i_{r_1}(\infty)}$$

$$r_1 = -\omega_0\left[\xi - \sqrt{\xi^2-1}\right] = -10^4\left[4'55 - \sqrt{4'55^2-1}\right] = -1112'5 \ s^{-1}$$

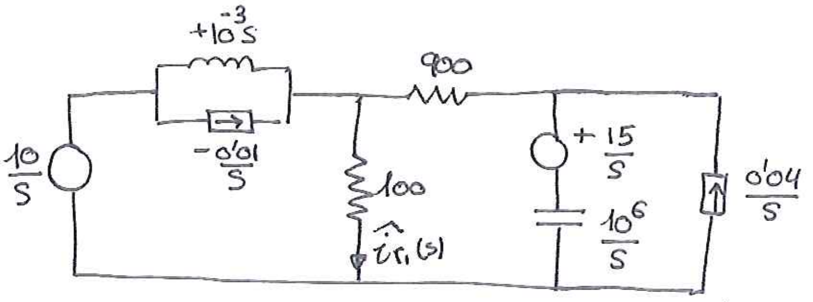
$$r_2 = -\omega_0\left[\xi + \sqrt{\xi^2-1}\right] = -89887'5 \ s^{-1}$$

$$\boxed{i'_{r_1}(t) = K_1 r_1 e^{r_1 t} + K_2 r_2 e^{r_2 t} + 0}$$

$$\left.\begin{array}{l} i_{r_1}(0) = 6\cdot 10^{-3} = K_1 + K_2 + 0'1 \\ i'_{r_1}(0) = 8484 = K_1\cdot(-1112'5) + K_2(-89887'5) \end{array}\right\} \Rightarrow \left\{\begin{array}{l}\text{Podemos resolve[...] la} \\ \text{fórmula d[...]o}\end{array}\right.$$

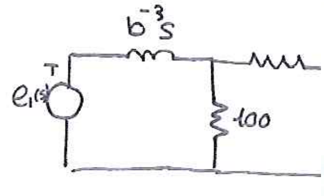
$$\Rightarrow \left|\begin{array}{l} K_1 = 3'8947\cdot 10^{-4} \\ K_2 = -0'094389 \end{array}\right. \Rightarrow \boxed{i_{r_1}(t) = 3'9\cdot 10^{-4} e^{-1112'5 t} - 0'094\cdot e^{-89[...]}}$$

Por Laplace habría que empezar así:



Despejar $\hat{i}_{r_1}$

$\approx$



// Problema

• Caso $\xi < 1$ → Aparecen complejos → $\underline{r} = -\xi\omega_0 + j\omega_0\sqrt{1-}$ ...

SUBAMORTIGUADO

$$\begin{cases} \alpha = - \ldots \\ \beta = \omega \ldots \end{cases}$$

$$X(t) = \underline{K}\, e^{\underline{r}t} + \underline{K}^* e^{\underline{r}^* t} + X_p(t) \qquad \underline{K} = K_r + jK_x = K e^{j\theta_K}$$

$$X(0) = \underline{K} + \underline{K}^* + X_p(0)$$

$$X'(t) = \underline{K}\,\underline{r}\, e^{\underline{r}t} + \underline{K}^*\underline{r}^* e^{\underline{r}^*t} + X'_p(t)$$

$$X'(0) = \underline{K}\,\underline{r} + \underline{K}^*\underline{r}^* + X'_p(0)$$

Se llega a: $\underline{K} = \dfrac{\underline{r}^*[X(0) \ldots - X'_p(0)}{\ldots}$

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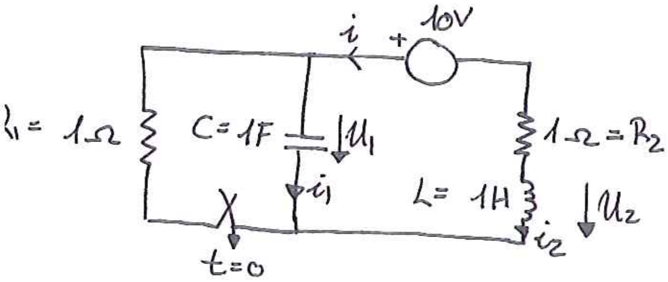
Que es: $\underline{K} = \dfrac{X(0)-X_p(0)}{2} + j\,\dfrac{\alpha[X \ldots) + X_p'(0}{\ldots}$

Empieza el ejercicio:



$$e = Z \cdot i$$

$$Z = R_2 + LD + \frac{R_1/CD}{R_1 + \frac{1}{CD}} = R \ldots$$

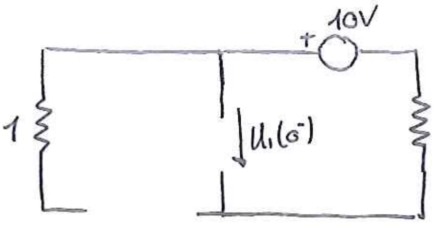
$$= \frac{(R_2+LD)(1+R_1CD) + \ldots}{1+R_1CD}$$

$$e = \frac{n}{d}\, i \Rightarrow ed = g(t) = n i \Rightarrow$$

$$\Rightarrow \frac{1}{R_1CL}\, g(t) = \left[D^2 + \frac{(L+R_1R_2C)}{LR_1C} D + \frac{R_1+R_2}{LR_1C}\right] i \Rightarrow \begin{cases} \omega_0^2 = \dfrac{R_1+ \ldots}{LR \ldots} \ldots \sqrt{2} \\ 2\xi\omega_0 = \dfrac{L+R \ldots}{LR \ldots} \ldots \dfrac{1}{\sqrt{2}} \end{cases}$$

Por tanto: $\begin{cases} \alpha = -\xi\omega_0 = -1 \\ \beta = \omega_0\sqrt{1-\xi^2} = 1 \end{cases} \Rightarrow \underline{r} = -1 + j$

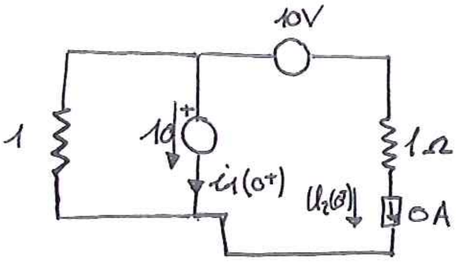
• $t=0^-$



$\boxed{u_1(0^-) = 10V}$

$\boxed{i_2(0^-) = 0\,A}$

• $t=0^+$



0A → Es un circuito abierto

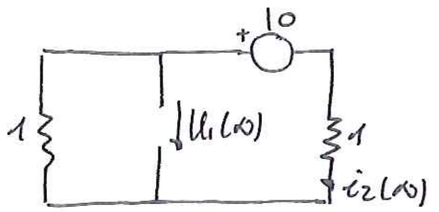
$\boxed{i_1(0^+) = -\frac{10}{1} = -10A}$

$\boxed{u_2(0^+) = 0V}$

$i_1(0^+) = C u_1'(0^+) \Rightarrow \boxed{u_1'(0}$

$u_2(0^+) = L\, i_2'(0^+) \Rightarrow \boxed{i_2'(0^+) =}$

• $t=\infty$



$\boxed{i_2(\infty) = \frac{-10}{2} = -5A}$

$\boxed{u_1(\infty) = \frac{1}{1+1}\cdot 10 = 5V}$

• SOLUCIÓN

$$u_1(t) = \underline{K}e^{rt} + \underline{K}^* e^{r^* t} + u_1(\infty) = 2\,\mathrm{Re}\{\underline{K}\,e^{rt}\} + u_1(\infty)$$

$$\underline{K} = \frac{X(0)-X_p(0)}{2} + j\,\frac{\alpha[X(0)-X_p(0)] - X'(0) + X_p'(0)}{2\beta} = \frac{10-5}{2} + j\,\frac{-1(10}{}\quad 1+$$

↑ pág 160

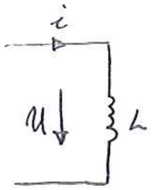
$$= \frac{5}{2}\sqrt{2}\,e^{j\frac{\pi}{4}} = \frac{5}{\sqrt{2}}\,e^{j\pi/4}$$

$$\underline{K}e^{rt} = \frac{5}{\sqrt{2}}e^{j\frac{\pi}{4}}e^{(-1+j)t} = e^{-t}\frac{5}{\sqrt{2}}\,e^{j(t+\frac{\pi}{4})} \qquad \mathrm{Re}\{\underline{K}e^{rt}\} = e^{-t}\frac{5}{\sqrt{2}}\,($$

$$\Rightarrow \boxed{u_1(t) = 5\sqrt{2}\,e^{-t}\cos\left(t+\frac{\pi}{4}\right) + 5 \quad V}$$

y análogamente: $\boxed{i_2(t) = 5\sqrt{2}\,e^{-t}\cos\left(t-\frac{\pi}{4}\right) - 5 \quad A}$

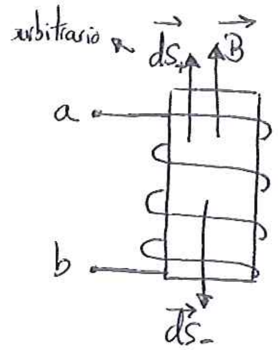
Acoplamientos inductivos sin movimiento.



$$u = L\frac{di}{dt}$$

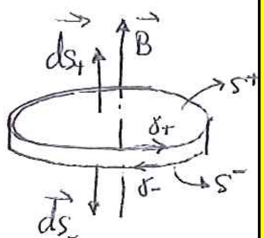
$$u = -L\frac{di}{dt}$$

- $\phi = \iint_S \vec{B}\,\vec{ds}$
- $u = \pm N\dfrac{d\phi}{dt}$ $\qquad \left[\operatorname{rot}\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}\right]$
- $\lambda = \sum_{j=1}^{N} \phi_j \approx N\phi$
  enlaces de flujo
- $N\phi = \pm L i$ $\qquad \left[\operatorname{rot}\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}\right]$



$$\phi_+ = \iint_{S+} \vec{B}\,\vec{ds}_+$$

$$\phi_- = \iint_{S-} \vec{B}\,\vec{ds}_-$$

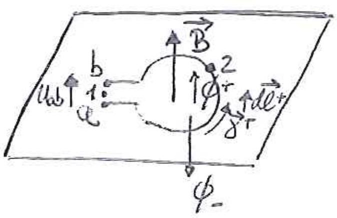


$$\operatorname{div}\vec{B} = 0 \Rightarrow \phi_+ + \phi_- = 0$$

$$\phi_+ = \iint_{S+} \vec{B}\,\vec{dS}_+ = \iint_{S_+} \operatorname{rot}\vec{A}\,\vec{dS}_+ = \oint_{\gamma+} \vec{A}\,\vec{dl}$$

$$\phi_- = \iint_{S-} \vec{B}\,\vec{dS}_- = \iint_{S-} \operatorname{rot}\vec{A}\,\vec{dS}_- = \oint_{\gamma-} \vec{A}\,\vec{dl}$$

Relación tensión-flujo.



$$u_{ab} = \frac{d\phi}{dt} \qquad \operatorname{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\phi = \iint \vec{B}\,\vec{ds} \Rightarrow \frac{\partial \phi}{\partial t} = \iint \frac{\partial \vec{B}}{\partial t}\,\vec{ds} = -\iint \operatorname{rot}\vec{E}\,\vec{ds}$$

$$\Rightarrow \oint_{\gamma} \vec{E}\,\vec{dl} = -\frac{d\phi}{dt}$$

$S_+ \rightarrow \gamma_+$

$S_- \rightarrow \gamma_-$

$$\int_{\gamma_+} \vec{E}\cdot\vec{dl}_+ = -\frac{d\phi}{dt}$$

$\vec{E} = \rho\vec{J} \Rightarrow \rho = 0 \Longleftrightarrow \vec{E} = 0$ (en el cond

$E \neq 0$ entre a y b

$$\Rightarrow \int_{\gamma_+} \vec{E}\,\vec{dl}_+ = \int_{b|a} \vec{E}\cdot\vec{dl} = -\frac{d\phi_+}{dt} = \int_{b|a} \vec{E_e}\,\vec{dl} = U_{ba} = -\frac{d\phi_+}{dt} \Rightarrow$$

↳ electrostático conservativo.

↳ $\vec{E} = 0 = \vec{E_e} + \vec{E_i}$

Fuera del conduc

Además
$$\oint_{\gamma_+} \vec{E_e}\,\vec{dl} = 0 = \underbrace{\int_{a|b} \vec{E_e}\,\vec{dl}_+}_{-U_{ba} \,=\, -\int \vec{E_i}\,\vec{dl}} + \underbrace{\int_{b|a} \vec{E_e}\,\vec{dl}_+}_{U_{ba}} = -\int_{a|b} \vec{E_i}\,\vec{dl}_+ + \underbrace{\int_{b|a} \vec{E_e}\,\vec{dl}}_{U_{ba}} = 0 \Rightarrow \qquad = 0$$

$$\Rightarrow \boxed{U_{ba} = \int \vec{E_i}\,\vec{dl}_+}$$

$$U_{ba} = \int_{b|a} \vec{E_e}\,\vec{dl} = -\frac{d\phi_+}{dt}$$

$$\boxed{U_{ab} = \int_{a|b} \vec{E_e}\,\vec{dl} = +\frac{d\phi_+}{dt}} = \overbrace{-\int_{a|b} \vec{E_i}\,\vec{dl}_+}^{e_{ab}}$$

De otra forma:
$$\oint_{\gamma} \vec{E}\,\vec{dl} = \underbrace{\int_{a|b} \vec{E_e}\,\vec{dl}}_{(1)} + \underbrace{\int_{a|b} \vec{E_i}\,\vec{dl}}_{(2)} + \underbrace{\int_{b|a} \vec{E_e}\,\vec{dl}}_{(3)} + \underbrace{\int_{b|a} \cancel{\vec{E_i}\,\vec{dl}}}_{(4)} \rightarrow 0$$

$$(1) + (3) = 0 \Rightarrow \int_{a|b} \vec{E_i}\,\vec{dl} = -\frac{d\phi}{dt} = e$$

Si $U_{ab} > 0 =$

$$(1) + (2) = 0 \Rightarrow \int_{b|a} \vec{E_e}\,\vec{dl} = -\frac{d\phi}{dt} = U_{ba} = -U_{ab}.$$

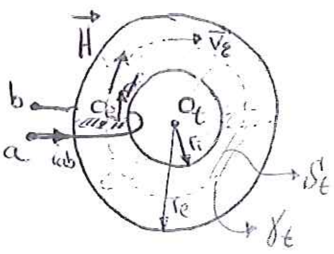
Para N espiras:

$$\oint_\gamma \vec{E}d\vec{l} = -\sum_{j=1}^{N} \frac{d}{dt}\iint_{S_j} \vec{B}\,d\vec{S} = -\frac{d\lambda}{dt} = -N\frac{d\phi}{dt}$$

$$\lambda = \sum_{j=1}^{N} \vec{B}\,d\vec{S} = \sum_{j=1}^{N} \phi_j \qquad \phi_j = \phi \;\; \forall j \Rightarrow \lambda = N\phi$$

Como $u_{ab} = N\dfrac{d\phi}{dt} = \dfrac{d\lambda}{dt} = \displaystyle\sum_{j=1}^{N} \dfrac{d\phi_j}{dt}$

Relación flujo-corriente



$$\text{rot}\,\vec{H} = \vec{J} + \cancel{\frac{\partial \vec{D}}{\partial t}}$$

$$\vec{H} = H_\varepsilon \vec{v}_\varepsilon$$

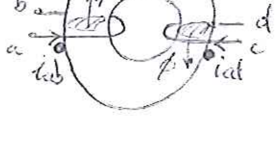
$$i_{ab} = \iint_{S_t} \vec{J}\,d\vec{S}_t = \iint_{S_t} \text{r}\ldots \; d\vec{l}$$

$$= \oint (H_\varepsilon \vec{v}_\varepsilon)(dl\,\vec{v}_\varepsilon) = \int \ldots$$

$$\phi = \iint \vec{B}\,d\vec{S} = \mu \iint \vec{H}\,d\vec{S} \Rightarrow \frac{\phi}{i_{ab}} = \mu \frac{\iint \vec{H}\,d\vec{S}}{\oint \vec{H}\,d\vec{l}_t}$$

Si $\vec{dS}$ y $\vec{H}$ tienen el mismo sentido, la relación entre φ ... .

Ahora colocamos otra bobina:

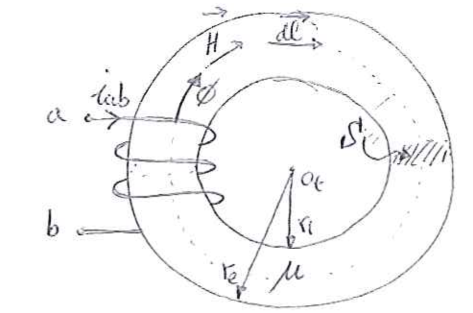


Unos terminales son correspondientes si al
intensidad del mismo signo generan camp
sentido (a y c y b y d son correspondi

$i_{ab} > 0 \Rightarrow \phi > 0$

$i_{cd} > 0 \Rightarrow \phi > 0$.

Ahora con varias espiras:



$i>0 \Rightarrow \phi>0$

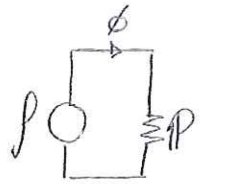
$$l=\pi(r_i+r_e) \qquad S=\pi r_0^2=\pi\left(\frac{r_e-\ldots}{2}\ldots\right.$$

$$S=\frac{\pi}{4}(r_e-r_i)^2 \qquad \phi=\mu\iint\vec{H}\,d\vec{S}$$

$$Ni=\oint\vec{H}\,d\vec{l} \qquad \vec{H}=H_\xi\vec{V}_\xi\,;\quad d\vec{l}=dl_\xi\vec{V}_\xi\,;\quad d\vec{S}=dS_\xi\vec{V}_\xi$$

Por tanto: $Ni=\oint H_\xi\,dl_\xi=H_\xi\, l$

$$\left.\begin{array}{l} Ni=\oint H_\xi\,dl_\xi=H_\xi\, l \\ d\phi=\mu\vec{H}\,d\vec{S}=\mu H_\xi\,dS_\xi \Rightarrow \phi=\mu H_\xi S \end{array}\right\} \qquad \phi=\mu S\frac{Ni}{l} \qquad \phi=\frac{\mu S}{l}Ni$$

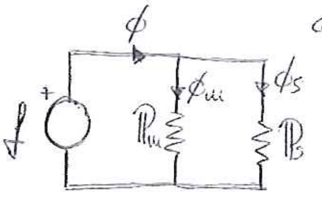


$\Rightarrow \phi=\mathbb{P}\mathcal{F}$

reluctancia: $\mathcal{R}=\dfrac{l}{\mu S}$

$$\vec{H}=\vec{H}_m+\vec{H}_s \Rightarrow \left\{\begin{array}{ll} \phi_m=\mu\int\vec{H}_m\,d\vec{S}=\mathbb{P}_m\mathcal{F} & \text{donde } \mathbb{P}_m=\mu_m\, l_{gm}\ldots \\ \phi_s=\mu_0\int\vec{H}_s\,d\vec{S}_a=\mathbb{P}_s\mathcal{F} & \text{donde } \mathbb{P}_s=\mu_0\, l_{gs} \end{array}\right.$$

$\vec{H}_s$ → por el aire



coef. acoplamiento

$$\left|\begin{array}{l} K_a=\dfrac{\phi_m}{\phi}=\dfrac{\mathbb{P}_m}{\mathbb{P}_m+\mathbb{P}_s} \\ K_s=\dfrac{\phi_s}{\phi}=\dfrac{\mathbb{P}_s}{\mathbb{P}_s+\mathbb{P}_m} \end{array}\right.$$

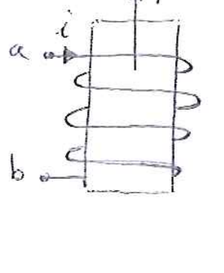
coef. dispersión

$$\phi=(\mathbb{P}_m+\mathbb{P}_s)\mathcal{F}=(\mathbb{P}_m+\mathbb{P}\ldots$$

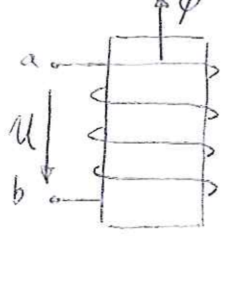
$\Rightarrow$ Multiplicando por N $\Rightarrow$ $N\phi=(\underbrace{\mathbb{P}_m N^2}_{L_m}+\underbrace{\mathbb{P}_s N^2}_{S})i = N\phi=(L_m+\ldots$

inductancia ← L_m

$L=L_m+S$



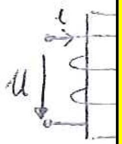
$N\phi=+Li$



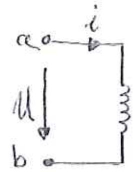
$V_A>V_B$

↳ $u=+N\dfrac{d\phi}{dt}$

$$\begin{cases} N\phi = Li \longrightarrow N\dfrac{d\phi}{dt} = L\dfrac{di}{dt} \\ u = N\dfrac{d\phi}{dt} \end{cases} \Longrightarrow \boxed{u = L\dfrac{di}{dt}}$$



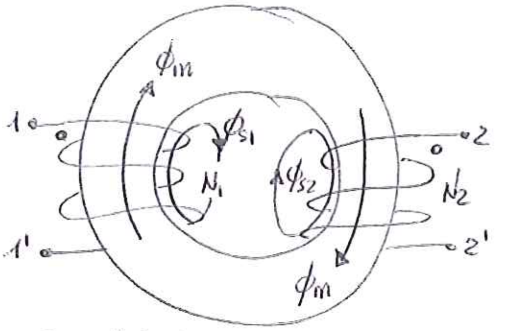
Como $N\phi = Li \iff \phi > 0 \iff i > 0$



$\phi > 0$
$i > 0$

$u = +N\dfrac{d\phi}{dt}$ ...

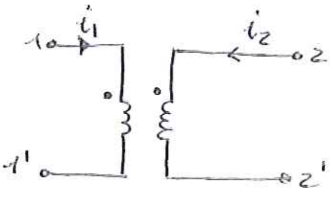
no equivalente a los ... 3D.



$\phi_1 = \phi_{s1} + \phi_m$

$\phi_2 = \phi_{s2} + \phi_m$

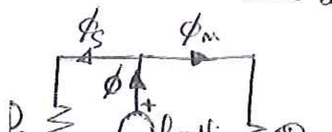
$a_{12} = \dfrac{N_1}{N_2} = \dfrac{1}{a_{21}}$



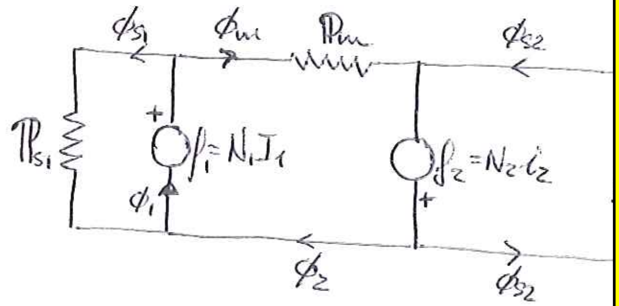
$\phi_1 > 0$ ...

$\phi_2 > 0$ ...

Si sólo había una bobina



Por haber dos bobinas:



$\mathbb{P}_m = \mu \dfrac{S_n}{l_n} = \mu\, l_{em}$

$\mathbb{P}_{s1} = \mu_0\, l_{es1}$

$\mathbb{P}_{s2} = \mu_0\, l_{es2}$

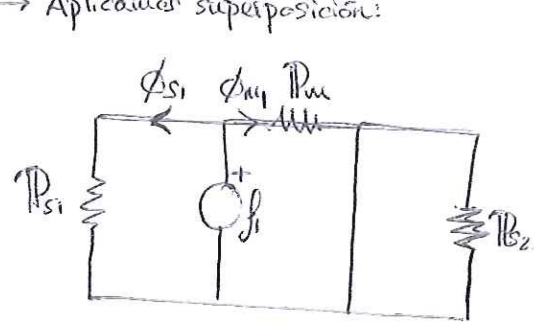
$\phi_{s1} = \mathbb{P}_{s1} f_1 = \mathbb{P}_{s1} N_1 i_1$

$\phi_{s2} = \mathbb{P}_{s2} f_2 = \mathbb{P}_{s2} N_2 i_2$

$\phi_m = \mathbb{P}_m (f_1 + f_2) = \mathbb{P}_m f = \mathbb{P}_m (N$ ...

$\oint \vec{H}\,d\vec{l} = N_1 i_1 + N_2 i_2$

→ Aplicamos superposición:



$\phi_{m1} = \mathbb{P}_m f_1 = \mathbb{P}_m N_1 i_1$

Análogamente

$\phi_{m2} = \mathbb{P}_m f_2 = \mathbb{P}_m N_2 i_2$

$$\rightarrow \phi_1 = \phi_{s1} + \phi_{m} = \underbrace{\phi_{s1} + \phi_{m1}}_{\phi_{11}} + \underbrace{\phi_{m12}}_{\phi_{12}} = \phi_{11} + \phi_{12}$$

↳ visto en 1 creado por corriente 2.

$$\rightarrow \phi_2 = \phi_{s2} + \phi_{m} = \underbrace{\phi_{s2} + \phi_{m2}}_{\phi_{22}} + \underbrace{\phi_{m1}}_{\phi_{21}} = \phi_{22} + \phi_{21}$$

$$K_{a1} = \frac{\phi_{m1}}{\phi_{11}} = \frac{\phi_{m1}}{\phi_{s1}+\phi_{m1}} = \frac{\mathbb{P}_{m}}{\mathbb{P}_{m}+\mathbb{P}_{s1}} \; ; \; K_{a2} = \frac{\phi_{m2}}{\phi_{22}} = \frac{\phi_{m2}}{\phi_{s2}+\phi_{m2}} \; ; \; K_{s1} = \frac{\phi_{s1}}{\phi_{11}} = \frac{\phi_{s}}{\phi_{m1}+} \quad \frac{\phi_{s2}}{\phi_{m2}+\phi}$$

$$K_a = \sqrt{K_{a1} \cdot K_{a2}}$$

$$\Downarrow$$

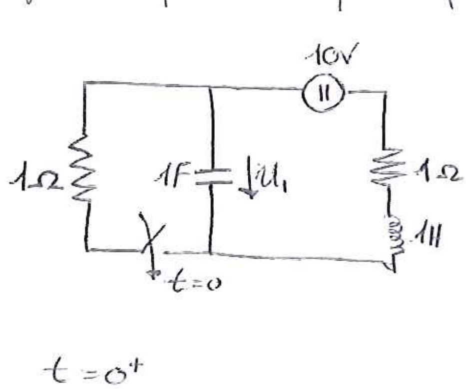
$$\boxed{K_a = \frac{\mathbb{P}_{m}}{\sqrt{(\mathbb{P}_{m}+\mathbb{P}_{s1})(\mathbb{P}_{m}+\mathbb{P}_{s2})}}}$$

Si $K_a = 1 \iff \mathbb{P}_{s1} = \mathbb{P}_{s2} = 0 \Rightarrow$ Acopla...

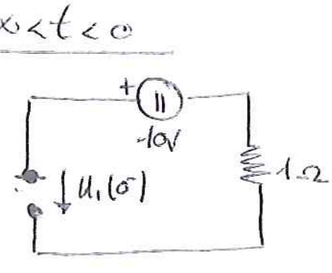
$$\begin{cases} \lambda_1 = N_1\phi_1 = \overbrace{N_1\phi_{s1}}^{\lambda_{s1}} + \underbrace{\overbrace{\phi_{m1}N_1}^{\lambda_{m11}} + \overbrace{N_1\phi_{m2}}^{\lambda_{m12}}}_{\lambda_{m1}} = \overbrace{N_1\phi_{11}}^{\lambda_{11}} + \overbrace{N_1\phi_{12}}^{\lambda_{12}} \\ \quad \cdots \\ \lambda_2 = N_2\phi_2 = \overbrace{N_2\phi_{s2}}^{\lambda_{s2}} + \underbrace{\overbrace{N_2\phi_{m2}}^{\lambda_{m22}} + \overbrace{N_2\phi_{m2}}^{\lambda_{m21}}}_{\lambda_{m2}} = \overbrace{N_2\phi_{22}}^{\lambda_{22}} + \overbrace{N_2\phi_{21}}^{\lambda_{21}} \end{cases}$$

$$\begin{cases} \lambda_{s1} = S_1 i_1 \\ \lambda_{s2} = S_2 i_2 \end{cases}$$

✓ Ejercicio pendiente por Laplace.

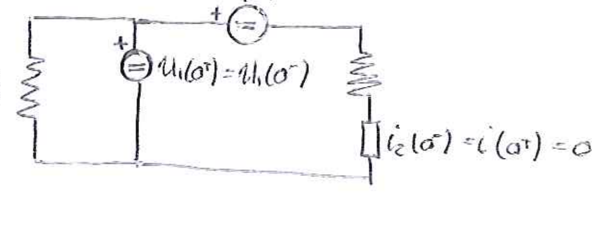


$-\infty < t < 0$



$u_1(0^-) = 1$

$t = 0^+$

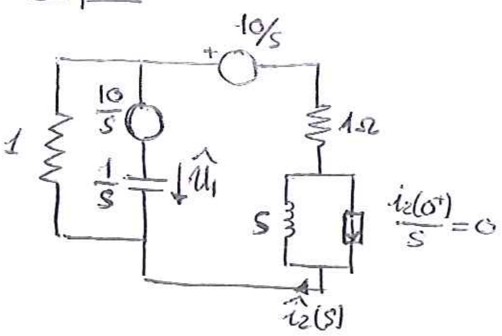


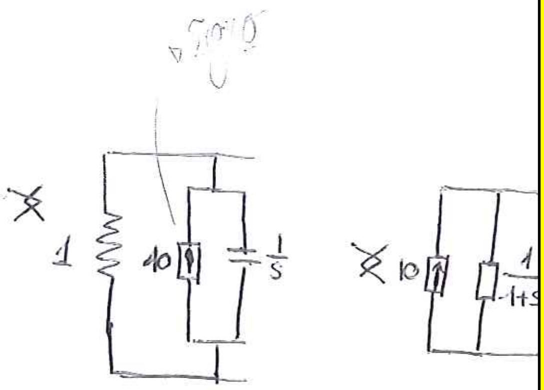
$$i_L(t) = K_1 e^{r_1 t} + K_2 e^{r_2 t} + i_f$$

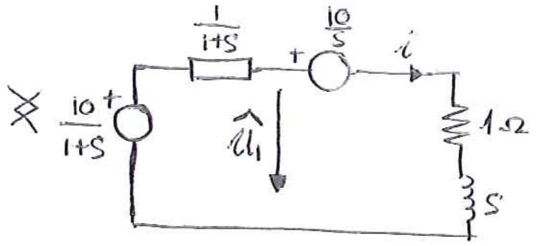
$r_1 = -0'5\, s^{-1}$ $\qquad K_1 = -5/3$

$r_2 = -2\, s^{-1}$ $\qquad K_2 = \frac{20}{3}$

Laplace:







$$\hat{i}_2 = \frac{\frac{10}{1+s}-\frac{10}{s}}{\frac{1}{1+s}+(1+s)} = \frac{-10}{[s^2+2s+2]}$$

$$r^2+2r+2=0 \longrightarrow r=\frac{-2\pm\sqrt{4-8}}{2} = -1\pm 2j\frac{1}{2} = -1\pm j$$

$$\underline{r} = \alpha + j\beta = -1+j \quad \underline{r}^* = -1-j \quad \Rightarrow \hat{i}_2 = \frac{K_0}{s} + \frac{\underline{K}}{s-\underline{r}} + \frac{\underline{K}^*}{s-\underline{r}^*} = \frac{-10}{s(s-\underline{r})}$$

$$K_0 = \hat{i}_2(s)\cdot s\Big|_{s=0} = \frac{-10}{2} = -5$$

$$\underline{K} = i_2(s)(s-\underline{r})\Big|_{s=\underline{r}} = \frac{-10}{\underline{r}(\underline{r}-\underline{r}^*)} = \frac{-10}{(-1+j)\cdot 2j} = \frac{5}{2}-\frac{5}{2}j = \frac{5}{2}\sqrt{2}\,e$$

$$\underline{K}^* = \frac{5}{2}(1+j) = \frac{5}{2}\sqrt{2}\,e^{j\frac{\pi}{4}}$$

Por tanto: $\hat{i}_2(s) = -\frac{5}{s} + \frac{5}{\sqrt{2}}e^{-j\frac{\pi}{4}}\frac{1}{s-\underline{r}} + \frac{5}{\sqrt{2}}e^{j\frac{\pi}{4}}\frac{1}{s-\underline{r}^*} \Rightarrow$

$$\Rightarrow i_2(t) = -5 + \frac{5}{\sqrt{2}}e^{-j\frac{\pi}{4}}e^{\underline{r}t} + \frac{5}{\sqrt{2}}e^{j\frac{\pi}{4}}e^{\underline{r}^*t} = -5 + 2\,\mathrm{Re}\left\{\frac{5}{\sqrt{2}}e^{-j\frac{\pi}{4}}e^{(-1+j)t}\right\} =$$

$$\Rightarrow i_2(t) = -5 + 5\sqrt{2}\,e^{-t}\cos\left(t-\frac{\pi}{4}\right)$$

$i_2(0) = 0$ ✓

otra forma

$$\hat{i}_2(s) = -\frac{10}{s(s^2+2s+2)} = -\frac{5}{s} + \frac{as+b}{s^2+2s+2} = \frac{-5(s^2+2s+2)+as^2+bs}{s(s^2+2s+2)}$$

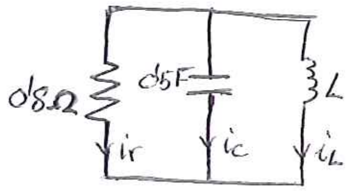
$s^2) \Rightarrow \boxed{5=a}$

$s^1)\ 0=-10+b \Rightarrow \boxed{b=10}$

$$\hat{i}_2(s) = -\frac{5}{s} + 5\,\frac{(s+2)}{s^2+2s+2}$$

Ant

// Problema 11. Junio 2012. Ejercicio 3.



$\xi = 1'25$ $\quad u_c(0)=4V$ $\quad$ a) ¿L?

$i_L(0) = 0A$

$$i_r + i_c + i_L = 0 \qquad \frac{u}{R} + \frac{u}{LD} + CDu = 0 \Rightarrow \frac{u}{R} + \frac{1}{LD}\left[u + LCD^2u\right.$$

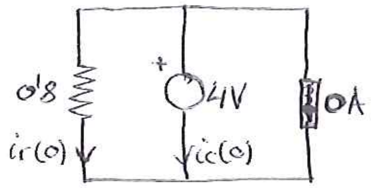
$$\Rightarrow \frac{1}{LD}\left[\frac{LDu}{R} + u + LCD^2u\right] = 0 \Rightarrow \frac{L}{R}u' + u + LCu'' = 0 \Rightarrow$$

$$\Rightarrow \boxed{u'' + \frac{1}{CR}u' + \frac{1}{LC}u = 0}$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow L = 2H$$

$$2\xi\omega_0 = \frac{1}{RC} \Rightarrow \omega_0 = \frac{1}{2\cdot 1'25\cdot 0'8\cdot 0}$$

b) $t=0$

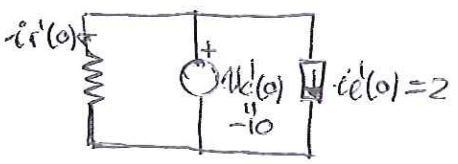


$$i_r(0) = \frac{4}{0'8} = 5A \Rightarrow i_c(0) = -5$$

$$i_c = Cu_c' \Rightarrow i_c(0) = C\cdot u_c'(0) \Rightarrow u_c'(0) = \frac{i_c(0)}{C} = \frac{-5}{0'5} = -10\ V/s$$

$$u_L = Li_L' \Rightarrow u_L(0) = L\cdot i_L'(0) \Rightarrow i_L'(0) = \frac{u_L(0)}{L} = \frac{4}{2} = 2\ A/s$$

→ Circuito derivado



$$\Rightarrow i_r'(0) = \frac{-10}{0'8} = -12'5\ A/s$$

Ejercicio 4 Junio 2012 Continuación y anterior.

$i_{cu} = 0'1 u_u + I_{cu}$ $\qquad I_{C_1} = -35A$; $I_{C_1} = 0'1A$

$i_{Lu} = 0'625 u_u + I_{Lu}$ $\qquad u_1 = 3'095V = u(t = 0'1s)$

4)

$i(0) = 5A$

$i'(0) = -2000\ A/s$

$i(t) = 5\sqrt{5}\cos\left(200t + 63'435^\circ \frac{\pi}{180^\circ}\right) = 5\cos 200t - 10\,\mathrm{Sen}\,200t$

5) Hecho

Solución Julio 2012.

4) $u(0) = 5V$

$u'(0) = -2000\ V/s$ $\qquad u(t) = 5\sqrt{5}\cos\left(200t + 63'435^\circ \frac{\pi}{180^\circ}\right)$

5) Hecho

Solución Enero 2013

4) $\omega_0 = \sqrt{\frac{R_1G_2+1}{LC}}$ $\qquad \xi = \frac{1}{2}\left(\frac{R_1}{L} + \frac{G_2}{C}\right)\sqrt{\frac{LC}{R_1G_2+1}}$ ✓

5) $\underline{i(t)} = 1 + 0'1699e^{-0'289t} - 1'17e^{-3'46167t}$ $\qquad \hat{i}_2 = \frac{4+16 s}{(s+0'289)(s+3'46)1S}$

BOBINAS ACOPLADAS.

• $\phi_1 = \underbrace{\phi_{s1} + \phi_{m11}}_{\phi_{11} \to \lambda_{11}} + \underbrace{\phi_{m12}}_{\phi_{12} \to \lambda_{12}}$ $\qquad$ • $\phi_2 = \underbrace{\phi_{s2} + \phi_{m2}}_{\phi_{22} \to \lambda_{22}} + \underbrace{\phi_{m1}}_{\phi_{21} \to \lambda_{21}}$

$$\begin{cases} \phi_{s1} = \mathbb{P}_{s1} N_1 i_1 \\ \phi_{m1} = \mathbb{P}_m N_1 i_1 \\ \phi_{m2} = \mathbb{P}_m N_2 i_2 \\ \phi_{s2} = \mathbb{P}_{s2} N_2 i_2 \end{cases}$$

Multiplicando por $N_1 \rightarrow \lambda_1 = \lambda_{s1} + \underbrace{\lambda_{m11} + \lambda_{m12}}_{\lambda_{m1}}$

Multiplicando por $N_2 \rightarrow \lambda_2 = \lambda_{s2} + \underbrace{\lambda_{m22} + \lambda_{m21}}_{\lambda_{m2}}$

$$N_1 \phi_{s1} = \left[\mathbb{P}_{s1} N_1^2\right] i_1 = S_1 i_1$$ → inductancia de dispersión.

$$N_2 \phi_{s2} = \left[\mathbb{P}_{s2} N_2^2\right] i_2 = S_2 i_2$$

$$\lambda_{m11} = N_1 \phi_{m1} = \left[\mathbb{P}_m N_1^2\right] i_1 = L_{m1} i_1 \qquad \boxed{L_{m1} = \mathbb{P}_m N_1^2}$$

$$\lambda_{m22} = N_2 \phi_{m2} = \left[\mathbb{P}_m N_2^2\right] i_2 = L_{m2} i_2 \qquad \boxed{L_{m2} = \mathbb{P}_m N_2^2}$$

$$\lambda_{12} = \lambda_{m12} = N_1 \phi_{m2} = \left[\mathbb{P}_m N_1 N_2\right] i_2 = M_{12} i_2 \qquad M_{12} = \mathbb{P}_m N_1 N_2 \rightarrow \text{Inductan}$$

$$\lambda_{21} = \lambda_{m21} = N_2 \phi_{m1} = \left[\mathbb{P}_m N_1 N_2\right] i_1 = M_{21} i_1 \qquad M_{21} = \mathbb{P}_m N_1 N_2 \Rightarrow \boxed{M_{12}}$$

$$\mathbb{P}_m = \frac{L_{m1}}{N_1^2} = \frac{L_{m2}}{N_2^2} = \frac{M}{N_1 N_2} \Rightarrow \boxed{L_{m1} = \left(\frac{N_1}{N_2}\right)^2 L_{m2} = a_{12}^2 L_{m2}} \qquad \boxed{L_{m}}$$

$$\lambda_{11} = N_1 \phi_{11} = L_1 i_1 = \lambda_{s1} + \lambda_{m11} = S_1 i_1 + L_{m1} i_1 \Rightarrow \boxed{L_1 = (S_1 + L_{m1}}$$

$$\lambda_{22} = N_2 \phi_{22} \Rightarrow \boxed{L_2 = S_2 + L_{m2}}$$

$$\boxed{M = \sqrt{L_{m1} L_{m2}}}$$

* Por tanto:

$$\begin{cases} \lambda_1 = L_1 i_1 + M i_2 = S_1 i_1 + N_1 \phi_m \\ \lambda_2 = L_2 i_2 + M i_1 = S_2 i_2 + N_2 \phi_m \end{cases}$$

$$\boxed{S_1 = L_1 - L_{m1} = L_1 - a}$$

Coeficientes de acoplamiento:

$$\left[K_{a1} = \frac{\phi_{m1}}{\phi_{m1} + \phi_{s1}} = \frac{\phi_{m1}}{\phi_{11}} = \frac{L_{m1}}{L_{m1} + S_1} = \frac{L_{m1}}{L_1} \right] \qquad \boxed{K_{a2} = \frac{L_{m2}}{L_2}}$$

$$\underline{\underline{K_a}} = \sqrt{K_{a1} K_{a2}} = \sqrt{\frac{L_{m1} L_{m2}}{L_1 L_2}} = \frac{M}{\sqrt{L_1 L_2}} < 1$$

$$\begin{matrix} K_{a1} + K_{s} \\ K_{a2} + K_{s2} \end{matrix}$$

$$\begin{cases} K_{s1} = \dfrac{\phi_{s1}}{\phi_{11}} = \dfrac{S_1}{L_1} \\ K_{s2} = \dfrac{S_2}{L_2} \end{cases} \qquad \begin{cases} S_1 = K_{s1} L_1 = (1 - K_{a1}) L_1 \\ S_2 = K_{s2} L_2 = (1 - K_{a2}) L_2 \end{cases}$$

$$\phi_m = \mathbb{P}_m\left(N_1 i_1 + N_2 i_2\right) = \mathbb{P}_m\left(N_1 i_{m1} + N_1 i_{r1} + N_2 i_2\right) = \mathbb{P}_m N_1 i_{m1}$$

$i_1 = i_{m1} + i_{r1}$

$N_1 i_{r1} + N_2 i_2 = 0$

$$\lambda_{m1} = N_1 \phi_m = \left(\mathbb{P}_m N_1^2\right) i_{m1} = L_{m1} i_{m1}$$

$$\frac{i_{r1}}{i_2} = -\frac{N_2}{N_1} = -\frac{1}{a_{12}} \Rightarrow \boxed{i_2 = -a_{12} i_{r1}}$$

$i_2 = i_{m2} + i_{r2}$

$N_1 i_1 + N_2 i_{r2} = 0$

$$\Rightarrow \phi_m = \mathbb{P}_m\left(N_1 i_1 + N_2 i_2\right) = \mathbb{P}_m N_2 i_{m2}$$

$$\lambda_{m12} = N_2 \phi_m = \left(\mathbb{P}_m N_2^2\right) i_{m2} = L_{m2} i_{m2}$$

$$\frac{i_1}{i_{r2}} = -\frac{1}{a_{12}} \Rightarrow \boxed{i_{r2} = -a_{12} i_1}$$

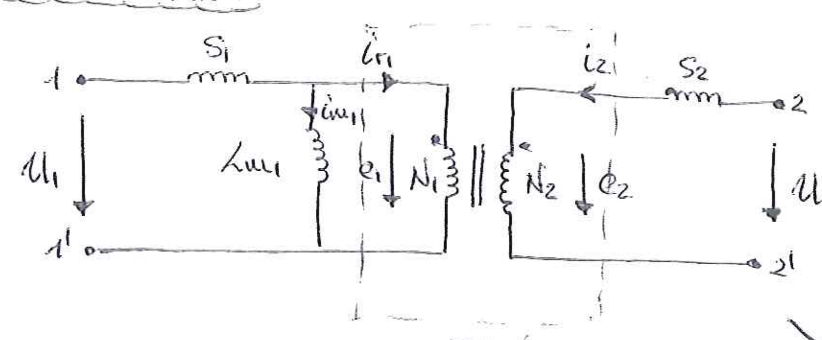
$$\lambda_1 = \lambda_{s1} + \lambda_{m1} \Rightarrow \lambda_1' = u_1 = \lambda_{s1}' + \lambda_{m1}' = S_1 i_1' + \underbrace{L_{m1} i_{m1}'}_{e_1}$$

e_1 → tensión indu[...] por induc[...]

$$\lambda_2 = \lambda_{s2} + \lambda_{m2} \Rightarrow \lambda_2' = u_2 = \lambda_{s2}' + \lambda_{m2}' = S_2 i_2' + \underbrace{L_{m2} i_{m2}'}_{e_2}$$

$$\frac{e_1}{e_2} = \frac{L_{m1} i_{m1}'}{L_{m2} i_{m2}'} = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{N_2}{N_1}\right) = \frac{N_1}{N_2} = a_{12}$$

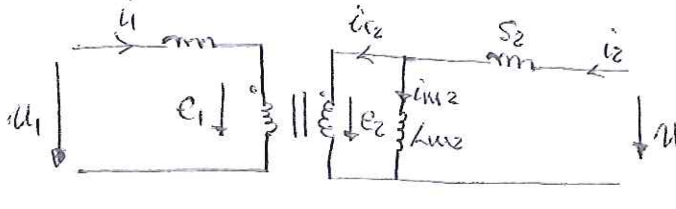
* CIRCUITO EQUIVALENTE



Transformad[...]

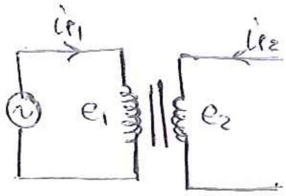
$$\boxed{\begin{array}{l} \dfrac{e_1}{e_2} = a_{12} \\ i_2 = -a_{12} i_{r1} \end{array}}$$

De otra forma:



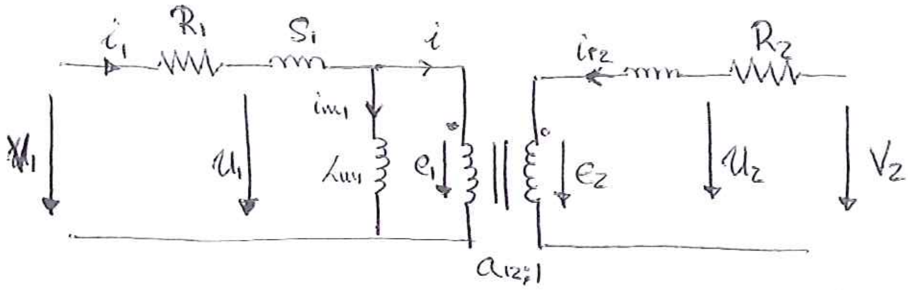
$$\boxed{\begin{array}{l} \dfrac{e_1}{e_2} = a_{12} \\ i_{r2} = -a_{12} i_1 \end{array}}$$

En un transformador ideal.



$$e_1 = N_1 \frac{d\phi_u}{dt} \rightarrow \underline{E}_1 = j\omega N_1 \underline{\phi}_u$$

$$e_1 \cdot i_{r1} + e_2 \cdot i_2 = 0 \rightarrow \text{Todo lo que e}$$

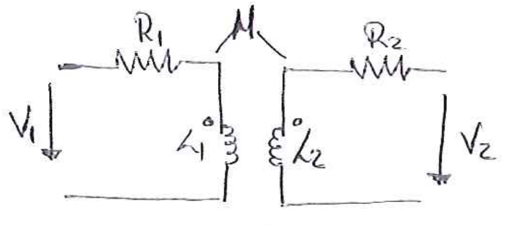


$$L_{\mu 1} = a_{12} M \longrightarrow M = \frac{L_{\mu 1}}{a_{12}}$$

$$L_1 = S_1 + a_{12} M \longrightarrow L_1$$

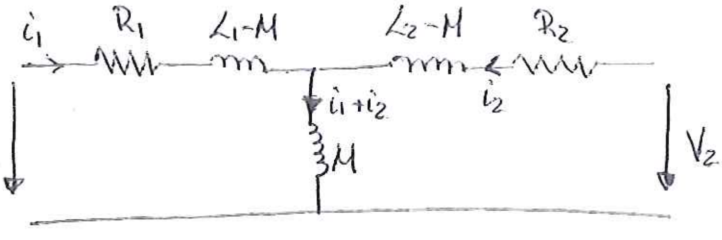
$$L_2 = S_2 + a_{21} M \longrightarrow L_2$$

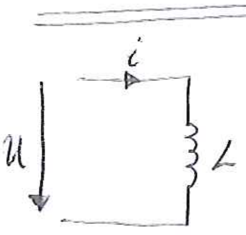
$$\begin{cases} V_1 = R_1 i_1 + L_1 i_1' + M i_2' + M i \\ V_2 = R_2 i_2 + L_2 i_2' + M i_1' + M i_2' \end{cases}$$



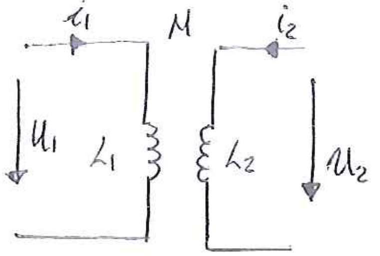
$$\begin{cases} (1) = R_1 i_1 + (L_1 - M) i_1' + M(i_1' + i_2') \\ (2) = R_2 i_2 + (L_2 - M) i_2' + M(i_1' + i_2') \end{cases}$$

$\Longrightarrow$





$$\begin{cases} P = u \cdot i \\ u = L \frac{di}{dt} \end{cases} \Longrightarrow P = L i \frac{di}{dt} \rightarrow W(t) = \int_{-\infty}^{t} P\,dt = \int_{i(-\infty)}$$



$$\begin{cases} P = u_1 i_1 + u_2 i_2 \\ u_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt} \\ u_2 = L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt} \end{cases} \Longrightarrow$$

$$\Longrightarrow p = u_1 i_1 + u_2 i_2 = L_1 i_1 \frac{di_1}{dt} + M i_1 \frac{di_2}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt}$$

$$W(t) = \int_{-\infty}^{t} L_1 i_1 \frac{di_1}{dt} + \int_{-\infty}^{t} L_2 i_2 \frac{di_2}{dt} + M \int_{-\infty}^{t} M \frac{d}{dt}(i_1 i_2)\,dt = \int_{i_1(-\infty)}^{i_1(t)} L_1 i_1 di_1 + \int_{i_2(-\infty)} L_2 \ldots$$

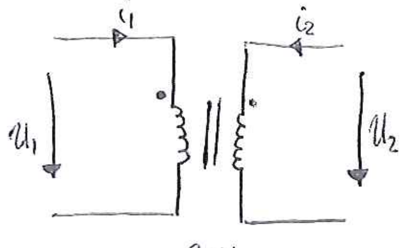
$$+ \int_{i_1 i_2(-\infty)}^{i_1 i_2(t)} M \frac{d}{dt}(i_1 i_2)\,dt = \boxed{\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + \underbrace{M i_1 i_2}_{\geq 0}} \geq 0$$

$i_1 i_2(-\infty)$

$i_1(-\infty) i_2(-\infty)$

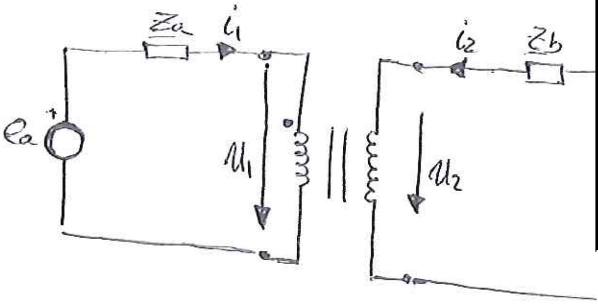
M

ka



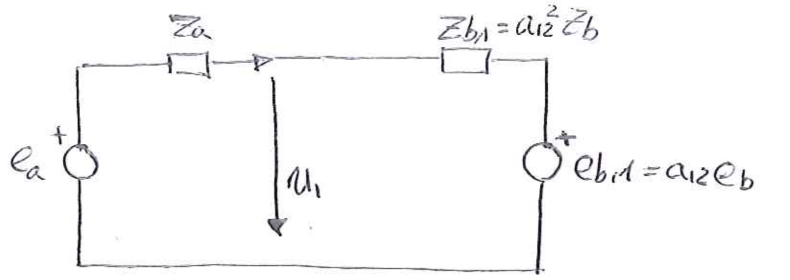
$a_{12}:1$

$$\boxed{\begin{array}{l} u_1 = a_{12} u_2 \\ i_2 = -a_{12} i_1 \end{array}}$$

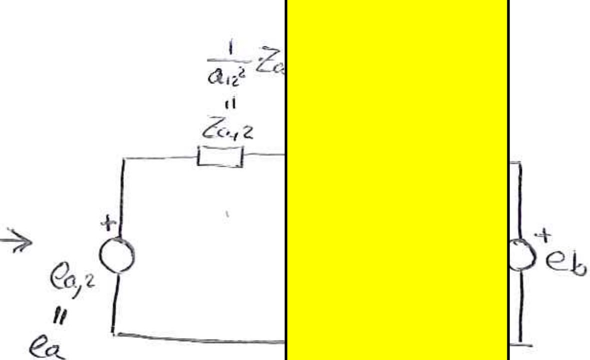


$$\begin{cases} u_1 = e_a - Z_a i_1 \\ u_2 = e_b - Z_b i_2 \longrightarrow \underbrace{a_{12} u_2}_{u_1} = \underbrace{a_{12} e_b}_{e_{b,1}} - \ldots \end{cases}$$

Se puede construir el siguiente circuito:

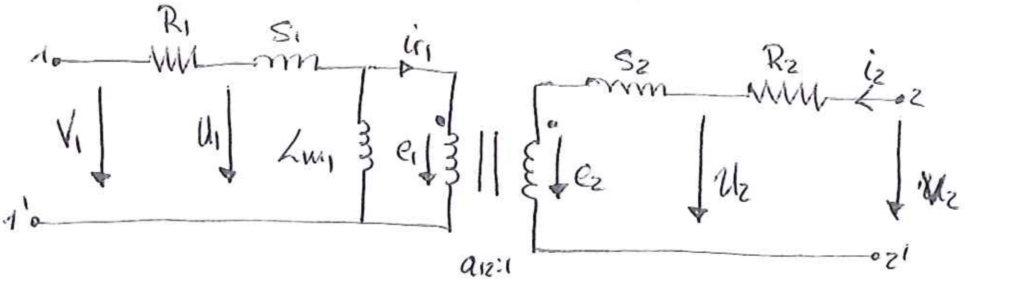


Referido al primario



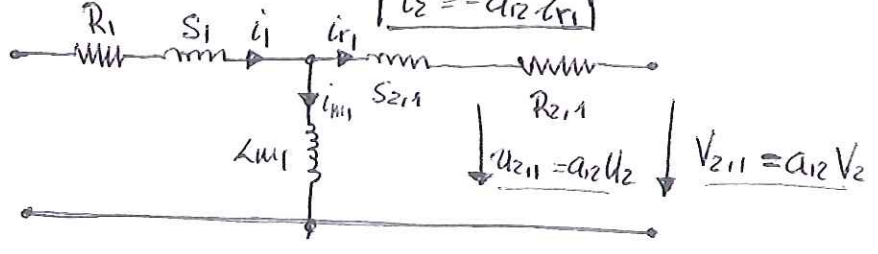
Refer... ...rio.

Transformador real



• Vemos a... secun... ...rio.

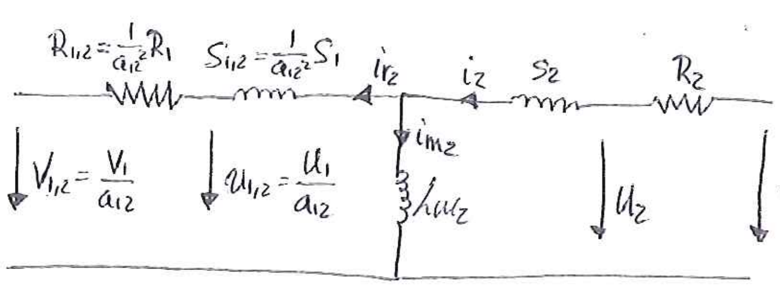
$$\boxed{\begin{aligned} e_1 &= a_{12} e_2 \\ i_2 &= -a_{12} i_{r1} \end{aligned}}$$



$S_{2,1} = a_{12}^2 S...$

$R_{2,1} = a_{12}^2 R_2$

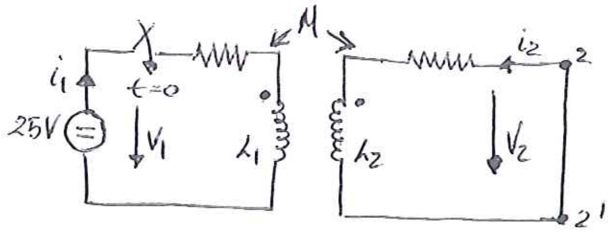
• Referido al secundario (poner L_m en el secundario previamente)



$i_{r2} = -c...$

Problema pág 183.

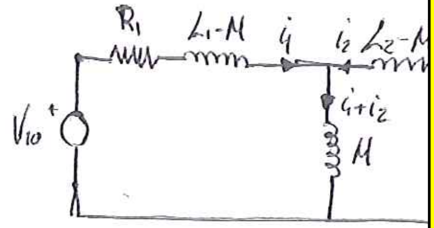
$N_1 = 100$ $N_2 = 500$ $L_1 = 100 H$ $L_2 = 25 H$ $M = 47'5 H$ $R_1 = R_2 =$ ... V.



$V_1 = i_1 R_1 + L_1 i_1' + M i_2'$

$V_2 = 0 = i_2 R_2 + L_2 i_2' + M$ ...

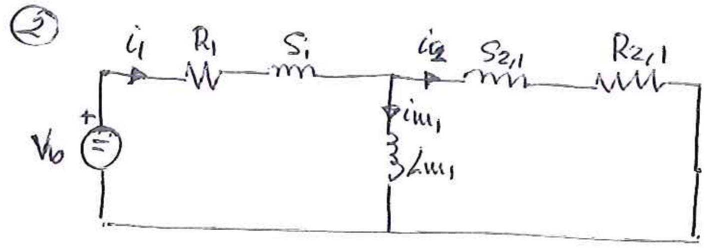
$$\Longrightarrow \begin{cases} V_1 = i_1 R_1 + (L_1 - M) i_1' + M(i_1' + i_2') \\ 0 = i_2 R_2 + (L_2 - M) i_2' + M(i_1' + i_2') \end{cases}$$



2º orden.

Otra forma posible sería:

(2)



$i_1 = -\frac{1}{a_{12}} i_2$ $a_{12} =$ ...

$L_{m1} = a_{12} M = 2 \cdot 47'5 =$ ...

$S_1 = L_1 - L_{m1} = 5H$ S_2 ... $5H$

$R_2 = 25\Omega \rightarrow R_{2,1} = a_{12}^2 R_2 = 4 \cdot 25 = 100\Omega$

En el caso (1), la ecuación diferencial se podría obtener sin haber ... circuito.

En ese circuito:

$$V_{10} = \left[R_1 + (L_1 - M)D + \frac{[(L_2 - M)D + R_2] MD}{R_2 + (L_2 - M)D + MD} \right] i_1$$

De otro modo:

$$\begin{cases} V_1 = (R_1 + L_1 D) i_1 + MD i_2 \longrightarrow V_1 = (R_1 + L_1 D) i_1 + M \ldots \\ 0 = (R_2 + L_2 D) i_2 + MD i_1 \longrightarrow i_2 = -\frac{MD}{R_2 L_2 D} i_1 \end{cases}$$

$$\Longrightarrow V_1 = \frac{(R_1 + L_1 D)(R_2 + L_2 D) - M^2 D^2}{R_2 + L_2 D} i_1 = \frac{R_1 R_2 + (R_1 L_2 + R_2 L_1) D - M^2 D^2}{R_2 + L_2 D} \ldots \Rightarrow$$

$$\Longrightarrow (R_2 + L_2 D) V_1 = \left[(L_1 L_2 - M^2) D^2 + (R_1 L_2 + R_2 L_1) D + R_1 R_2 \right] i_1 =$$

$$= \left[D^2 + \frac{R_1 L_2 + R_2 L_1}{L_1 L_2 - M^2} D + \frac{R_1 R_2}{L_1 L_2 - M^2} \right] i_1$$

Por tanto:

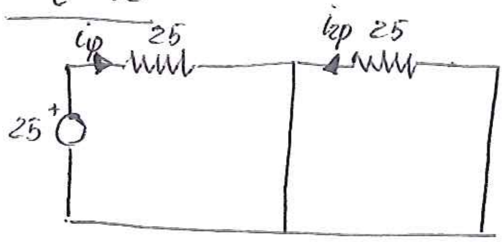
$$\omega_0 = \sqrt{\frac{R_1R_2}{L_1L_2-M^2}} = 1'60128\ \frac{rad}{s} \qquad 2\xi\omega_0 = \frac{R_1L_2+R_2L_1}{L_1L_2-M^2} \Rightarrow \quad > 1$$

Las bobinas estaban descargadas previamente.

$$r_1 = -\omega_0\left(\xi - \sqrt{\xi^2-1}\right) = -0'203221\ \frac{rad}{s} \qquad r_2 = -\omega_0\left(\xi + \sqrt{\xi^2 \quad\right. \quad 73\ \frac{rad}{s}$$

$$\begin{cases} i_1(t) = i_{1p}(t) + K_1e^{r_1t} + K_2e^{r_2t} \\ i_2(t) = i_{2p}(t) + K_3e^{r_1t} + K_4e^{r_2t} \end{cases}$$

• $t = \infty$



$$i_{1p}(t) = \frac{25}{25} = 1A$$

$$i_{2p}(t) = 0A$$

Condiciones iniciales:

$$\left.\begin{array}{l} i_1(0) = 0 \\ i_2(0) = 0 \end{array}\right\} \begin{array}{l} V_{10} = R_1i_1(0) + L_1i_1'(0) + Mi_2'(0) \rightarrow 25 = 0 + 100i_1'(0) + \\ 0 = R_2i_2(0) + L_2i_2'(0) + Mi_1'(0) \rightarrow 0 = 0 + 25i_2'(0) + \end{array}$$

$$\Rightarrow \begin{cases} i_1'(0) = 2'5641\ A/s \\ i_2'(0) = -4'87179\ A/s \end{cases} \rightarrow \begin{cases} 0 = 1 + K_1 + K_2 \\ i_1'(0) = r_1K_1 + r_2K_2 \end{cases} \Rightarrow \begin{cases} K \\ K \\ K_3 \\ K_4 \end{cases}$$

Se va a comprobar las tensiones.

$$25 = 0 + 52'5\, i_1'(0) + 47'5\left[i_1'(0) + i_2'(0)\right] = 0 + 100\, i_1'(0) + 47'5$$

Por Laplace:

$$\begin{cases} \frac{V_{10}}{s} = R_1 \hat{i}_1 + L_1\left[\hat{i}_1 \cdot s - \cancel{i_1(0)}^{0}\right] + M\left[s\hat{i}_2 - \cancel{i_2(0)}^{0}\right] \\ 0 = R_2\hat{i}_2 + L_2\left[s\hat{i}_2 - \cancel{i_2(0)}^{0}\right] + M\left[s\hat{i}_1 - \cancel{i_1(0)}^{0}\right] \end{cases} \Longrightarrow$$

$$\Longrightarrow \begin{cases} \frac{25}{s} = 25\,\hat{i}_1 + 100s\,\hat{i}_1 + 47'5 \cdot s\,\hat{i}_2 \\ 0 = 25\hat{i}_2 + 25 \cdot s\,\hat{i}_2 + 47'5 \cdot s \cdot \hat{i}_1 \longrightarrow \hat{i}_2 = -\frac{47'5 \cdot s}{25(1+s)}\hat{i}_1 \end{cases}$$

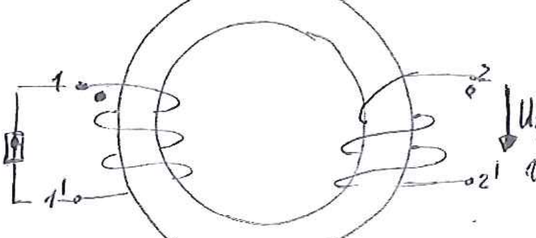
$$\Longrightarrow \frac{25}{s} = 25\left[1+4s\right]\hat{i}_1 + 47'5 \cdot s\left(-\frac{47'5 \cdot s}{25(s+1)}\right)\hat{i}_1 \Rightarrow$$ Despejar $\hat{i}$ ... antitrans...

$$\Longrightarrow \frac{25}{s} = \frac{25^2\left[0'39s^2 + 5 \cdot s + 1\right]}{25(1+s)}\hat{i}_1 \longrightarrow \quad \omega_0 = \sqrt{\frac{1}{0'39}} \quad 2\xi\omega_0$$

Problema 3 de Mayo 2012

✓ Ejercicio 2.

¿no ... correspondientes?



$N_1 = 50 \quad i_{11'} = 2 + 3\cos 100\pi t = ...$

$N_2 = 20$

$r_1 = 8\,cm \quad r_2 = 10\,cm$

$\mu_r = 2...$

$\mu_0 = 4...$

$R = 0$ si no se dice nada.

$$\begin{cases} u_1 = L_1 \cdot i_1' + \cancel{M i_2'}^{0} \\ u_2 = M i_1' + \cancel{L_2 i_2'}^{0} \end{cases} \quad i_2 = 0 \Rightarrow i_2' = 0$$

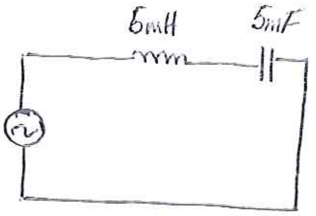
$$M = \mathbb{P}_m N_1 N_2 = \frac{\mu \cdot S}{l} N_1 N_2 = \frac{4\pi}{9}\,m...$$

$$l = 2\pi\left(\frac{r_1 + r_2}{2}\right) \qquad S = \pi\left(\frac{r_2 - r_1}{2}\right)^2$$

$$u_2 = \frac{4\pi}{9} \cdot 10^{-3}\left[0 - 3 \cdot 100 \cdot \pi \operatorname{Sen}(100\pi t)\right] = \underline{\underline{-1'3159 \operatorname{Sen}(100\pi t)}}$$

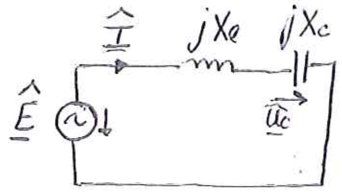
Problema Junio 2012

$e(t) = 7'5\sqrt{2}\,Cos\left(100t - \frac{\pi}{4}\right)$



$\Rightarrow$

$\Rightarrow$



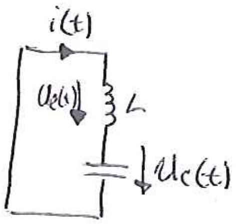
$\hat{I} = \dfrac{\hat{E}}{j(X_e + X_c)} \qquad \hat{U}_c = jX_c\hat{I}$

$X_e = 0'5\,\Omega \;;\; X_c = -2 \;;\; \hat{E} = 7'5\sqrt{2}\,\angle{-\pi/4} \Rightarrow$

$\Rightarrow \hat{I} = \dfrac{7'5\sqrt{2}\,e^{-j\frac{\pi}{4}}}{j(0'5-2)} = j\cdot 5\sqrt{2}\,e^{-j\frac{\pi}{4}}\,A \Rightarrow \hat{U}_c = jX_c\hat{I} = 10\sqrt{2}\,e^{-j\frac{\pi}{4}}$

$i(t) = Re\left\{\hat{I}e^{j\omega t}\right\} \Rightarrow i(0) = Re\left\{\hat{I}\right\} = Re\left\{j5\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\right\} = 5$

$u(t) = Re\left\{\hat{U}e^{j\omega t}\right\} \Rightarrow u_c(0) = Re\left\{\hat{U}\right\} = Re\left\{10\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\right\} = 10$



$u_L = Li' \Rightarrow u_L(0) = Li'(0)$

$i = Cu_c' \Rightarrow i(0) = Cu_c'(0) \Rightarrow u_c'(0) = \dfrac{5}{5\cdot 10^{-3}} = 10$

$\Rightarrow u_L(0) = -u_c(0) = -1000\,\frac{V}{s} \Rightarrow i'(0) = -\dfrac{u_c'(0)}{L} = -$

$LDi + \frac{1}{CD}i = 0 \Rightarrow LCD^2i + i = 0 \Rightarrow D^2i + \frac{i}{LC} = 0 \Rightarrow \left|\begin{array}{l} \omega_0 = \sqrt{\frac{1}{LC}} \\ \xi = 0 \end{array}\right.$

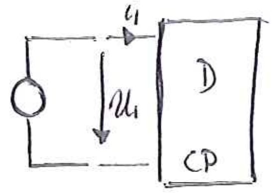
$i(t) = i_p\overset{0}{(t)} + \underline{K}e^{j\omega_0 t} + \underline{K}^* e^{-j\omega_0 t} \Rightarrow i(0) = 5 = \underline{K} + \underline{K}^*$

$i'(t) = j\omega_0\underline{K}e^{j\omega_0 t} - j\omega_0\underline{K}^* e^{-j\omega_0 t} \Rightarrow i'(0) = -2000 = 200j\underline{K} - 200j\underline{K}^*$

$i(t) = 2Re\left\{\underline{K}e^{j\omega_0 t}\right\} = 5\sqrt{5}\,Cos\left(200t + 63'435^\circ\right)$

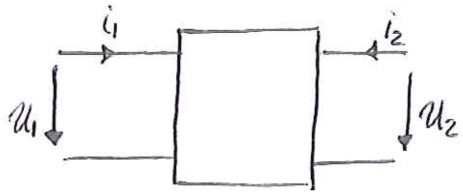
i)

CUADRIPOLOS.



Dipolo.

$$\frac{u_1}{i_1} = Z \qquad \frac{i_1}{u_1} = Y$$



Ecuaciones del cuadripolo en impedanc[ias]

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Ecuaciones del cuadripolo en admitanci[as]

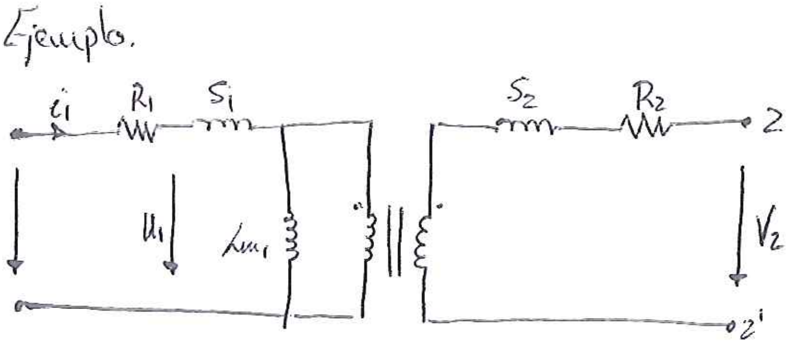
$$\left| \begin{array}{l} u_1 = Z_{11} i_1 + Z_{12} i_2 \\ u_2 = Z_{21} i_1 + Z_{22} i_2 \end{array} \right. \Rightarrow i_2 = 0 \Rightarrow \left| \begin{array}{l} u_1 = Z_{11} i_1 \\ u_2 = Z_{21} i_1 \end{array} \right. \Rightarrow Z_{11} = \left.\frac{u_1}{i_1}\right|_{i_2=0}$$

Análogamente $\Rightarrow$ $Z_{22} = \left.\frac{u_2}{i_2}\right|_{i_1=0}$

$$\left| \begin{array}{l} i_1 = Y_{11} u_1 + Y_{12} u_2 \\ i_2 = Y_{21} u_1 + Y_{22} u_2 \end{array} \right. \Rightarrow$$

$$\boxed{\begin{array}{ll} Y_{11} = \left.\frac{i_1}{u_1}\right|_{u_2=0} & Y_{21} = \left.\frac{i_2}{u_1}\right|_{u_2=0} \\ Y_{12} = \left.\frac{i_1}{u_2}\right|_{u_1=0} & Y_{22} = \left.\frac{i_2}{u_2}\right|_{u_1=0} \end{array}}$$

Ejemplo.



$e_1/e_2 = a_{12}$

$i_2 = -a_{12} i_r$

$$Z_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=0} = R_1 + S_1 + \lambda_{m1} \qquad Z_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=0} = ?$$

$$V_1 = \left[R_1 + (S_1 + L_{m1})D\right] i_1$$

$$V_2 = e_2 = \frac{e_1}{a_{12}} = \frac{L_{m1} D\, i_1}{a_{12}} \Rightarrow \left.\frac{V_2}{i_1}\right|_{i_2=0} = \frac{L_{m1} D}{a_{12}} = Z_{21}$$

$$Z_{22} = R_2 + S_2 D + \frac{1}{a_{12}^2} L_{m1} D \;//$$

Por ecuaciones: $\begin{cases} V_2 = \left[R_2 + S_2 D\right] i_2 + e_2 \\ e_1 = a_{12} e_2 = L_{m1} D\, i_{m1} = L_{m1} D\,(-i_{r1}) = L_{m1} D\,(i_2/a_{12}) \Rightarrow e_2 = L \end{cases}$

$$Z_{12} = \left.\frac{V_1}{i_2}\right|_{i_1=0} = \frac{e_1}{i_2} = \frac{\frac{L_{m1} D}{a_{12}} i_2}{i_2} = \frac{L_{m1} D}{a_{12}} \;//$$

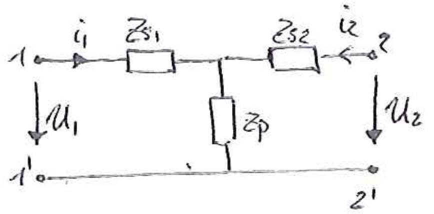
* PROPIEDADES CUADRIPOLO

- Recíproco $\begin{cases} Y_{12} = Y_{21} \\ Z_{12} = Z_{21} \end{cases}$ No todos las cumplen.

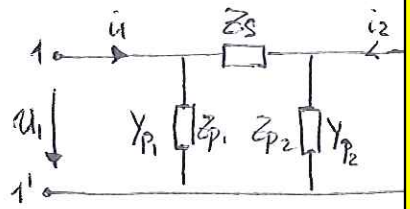
- Simétrico $\begin{cases} Z_{11} = Z_{22} \\ Y_{11} = Y_{22} \end{cases}$

* EQUIVALENTE CUADRIPOLOS RECÍPROCOS.

① • Equivalente en T.



② • Equivalente en π.



①

$$Z_{11} = \left.\frac{u_1}{i_1}\right|_{i_2=0} = Z_{S1} + Z_p$$

$$Z_{21} = \left.\frac{u_2}{i_1}\right|_{i_2=0} = Z_p$$

$$\left[Z\right] = \begin{bmatrix} Z_{S1} + Z_p & Z_p \\ Z_p & Z_{S2} + Z_p \end{bmatrix}$$

$$Z_p = Z_{12} = Z_{21}$$

$$Z_{S1} = Z_{11} - Z_p = Z_{11} - Z_{12}$$

$$Z_{S2} = Z_{22} - Z_p = Z_{22} - Z_{12}.$$

En el ejemplo de ayer:

$$\begin{cases} Z_{11} = R_1 + (S_1 + L_{m1})D = R_1 + L_1 D \\ Z_{22} = R_2 + \left(S_2 + \frac{1}{a_{12}^2} L_{m1}\right) D = R_2 + L_2 D \\ Z_{12} = Z_{21} = \frac{L_{m1} D}{a_{12}} = MD \end{cases} \Rightarrow \left| \begin{array}{l} Z_p = MD \\ Z_{s1} = Z_{11} - Z_{12} = R_1 + (L_1 - M \\ Z_{s2} = Z_{22} - Z_{12} = R_2 + (L_2 - \end{array} \right.$$

(2)

$$Y_{11} = \frac{i_1}{u_1}\Big|_{u_2=0} = Y_s + Y_{p_1}$$

$$Y_{21} = \frac{i_2}{u_1}\Big|_{u_2=0} = -Y_s$$

$$i_2 = \frac{-Y_s}{Y_s + Y_{p_1}} i_1$$

$$[Y] = \begin{bmatrix} Y_s + Y_{p_1} & -Y_s \\ -Y_s & Y_s + Y_{p_2} \end{bmatrix}$$

$$Y_s = -Y_{12}$$

$$Y_{p1} = Y_{11} - Y_s = Y_{11} + Y_{12}$$

$$Y_{p2} = Y_{22} - Y_s = Y_{22} + Y_{12}$$

* Cambio $\Delta - Y \Rightarrow$ Cambio $T - \Pi$

$$Z_s = Z_{s1} + Z_{s2} + \frac{Z_{s1} Z_{s2}}{Z_p}$$

$$Z_{p1} = Z_{s1} + Z_p + \frac{Z_{s1} Z_p}{Z_{s2}}$$

$$Z_{p2} = Z_{s2} + Z_p + \frac{Z_{s2} Z_p}{Z_{s1}}$$

Si $\left.\begin{array}{l} Z_{s1} = Z_{s2} = Z_p = Z_Y \\ Z_{p1} = Z_{p2} = Z_s = Z_\Delta \end{array}\right\} Z_\Delta$

$$[Z]_T^{-1} = [Y]_T = \frac{1}{\det Z} \begin{bmatrix} Z_{s2} + Z_p & -Z_p \\ -Z_p & Z_{s1} + Z_p \end{bmatrix}$$

$$\det(Z) = Z_{s1} Z_{s2} + Z_{s1} Z_p + Z_{s2} Z_p$$

Identificando:

$$-Y_s = \frac{-Z_p}{\det(Z)} = Z_{s1} + Z_{s2} + \frac{Z_{s1} Z_{s2}}{Z_p} \quad \checkmark$$

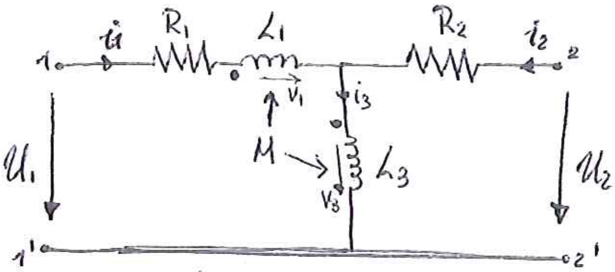
y así con el resto.

$$Z_{s1} = \frac{Z_{p1} \cdot Z_s}{Z_{p1} + Z_{p2} + Z_s} \qquad Y_s = \frac{Y_{s1} Y_{s2}}{Y_{s1} + Y_{s2} + Y_p}$$

$$Z_{s2} = \frac{Z_{p2} \cdot Z_s}{Z_{p1} + Z_{p2} + Z_s}$$

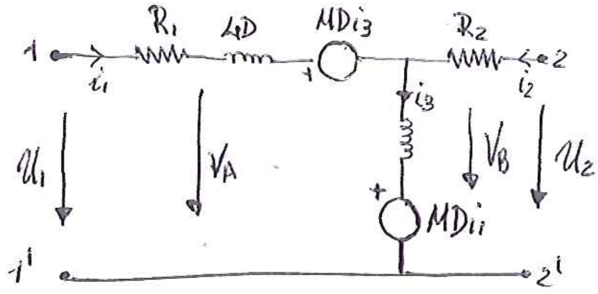
$$Z_p = \frac{Z_{p1} Z_{p2}}{Z_{p1} + Z_{p2} + Z_s}$$

Ejemplo.



$$V_1 = L_1 D i_1 + M D i_3$$

$$V_3 = L_3 D i_3 + M D i_1$$



$$Z_{11} = \left.\frac{U_1}{i_1}\right|_{i_2=0} \qquad Z_{21} = \left.\frac{U_2}{i_1}\right|_{i_2=0} \qquad \begin{cases} U_1 = [R_1 + (L_1 \dots \\ U_2 = (L_3 + \dots \end{cases}$$

$$Z_{11} = R_1 + (L_1 + L_3 + 2M) D \quad ; \quad Z_{21} = (L_3 + M) D$$

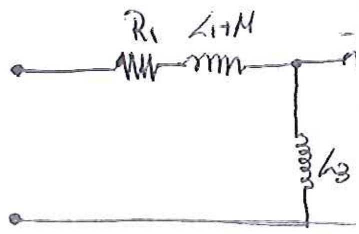
$$\left.\begin{aligned} Z_{22} &= \left.\frac{U_2}{i_2}\right|_{i_1=0} \\ Z_{12} &= \left.\frac{U_1}{i_2}\right|_{i_1=0} \end{aligned}\right\} \quad \begin{aligned} U_2 &= [R_2 + (L_3 + M) D] \longrightarrow Z_{22} = R_2 + (L_3 + M) D \\ U_1 &= (L_3 + M) D i_2 \longrightarrow Z_{12} = \frac{U_1}{i_2} = (L_3 + M) \end{aligned}$$

Se puede sacar el cuadripolo en T:

$$Z_{11} = Z_{s1} + Z_p \rightarrow Z_{s1} = R_1 + (L_1 + M) D$$

$$Z_{22} = Z_{s2} + Z_p \rightarrow Z_{s2} = R_2 - MD$$

$$Z_{12} = Z_p$$



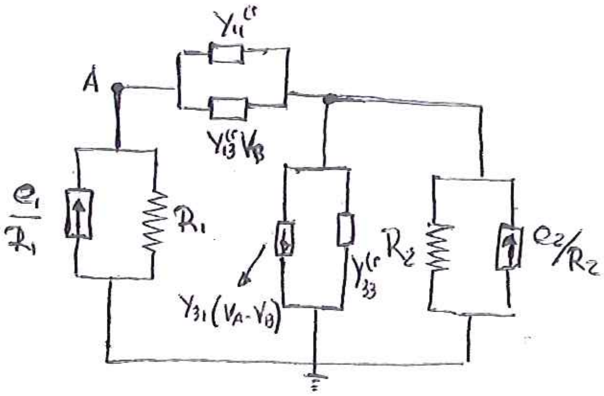
Si se quisiera resolver por mallas el ejemplo: (Poniendo fuentes en M...

$$\begin{bmatrix} R_1+(L_1+L_3) & -L_3D \\ -L_3D & R_2+L_3D \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} u_1-MDi_3-MDi_1 \\ MDi_1-u_2 \end{bmatrix}$$

Despejan... de la izqui... $\longrightarrow$ esc... y i_b como i_a = ... obtendría ...

$$\begin{cases} i_1 = i_a \\ i_3 = i_a - i_b \end{cases}$$

Transformamos a Norton:
(No sabemos transformar bobinas acopladas)



$$\begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_3 \end{bmatrix} D \begin{bmatrix} \ldots \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \frac{1}{D(L_1L_3-M^2)} \begin{bmatrix} L_3 & \ldots \\ -M & \ldots \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11}^{(r} & Y_{13}^{(r} \\ Y_{31}^{(r} & Y_{33}^{(r} \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix} \Longrightarrow Y_{11}^{(r} = \frac{L_3}{(L_1L_3-M^2)D} \qquad Y_{33}^{(r} = \frac{\ldots}{(L_1L_3 \ldots}$$

$$Y_{13}^{(r} = Y_{31}^{(r} = -\frac{M}{(L_1L_3-M \ldots}$$

$$i_1 = Y_{11}^{(r}V_1 + Y_{13}^{(r}V_3 = Y_{11}^{(r}V_1 + Y_{13}^{(r}V_B$$

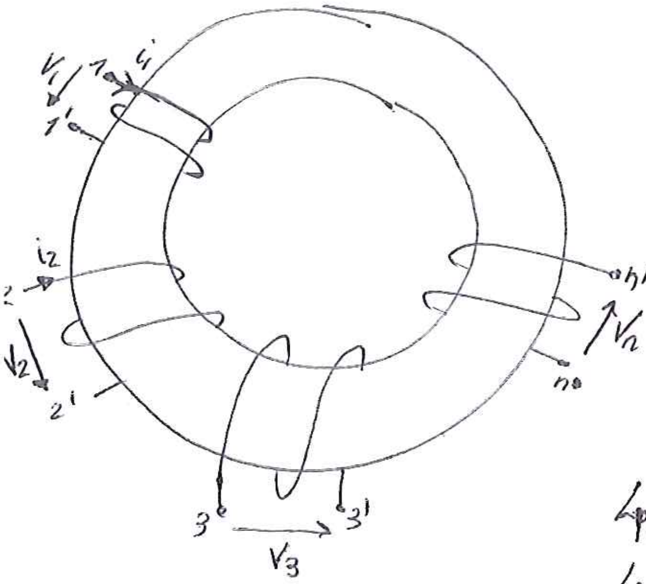
$$i_3 = Y_{33}^{(r}V_3 + Y_{31}^{(r}V_1 = Y_{33}^{(r}V_3 + Y_{31}^{(r}(V_A-V_B)$$

$$\begin{bmatrix} \frac{1}{R_1}+Y_{11}^{(r} & -Y_{11}^{(r} \\ -Y_{11}^{(r} & \frac{1}{R_2}+Y_{33}^{(r}+Y_{11}^{(r} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{e_1}{R_1} - Y_{13}^{(r}V_B \\ \frac{e_2}{R_2} + Y_{13}^{(r}V_B - Y_{31}^{(r}(V_A-V_B) \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{R_1} + Y_{11}^{(r} & -Y_{11}^{(r} + Y_{13}^{(r} \\ -Y_{11}^{(r} + Y_{13}^{(r} & \frac{1}{R_2} + Y_{11}^{(r} + Y_{33}^{(r} - 2Y_{13}^{(r} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{e_1}{R_1} \\ \frac{e_2}{R_2} \end{bmatrix}$$

Bobinas acopladas

pág 184.



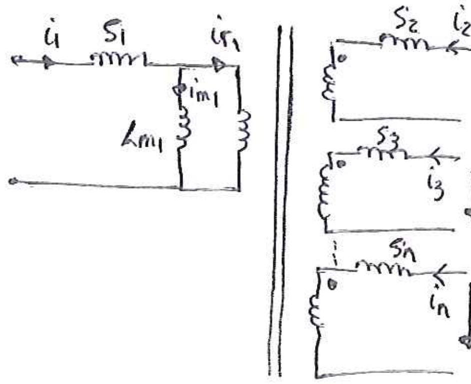
$$u_p = \sum_{q=1}^{n} L_{pq}\, i_q' = \sum_{q=1}^{n} (L_{pq}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ & \ddots & & \\ & & & L_{nn} \end{bmatrix}$$

$L_{pp} = L_p = S_p + L_{mp}$

$L_{pq} = M_{pq}$

$a_{pq} =$



$N_1 i_{m1} = \mathcal{F}$

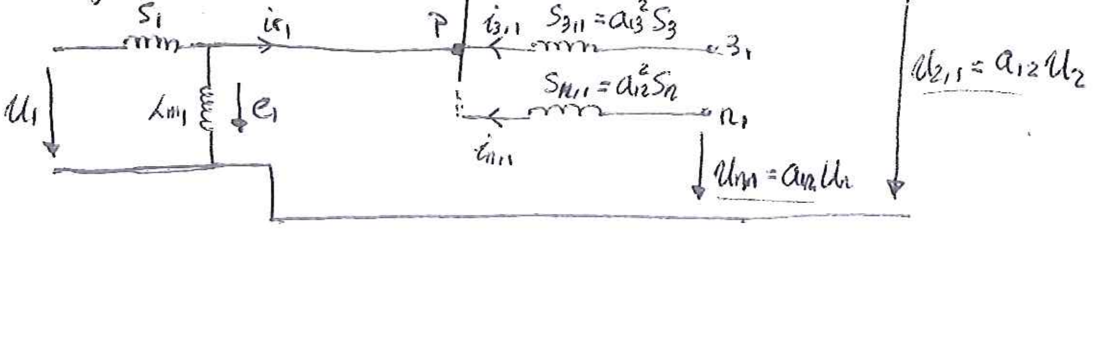
$\phi_m = \mathbb{P}_m \mathcal{F}$

$N_1 i_{r1} + N_2 i_{r2} + \cdots + N_n i_{rn}$

$\boxed{i_{r1} + a_{21} i_2 + a_{31} i_3 + \cdots +}$

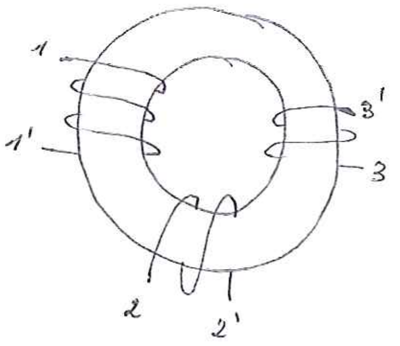
$$\boxed{\frac{e_1}{N_1} = \frac{e_2}{N_2} = \frac{e_3}{N_3} = \frac{e_n}{N_n}} \; ; \quad e_2 = a_{21} e_1 \quad e_n = a_{n1} e_1$$

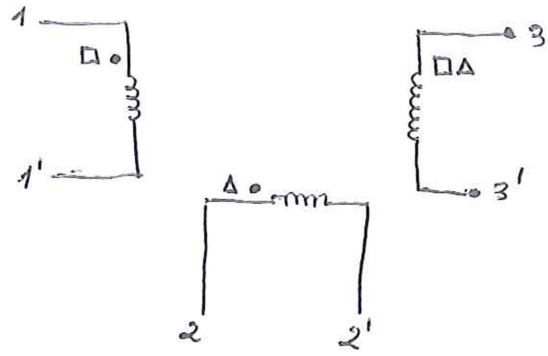
* Transferencia al primario



Tres bobinas acopladas

*Caso 1.





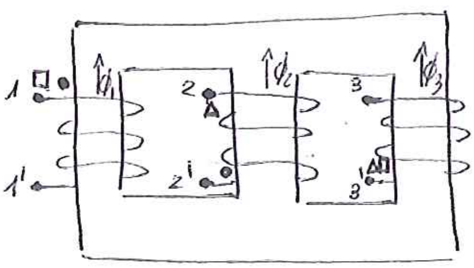
1-2 •

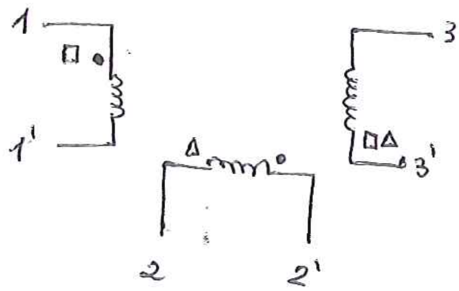
1-3 □

2-3 Δ

Otro caso:

*Caso 2





1-2 •

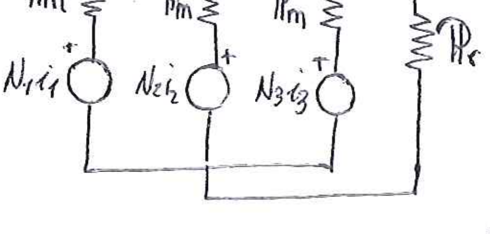
1-3 □

Caso 1

Caso 2

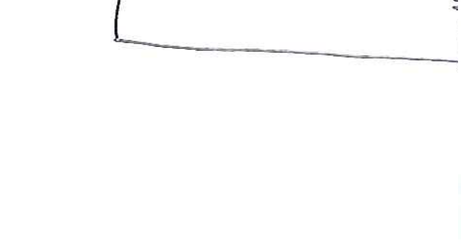
$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} +L_1 & \mp M_{12} & \mp M_{13} \\ \mp M_{12} & +L_2 & \mp M_{23} \\ \mp M_{13} & \mp M_{23} & +L_3 \end{bmatrix} \begin{bmatrix} i_1' \\ i_2' \\ i_3' \end{bmatrix}$$

*Esquema Caso 2

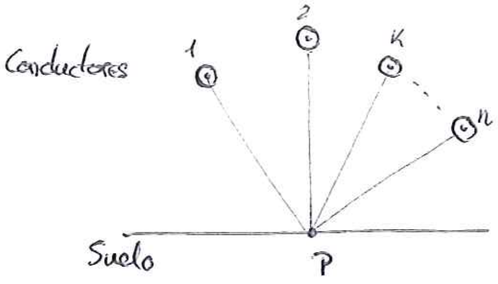


$$\alpha = \frac{\mathbb{R}_r}{\mathbb{R}_m}$$

*Esquema caso -1



Acoplamientos

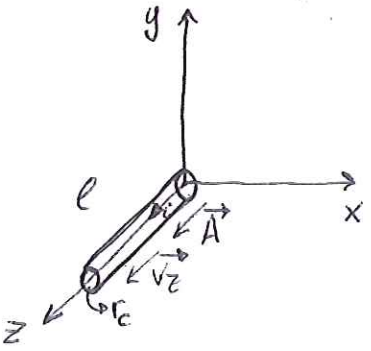


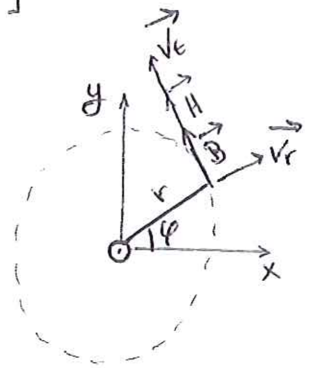
Caso sencillo: $i_1+i_2+\cdots+i_n=0$

Caso más complicado: $i_1+i_2+\cdots+$ … $\neq 0)$

$$\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_n\end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1n}\\ L_{21} & L_{22} & \cdots & L_{2n}\\ & & & \\ L_{n1} & & \cdots & L_{nn}\end{bmatrix}\begin{bmatrix} i_1\\ i_2\\ \vdots\\ i_n\end{bmatrix}$$

$$\Lambda_{kp} = \lambda_{kp}\, l$$





$\vec{H} = H_t \vec{V_t}$

$\vec{B} = B_t \vec{V_t}$

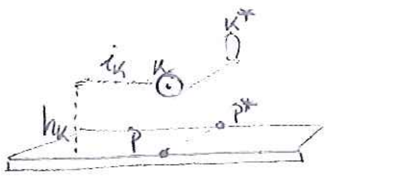
$\mu = \mu_0 = 4$…

$$\oint \vec{H}\cdot d\vec{l} = i \quad ; \quad \oint \left(H_t \vec{V_t}\right)\left(dl_t \vec{V_t}\right) = H_t \cdot 2\pi r = i \Rightarrow H_t = \ldots$$

$$\Rightarrow B_t = \frac{\mu \cdot i}{2\pi r} \qquad \vec{A} = A_z \vec{V_z} \Rightarrow A_z = -\frac{\mu i}{2\pi} \ln \ldots \quad \vec{A}$$

$$\Lambda = \iint_S \vec{B}\, d\vec{S} = \oint_\gamma \vec{A}\cdot d\vec{l}$$

$$\vec{B} = -\frac{1}{r}\, r\, \vec{V_t}\left[\frac{\partial A_z}{\partial r} - \frac{\partial A_r}{\partial z}\right] = -\frac{dA_z}{dr}\vec{V_t} = \frac{\mu i}{2\pi r}\vec{V_t}$$



$$\lambda_{Kp} = \oint_\gamma \vec{A}\,\vec{dl} = \int_{K-K^*} \vec{A}\,\vec{dl} + \cancel{\int_{K^*-p^*} \vec{A}\,\vec{dl}} + \int_{p^*-p} \vec{A}\,\vec{dl}$$

i_K ⊙ K' → carga imagen

$i_K + i_p = 0$

Suposición: $h_K \gg r_c$

$$A_K^{(i_K} + A_K^{(i_K'} = A_K$$

$$A_p^{(i_K} + A_p^{(i_K'} = A_p$$

$$A_K^{(i_K} = -\frac{\mu \cdot i_K}{2\pi} \ln r_c$$

$$A_K^{(i_K'} = -\frac{\mu \cdot \overset{-i_K}{i_K'}}{2\pi} \ln 2h_K$$

$$\rightarrow \boxed{A_K = \mu \frac{i_K}{2\pi} \ln \ldots}$$

$$A_p^{(i_K} = -\frac{\mu i_K}{2\pi} \ln h_K$$

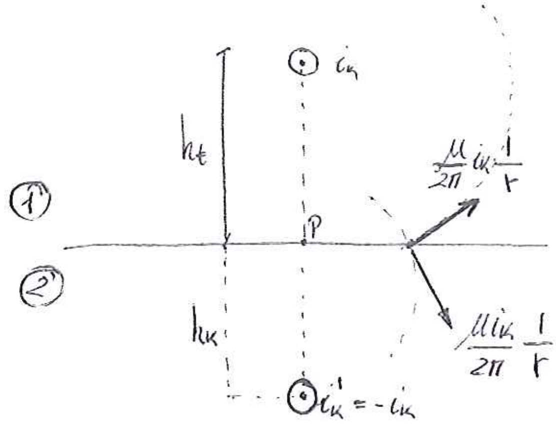
$$A_p^{(-i_K} = -\frac{\mu i_K'}{2\pi} \ln h_K$$

$$\Rightarrow \boxed{A_p = 0}$$

$$\lambda_{Kp} = A_K = \frac{\mu i_K}{2\pi} \ln \ldots$$

$$\boxed{L_{Kp} = \frac{\mu}{2\pi} \ldots}$$

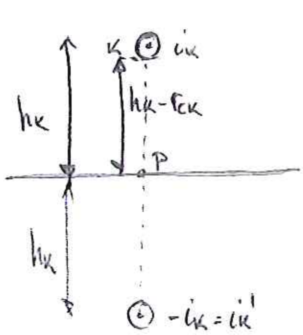
$$E_{ip} = \frac{\partial \vec{A_p}}{\partial t} = 0 \rightarrow \vec{E_e} + \vec{E_i} = 0 \Rightarrow \vec{E_e} = 0$$



$\vec{B} = 0$ en la superficie

$$\boxed{B_{n1} = B_{n2}}$$

↳ c. perfecto ↳ aire



$$\lambda_{Kp} = A_K - A_p \underset{A}{=} \mu \frac{i_K}{2\pi} \ln \frac{(2h_K - r_{cK})}{r_{cK}} = L_{Kp}\, i \ldots$$

$$A = -\frac{\mu i}{2\pi} \ln r$$

$$A_K = -\frac{\mu i_K}{2\pi} \ln r_{cK} - \mu \ldots$$

$$A_p = -\frac{\mu i_K}{2\pi} \ln h_K - \mu \ldots$$

$$L_{Kp} = \frac{\mu}{2\pi} \ln \frac{2h_K - r_{cK}}{r_{cK}} \quad (H/m)$$

$$A_K \simeq \frac{\mu}{2\pi} \ln \frac{2h_K}{r_{cK}}$$

Si hay varios conductores: $\lambda_{kp} = A_k - \cancel{A_p} = \sum_{m=1}^{n} \left[A_k^{(im} + A_k^{(im'} \right] =$

$$= \sum_{m=1}^{n} \left[-\mu \frac{i_m}{2\pi} \ln d_{km} - \mu \frac{i_m}{2\pi} \ln d_{km'} \right] = \sum_{m=1}^{n} \left(\frac{\mu}{2\pi} \ln \frac{d_{km'}}{d_{km}} \right) i_m = \sum_{m=1} \ldots_{kp}$$

$$\boxed{L_{km}^{(t} = \frac{\mu}{2\pi} \ln \frac{d_{km'}}{d_{km}}}$$

$$L_{km}^{(t} = L_{km} + \Delta L_{km} \quad \left\{ \begin{array}{l} L_{km} = \ldots \\ \Delta L_{km} = \ldots \end{array} \right.$$

$$i_p = \sum_{m=1}^{n} i_m \neq 0$$

$$L_{kk} = \frac{\mu}{2\pi} \ln \frac{1}{r_{ck}}$$

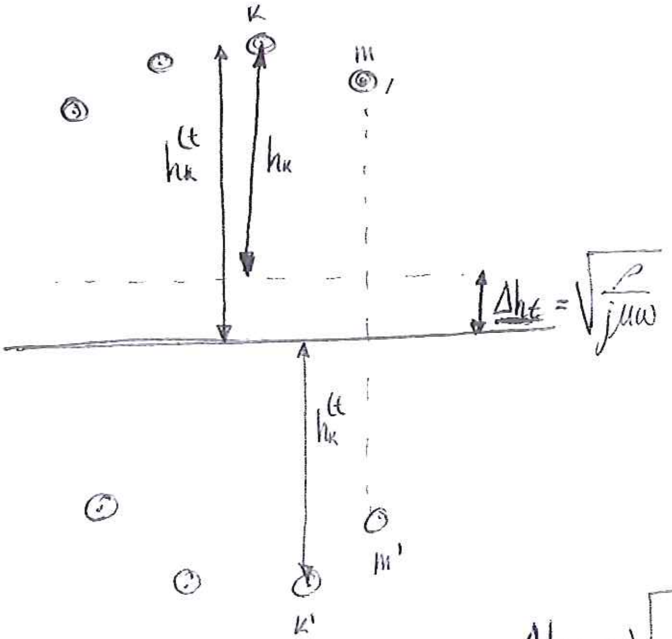
Si $i_p = 0 \Rightarrow \sum_{m}^{n} i_m = 0$
y
Si $h_k \to \infty$

$$\Rightarrow \lambda_{kp} = \sum_{m=1}^{n} L_{km} \cdot i_m + \sum_{m=1}^{n} \Delta L_{km} \cdot i_m$$

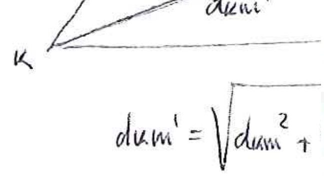
$$\sum_{m}^{n} \underbrace{\Delta L_m}_{\to\infty} \cdot i_m \approx \underbrace{\Delta L}_{\to\infty} \sum_{m}^{n} i_m$$

---

* Si $\rho_{terreno} \neq 0$ (No actúa)



$$h_k^{(t} = h_k + \Delta h_t$$



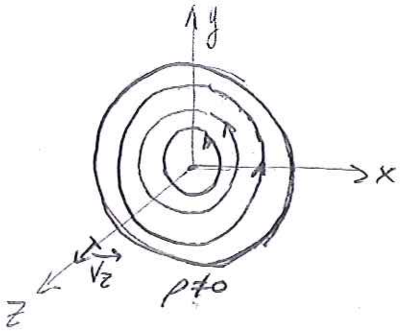
$$d_{km'} = \sqrt{d_{km}^2 + \ldots}$$

Si $h_k = h_m \Rightarrow d_{km'} = \ldots$

$$\Delta h_t = \sqrt{\frac{\rho}{\mu\omega} e^{-j\frac{\pi}{2}}} = \sqrt{\frac{\rho}{\mu\omega}}\, e^{-j\frac{\pi}{4}} = \Delta h_t$$

$$\Delta h_t = \sqrt{\frac{\rho}{\mu\omega}} \approx \sqrt{\frac{100\,\Omega m}{4\pi 10^{-7} \frac{H}{m} \cdot 100\pi \frac{rad}{s}}} = 50\ldots$$

En el interior del conductor:



$\vec{J} = J_z \vec{V_z}$ En cc $\Rightarrow J_z(r) = cte =$

Si ρ=c

en

Mayo 2012

3 conductores

$r_c = 5mm$

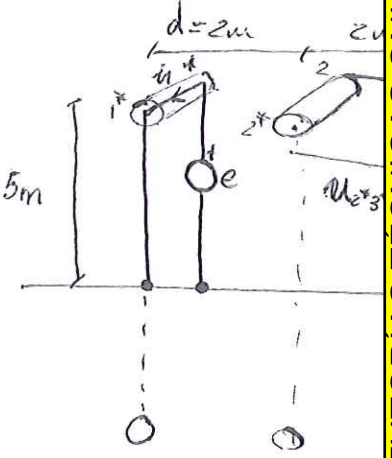
$\ell = 1 km$

$h = 5m$

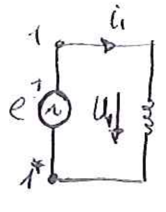
$d_{12} = d_{23} = 2m$

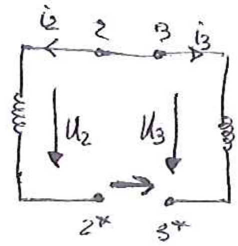
$d_{13} = 4m$

$e = 200\cos(100\pi t)$ V



Se puede escribir como:





$u_1 = e = L_{11} i_1' + L_{12} i_2' + L$

$u_2 = L_{12} i_1' + L_{22} i_2' + L_{23} i_3'$

$u_3 = L_{13} i_1' + L_{23} i_2' + L_{33} i_3'$

$i_2 = i_3 = 0 \Rightarrow i_2' = i_3' = 0 \Rightarrow \begin{cases} u_1 = e = \overset{H/m}{L_{11}} i_1' \cdot \overset{m}{\ell} \\ u_2 = L_{12} i_1' \cdot \ell \\ u_3 = L_{13} i_1' \cdot \ell \end{cases}$

$$L_{11} = \frac{\mu}{2\pi} \ln \frac{d_{11'}}{r_c} = \frac{\mu}{2\pi} \ln \frac{2h}{r_c} = \frac{4\pi \cdot 10^{-7}}{2\pi} \ln \frac{10000}{5} \quad H/m$$

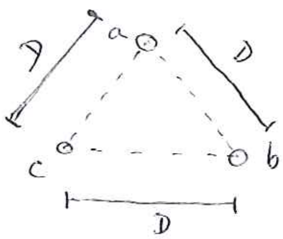
$$L_{12} = \frac{\mu}{2\pi} \ln \frac{d_{12'}}{d_{12}} = \frac{\mu}{2\pi} \ln \frac{\sqrt{d^2+(2h)^2}}{d} = 2 \cdot 10^{-7} \ln \frac{\sqrt{2^2+10^2}}{2} \quad H/m$$

$$L_{13} = \frac{\mu}{2\pi} \ln \frac{d_{13'}}{d_{13}} = \frac{\mu}{2\pi} \ln \frac{\sqrt{(2d)^2+(2h)^2}}{2d} = 2 \cdot 10^{-7} \ln \frac{\sqrt{4^2+10^2}}{2} \quad H/m$$

$$\hat{E} = 200\angle 0° = j\omega L_{11}\hat{I}_1 l_1 \Rightarrow \hat{I}_1 = \frac{\hat{E}}{j\omega L_{11} l_1} = \frac{200\angle 0}{j\cdot 100\pi \cdot L_{11}\cdot l_1} \rightarrow$$

$$\rightarrow i_1 = \hat{I}_1 \cos\left(100\pi t - \frac{\pi}{2}\right)$$

$$u_{2'3'}^{**} = u_3 - u_2 = (L_{13}-L_{12})\, l\cdot i' = (L_{13}-L_{21})\cdot l' \cdot \frac{e}{L_{11}\cdot l'} = e\,\frac{(L_{13}-\ldots}{L\ldots}$$



$$\lambda_{kp} = \sum_{m=1}^{n} L_{km}\, i_m$$

$$L_{km} = \frac{\mu}{2\pi}\ln\frac{1}{d_{km}}$$

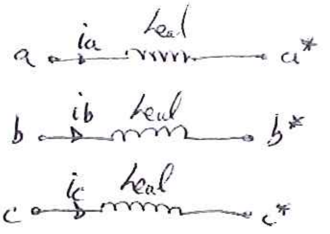
$$\lambda_{ap} = L_{aa}\, i_a + L\ldots$$

$$i_a + i_b + i_c = 0$$

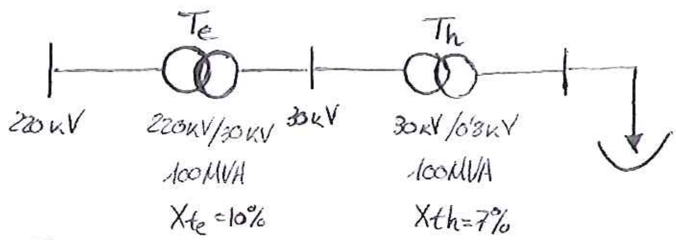
$$L_{aa} = \frac{\mu}{2\pi}\ln\frac{1}{r_c} \quad ; \quad L_{ab} = L_{ac} = \frac{\mu}{2\pi}\ln\frac{1}{D} \Rightarrow \lambda_{ap} = L_{aa}\cdot i_a + L_{a\ldots}$$

$$= \lambda_{ap} = (L_{aa} - L_{ab})\, i_a = L_{eq}\, i_a \,, \quad \text{donde } L_{eq} = \frac{\mu}{2\pi}\ln\frac{D}{r_c}$$

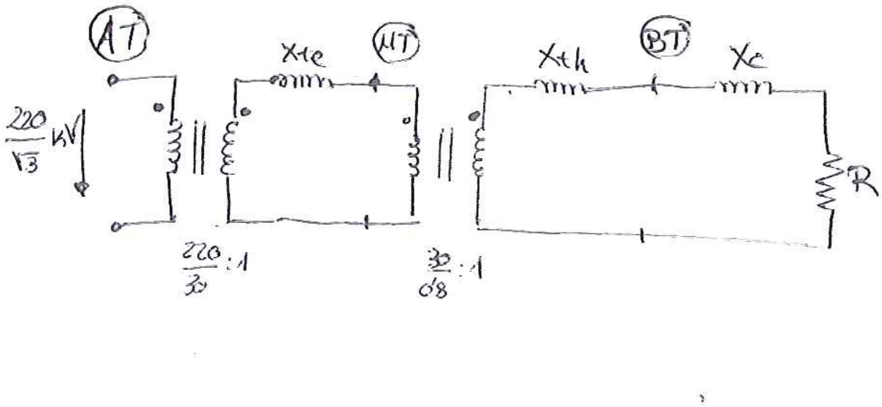
$$\frac{d\lambda_{ap}}{dt} = L_e\cdot\frac{di_a}{dt}$$



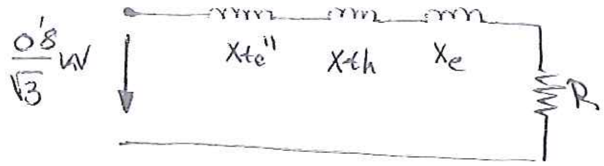
Ejemplo del libro



$r_c = 10\,cm$
$D = 2\,m$
$l = 16\,m$



Circuito equivalente



$$X_{te} = \frac{30kV}{100MVA} \times 0'1 =$$

$$X_{te}'' = 0'9 \cdot \frac{1}{a^2_{12th}} =$$

$$X_{th} = \frac{0'8^2}{100MVA} \cdot 0'07$$

$$X_{te}'' + X_{th} = \frac{(0'8\,kV)^2}{100MVA} \cdot 0'17 = 0'001088\,\Omega$$

$$X_c = \omega L_e = \omega L_{eu} \cdot l = 100\pi \frac{\mu}{2\pi} \ln\left(\frac{D}{R_c}\right) \cdot l = 0'003012\,\Omega$$

ANÁLISIS EN FRECUENCIA.

Una onda periódica a(t) cumple: $\boxed{a(t) = a(t+T) = \sum_{k=0}^{\infty} a_k(t)}$

$$a_k = \hat{A}_k \operatorname{Sen}(k\omega_1 t + \alpha_k) = \hat{A}_k \operatorname{Cos}(k\omega_1 t + \gamma_k) \qquad \boxed{\alpha_k = \gamma_k +}$$

k = 1 → componente fundamental
k > 1 → componentes armónicas
k = 0 → componente de continua → $a_0 = \hat{A}_0 \operatorname{Sen}\alpha_0 = \hat{A}_0 \operatorname{Cos}\gamma_0$

$$\underline{\hat{A}_0} = jA_0 \; ; \; \hat{A}_0' = A_0$$

$$a_k(t) = \operatorname{Im}\left\{\underline{\hat{A}}_k e^{j\omega_1 t}\right\} = \operatorname{Re}\left\{\underline{\hat{A}}_k' e^{j\omega_1 t}\right\} \; ; \; \left| \begin{array}{l} \underline{\hat{A}}_k = \hat{A}_k e^{j\alpha_k} \\ \underline{\hat{A}}_k' = \hat{A}_k e^{j\gamma_k} \end{array} \right.$$

Utilizaremos el seno:

$$a_k(t) = \hat{A}_k \operatorname{Sen}(k\omega_1 t + \alpha_k) = \hat{A}_k \operatorname{Cos}\alpha_k \operatorname{Sen} k\omega_1 t + \hat{A}_k \operatorname{Sen}\alpha_k \operatorname{Cos} k\omega_1 t$$

$$= \hat{A}_{rk} \operatorname{Sen} k\omega_1 t + \hat{A}_{xk} \operatorname{Cos} k\omega_1 t$$

Otra señal periódica: $b(t) = \sum_{m=0}^{\infty} b_m(t)$. Periodo T. $b_m(t) = \hat{B}$

$c(t) = a(t) \cdot b(t)$ ~ Uno de los productos será $c_{km}(t) = a_k(t) \cdot$

$$= \hat{A}_k \hat{B}_m \operatorname{Sen}(k\omega_1 t + \alpha_k) \operatorname{Sen}(m\omega_1 t + \beta_m) = \hat{A}_k \hat{B}_m \cdot \frac{1}{2}\left[\operatorname{Cos}[(k-m)\omega_1 t\right.$$

$$\operatorname{Sen}a \operatorname{Sen}b = \frac{1}{2}\left[\operatorname{Cos}(a-b) - \operatorname{Cos}(a+b)\right]$$

Para el caso particular $k=0$ y $m \neq 0$:

$$C_{0m} = a_0(t)\, b_m(t) = A_0 \hat{B}_m \operatorname{Sen}(m w_1 t + \beta_m)$$

Para $k \neq 0$ y $m=0$:

$$C_{k0} = a_k(t)\, b_0(t) = B_0 \hat{A}_k \operatorname{Sen}(k w_1 t + \alpha_k)$$

Para $k=0$ y $m=0$:

$$C_{00} = A_0 B_0$$

$\cos(k+m)$ no co... med

$\cos(k-m)$ sólo co... $=m$

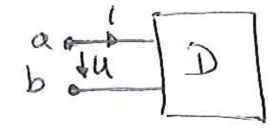
$$C_{medio} = \frac{1}{T}\int_0^T c(t)\,dt = \boxed{A_0 B_0 + \sum_{k=1}^{\infty} A_k B_k \cos(\alpha_k - \beta_k) = \bar{C}}$$

valores eficaces

Otra señal:

$$c(t) = a^2(t) \Rightarrow \boxed{C_{med} = A_0^2 + \sum_{k=1}^{\infty} A_k^2}$$

* DIPOLO



$$u = \sum_{k=0}^{\infty} \operatorname{Im}\left\{\hat{U}_k e^{k j w_1 t}\right\} ; \quad \hat{U}_k = \hat{U}_k e^{j\theta_k} = \sqrt{2} \ldots$$

$$i = \sum_{k=0}^{\infty} \operatorname{Im}\left\{\hat{I}_k e^{j k w_1 t}\right\} ; \quad \hat{I}_k = \hat{I}_k e^{j\phi_k} = \sqrt{2}\, I \ldots$$

$$p = u \cdot i \longrightarrow \boxed{P = \frac{1}{T}\int_0^T p(t)\,dt = U_0 I_0 + \sum_{k=1}^{\infty} U_k I_k \cos(\underbrace{\theta_k - \varphi \ldots}_{\varphi_k}}$$

$$U_{ef}^2 = \frac{1}{T}\int_0^T u^2\,dt = \boxed{U_{ef}^2 = U_0^2 + \sum_{k=1}^{\infty} U_k^2} \qquad \boxed{I^2 = I_0^2 + \sum_{k=1}^{\infty} I_k \ldots}$$

$$S = U I \longrightarrow \quad \lambda = \frac{P}{S} = \text{Factor de potencia.}$$

Caso particular: $u(t) = \text{Im}\left\{ \hat{U}_1 e^{j\omega_1 t} \right\} \Rightarrow$

$$\Rightarrow \begin{cases} P = U_1 I_1 \cos(\theta_1 - \varphi_1) = \text{Re}\left\{ \underline{U}_1 \underline{I}_1^* \right\} = U_1 I_p \\ Q = \text{Im}\left\{ \underline{U}_1 \underline{I}_1^* \right\} = U_1 I_1 \operatorname{Sen}(\theta_1 - \phi_1) = U_1 I_q \end{cases} \rightarrow \underline{S}_1 = \underline{U}_1 \underline{I} \ldots$$

$$\Longrightarrow \underline{I}_1^* = U_1 \frac{(I_p + jI_q)}{\underline{U}_1} \quad ; \quad I_1 = \frac{U_1 (I_p - jI_q)}{\underline{U}_1^*} \Rightarrow \boxed{\underline{I}_1 = (\ldots}$$

$$\underline{I}_1 = I_{r1} + jI_{x1} = (I_p - jI_q)(\cos\theta_1 - j\operatorname{Sen}\theta_1) \quad \begin{cases} I_{r1} = I_p \cos \ldots \theta_1 \\ I_{x1} = I_p \operatorname{Sen} \ldots \theta_1 \end{cases}$$

$$\text{Si } \theta_1 = 0 \Rightarrow \begin{cases} I_{r1} = I_p \\ I_{x1} = -I_q \end{cases}$$

$$I^2 = I_1^2 + \underbrace{\sum_{k=2}^{n} I_k^2}_{\text{componente distorsionante}} = I_1^2 + I_h^2 = I_p^2 + I_q^2 + I_h^2 \Rightarrow$$

$$\Rightarrow \underbrace{U_1^2 I^2}_{} = \underbrace{(U_1 I_p)^2}_{} + \underbrace{(U_1 I_q)^2}_{} + \underbrace{(U_1 I_h)^2}_{}$$

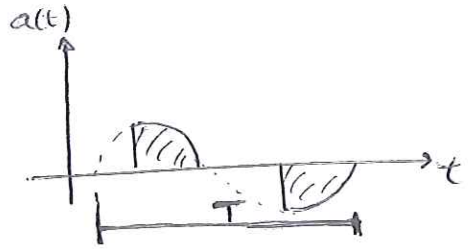
$$\boxed{S^2 = P^2 + Q^2 + S_h^2}$$

Componente distorsio… de la potencia…

$$\lambda = \frac{P}{S} = \underbrace{\frac{P}{S_1}}_{\lambda_q} \cdot \underbrace{\frac{S_1}{S}}_{\lambda_h} = \lambda_q \cdot \lambda_h$$

$$\begin{cases} \lambda_q = \dfrac{P}{S_1} = \dfrac{I_p}{I_1} = \dfrac{I_p}{\sqrt{I_p^2 + I_q^2}} = \dfrac{\ldots}{\sqrt{P \ldots}} \\ \lambda_h = \dfrac{S_1}{S} = \dfrac{I_1}{I} = \dfrac{I_1}{\sqrt{I_1^2 + I_h^2}} = \dfrac{S_1}{\sqrt{S_1^2 + S_h \ldots}} \end{cases}$$

Cálculo de armónicos.



$$A_0 = \frac{1}{T}\int_0^T a(t)\,dt \rightarrow \text{compone}\ldots \text{a}$$

$$a(t)\cdot e^{-jm\omega_1 t} \quad \text{donde} \quad a(t) = \sum_{k=0}^{\infty} a_k(t) = A_0 + \sum_{k=1}^{\infty} \mathrm{Im}\left\{\hat{\underline{A}}_k e^{jk\omega_1 t}\right.$$

$$= A_0 + \sum_{k=1}^{\infty}\left[\frac{\hat{\underline{A}}_k e^{jk\omega_1 t} - \hat{\underline{A}}_k^{*} e^{-jk\omega_1 t}}{2j}\right] \Rightarrow$$

$$c - c^{*} = 2j\,\mathrm{Im}(c)$$

$$\Rightarrow a(t)e^{-jm\omega_1 t} = A_0 e^{-jm\omega_1 t} + \sum_{k=1}^{\infty}\frac{1}{2j}\left[\hat{\underline{A}}_k e^{-j(m-k)\omega_1 t} - \hat{\underline{A}}_k^{*} e^{-j(m+k)\ldots}\right.$$

$$\int_0^T f(t)\,dt = \frac{1}{2j}\hat{\underline{A}}_k\cdot T \Rightarrow \boxed{\hat{\underline{A}}_k = \frac{2j}{T}\int_0^T a(t)\,e^{-jk\omega_1 t}\,dt}$$

Si hay simetría de semionda $\rightarrow a(t) = -a(t - T/2) \rightarrow \left|\begin{array}{l} A_0 = 0 \\ \hat{A}_k = 0 \ \text{si} \end{array}\right.$

$$\hookrightarrow \hat{\underline{A}}_k = \frac{2j}{T/2}\int_0^{T/2} a(t)\,e^{-jk\omega_1 t}\,dt$$

Se prefiere trabajar con ángulos $\Rightarrow$ Cambio de variable $\boxed{\varepsilon = \omega_1 t}$

$d\varepsilon = \omega_1 dt$

$\omega_1 T = 2\pi$

$$\Rightarrow \hat{\underline{A}}_k = \frac{2j}{T}\int_0^T a(t)\,e^{-jk\omega_1 t}\,dt = \frac{j}{\pi}\int_0^{2\pi} a(\varepsilon)\,e^{-\varepsilon k j}\,d\varepsilon$$

Simetría de semionda: $\hat{\underline{A}}_k = \frac{2j}{\pi}\int_0^{\pi} a(\varepsilon)\,e^{-jk\varepsilon}\,d\varepsilon$

Señales desplazadas

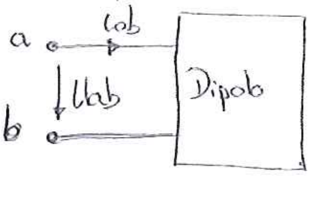
$$a_d(t) = a(t + \Delta t)$$

$$a_d(t) = \sum_{m=0}^{\infty} I_m \left\{ \hat{\underline{A}}_m e^{jm\omega_1(t+\Delta t)} \right\} = \sum_{m=0}^{\infty} I_m \left\{ \left(\hat{\underline{A}}_m e^{jm\omega_1 \Delta t} \right) e^{jm\omega_1 t} \right\}$$

$$\hat{\underline{A}}_m^{(d} = \hat{\underline{A}}_m e^{jm\Delta x} \qquad \left| \begin{array}{l} \hat{A}_m^{(d} = \hat{A}_m \\ \boxed{x_m^{(d} = x_m + m\Delta x} \end{array} \right.$$

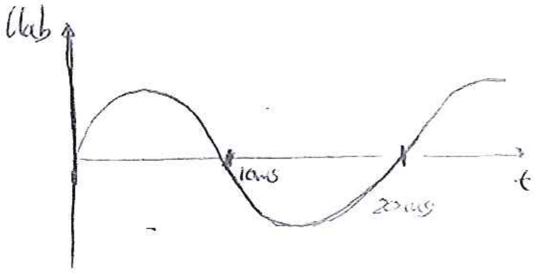
$x_1^{(d} = x_1 + \Delta x$ quiero un desplazamiento tal que el fundamental [...] fases $\Rightarrow \Delta x = -x_1$

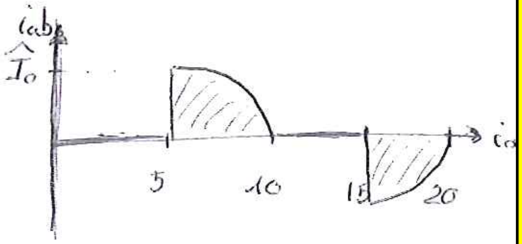
Problema Julio 2012.



$$i_{ab} = \begin{cases} 0 & 0 < t < 5\,ms \\ \hat{I}_o \operatorname{Sen} \omega_1 t & 5\,ms < t < 10\,ms \end{cases}$$

$u_{ab} = \hat{E} \operatorname{Sen} \omega t \qquad \omega_1 = 100\pi \ \frac{rad}{s}$





$\hat{\underline{E}} = \hat{E} e^{\dots}$

$$i_{ab} = \sum_{k=1}^{\infty} I_m \left\{ \hat{\underline{I}}_k e^{jk\omega_1 t} \right\} = \sum_{k=1}^{\infty} \hat{I}_k \operatorname{Sen}(k\omega_1 t + \phi_k)$$

$$\hat{\underline{I}}_k = \hat{I}_k e^{j\phi_k}$$

$$i_{ab} = I_o \operatorname{Sen} \omega_1 t = \hat{I}_o \operatorname{Sen} \varepsilon = \hat{I}_o \frac{e^{j\varepsilon} - e^{-j\varepsilon}}{2j} \longleftarrow \left| \begin{array}{l} e^{j\varepsilon} = \cos\varepsilon + j\operatorname{Se}\dots \\ e^{-j\varepsilon} = \cos\varepsilon - j\operatorname{S}\dots \end{array} \right.$$

$$\hat{\underline{I}}_k = \frac{2j}{\pi} \int_0^{\pi} i_{ab}(\varepsilon) e^{-jk\varepsilon} d\varepsilon = \frac{2j}{\pi} \int_{\frac{\pi}{2}}^{\pi} \frac{\hat{I}_o}{2j} \left[e^{-j(k-1)\varepsilon} - e^{-j(k+1)\varepsilon} \right] d\varepsilon =$$

$$= \frac{\hat{I}_o}{\pi} \left[\frac{e^{-j(k-1)\varepsilon}}{-j(k-1)} - \frac{e^{-j(k+1)\varepsilon}}{-j(k+1)} \right]_{\frac{\pi}{2}}^{\pi} = j \frac{\hat{I}_o}{\pi} \left[\underbrace{\frac{e^{-j(k-1)\pi} - e^{-j(k-1)\pi/2}}{k-1}}_{\text{1 por ser k impar}} - \frac{e^{-j(k+1)\pi} \dots}{k+1} \right.$$

Para $K=3 \rightarrow \hat{I}_3 = j\,\frac{\hat{I}_o}{\pi}\left[\frac{1-e^{-j\pi}}{3-1} - \frac{1-e^{-j2\pi}}{3+1}\right] = j\,\frac{\hat{I}_o}{\pi}\left[1 - 0\right.$

$K=5 \rightarrow \hat{I}_5 = j\,\frac{\hat{I}_o}{\pi}\left[\frac{1-1}{5-1} - \frac{1-(-1)}{5+1}\right] = -j\,\frac{\hat{I}_o}{3\pi}$

$K=1 \longrightarrow \hat{I}_1 = j\,\frac{\hat{I}_o}{\pi}\left[\frac{e^{-j0}-e^{-j0}}{1-1} - \frac{1-e^{-j\pi}}{2}\right] = j\,\frac{\hat{I}_o}{\pi}\left[-j\frac{\pi}{2} - \frac{1-(-1)}{2}\right.$

↳ hay que aplicar L'hopital derivando respecto numerador y denominador.

$$\lim_{x\to 0} \frac{e^{-j\pi x} - e^{-j\frac{\pi}{2}x}}{x} = \frac{-j\pi e^{-j\pi x} - (-j\frac{\pi}{2})e^{-j\frac{\pi}{2}}}{1} \ldots \frac{\pi}{2}$$

$$S_1 = E\,I_1^{*} = \frac{1}{2}\hat{E}\cdot\hat{I}_1^{*} = \frac{1}{2}\hat{E}\,\frac{\hat{I}_o}{\pi}\left[\frac{\pi}{2}+j\right] = P + jQ \quad \begin{cases} P = \frac{1}{4}\hat{E}\cdot\hat{I}_o \\ Q = \frac{\hat{E}\hat{I}_o}{2\pi}\end{cases}$$

$$S_1 = E\,I_1 = \sqrt{P^2+Q^2} = \frac{\hat{E}\hat{I}_o}{2\pi}\sqrt{1+\frac{\pi^2}{4}}$$

$$S = E\,I$$

$$I^2 = \frac{1}{T}\int_0^T i_{ab}^2\,dt = \frac{1}{\pi}\int_0^{\pi} i_{ab}^2(\varepsilon)\,d\varepsilon = \frac{1}{\pi}\int_{\frac{\pi}{2}}^{\pi}\hat{I}_o^2 \,\mathrm{Sen}^2\varepsilon\, d\varepsilon = \frac{\hat{I}_o^2}{4} \Rightarrow$$

$$\Rightarrow S = \frac{\hat{E}}{\sqrt{2}}\,\frac{\hat{I}_o}{2}$$

$$\lambda_q \text{ (factor de potencia por reactiva)} = \frac{P}{S_1} = \frac{I_P}{I_1} = \frac{\hat{I}_P}{\hat{I}_1} = \frac{\hat{I}_o/2}{\frac{\hat{I}_o}{\pi}\sqrt{1+\frac{\pi^2}{4}}} =$$

$$\lambda_h \text{ (por distorsión)} = \frac{S_1}{S} = \frac{I_1}{I} = \frac{\hat{I}_1/\sqrt{2}}{I} = \frac{\frac{\hat{I}_o}{\pi}\sqrt{1+\frac{\pi^2}{4}}}{\sqrt{2}\cdot\hat{I}_o}\cdot 2 = \frac{2}{\sqrt{2}\cdot\pi}\sqrt{1+\frac{\pi^2}{4}}$$

$$\lambda = \lambda_q \cdot \lambda_h$$

Si el problema hubiera sido:

$u_{ab} = \hat{E}\,\mathrm{Sen}(\omega t + \pi/4) \qquad \frac{\pi}{4} < \varepsilon < \frac{3\pi}{4}$

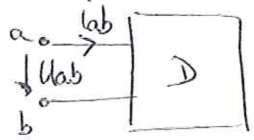
$i_{ab} = \hat{I}_o\,\mathrm{Sen}(\omega_1 t + \frac{\pi}{4}) \rightarrow i_{ab} = \mathrm{Im}\left\{\underbrace{\hat{I}_o e^{j\theta}}_{\hat{I}_o} e^{j\omega_1 t}\right\} = \frac{\hat{I}_o e^{j\varepsilon} - \ldots}{2j}$

$\hat{E} = \hat{E}\,e^{j\theta} = \hat{E}\,e^{j\frac{\pi}{4}}$

Se pue

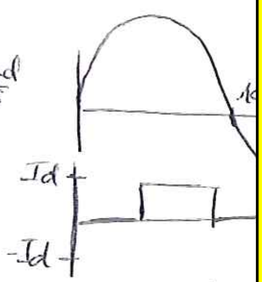
$$\hat{\underline{I}}_k = \frac{2j}{\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\hat{\underline{I}}_0 e^{-j(k-1)\varepsilon} - \hat{\underline{I}}_0^{*} e^{-j(k+1)\varepsilon}}{2j}\, d\varepsilon$$

Ver Problema 2 Junio 2012



$u_{ab} = \hat{E}\,\mathrm{Sen}\,\omega_1 t \qquad \omega_1 = 100\pi \frac{rad}{s}$

$$i_{ab} = \begin{cases} 0 & 0 \le t \le 5ms \\ I_d & 5 \le t \le 10 \end{cases}$$



$$\hat{\underline{I}}_k = \frac{2j}{\pi}\int_0^{\pi} i_{ab}(\varepsilon) e^{-jk\varepsilon} d\varepsilon = \frac{2j}{\pi}\int_{\frac{\pi}{2}}^{\pi} I_d e^{-jk\varepsilon} d\varepsilon = 2j\frac{I_d}{\pi}\left(+\frac{e^{-jk\varepsilon}}{-jk}\right.$$

$$= \frac{2jI_d}{\pi}\left(\frac{e^{-jk\pi} - e^{-jk\frac{\pi}{2}}}{-jk}\right) = \frac{2I_d}{\pi k}\left[1 + e^{-jk\frac{\pi}{2}}\right]$$

$$\hat{\underline{I}}_1 = \frac{2I_d}{\pi}\left[1 - j\right] \qquad \underline{S}_1 = \frac{1}{2}\hat{\underline{E}}\,\hat{\underline{I}}_1^{*} = \frac{1}{2}\hat{E}\cdot\frac{2I_d}{\pi}(1+j)$$

$$\hookrightarrow P = Q = \frac{\hat{E} I_d}{\pi}$$

$$\hat{\underline{I}}_3 = \frac{2I_d}{3\pi}\left[1 + j\right]$$

$$\hat{\underline{I}}_5 = \frac{2I_d}{5\pi}\left[1 - j\right]$$

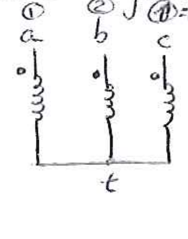
$$I^2 = \frac{1}{\pi}\int_0^{\pi} i_{ab}^2 d\varepsilon = \frac{1}{\pi}\int_{\pi/2}^{\pi} I_d^2 d\varepsilon = \frac{I_d^2}{2}$$

$$S = E\cdot I = \frac{\hat{E}}{\sqrt{2}}\cdot\frac{I_d}{\sqrt{2}} = \frac{1}{2}\hat{E} I_d$$

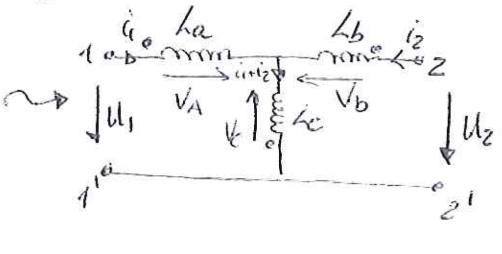
$$\hat{\underline{I}}_1 = \hat{I}_{r1} + j\,\hat{I}_{x1}$$
$$\downarrow \qquad \downarrow$$
$$\hat{I}_p \qquad -\hat{I}_q$$

$$\lambda_q = \frac{1}{\sqrt{2}} \; ; \; \lambda_h = \frac{S_1}{S} = \frac{I_1}{I} = \frac{2I_d}{\pi}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

Problema junio 2012



$\rightsquigarrow$



$V_A = L_a\cdot i_1' + M_{ab} i_2' - M_{ca}$

$V_B = L_b\cdot i_2' + M_{ba}\cdot i_1' - M_{bc}$

$V_c = -L_c(i_1' + i_2') + M_{ca}$

$V_A = (L_a - M_{ca})i_1' + (M_{ab} - M_{ca})i_2'$

$V_B = (M_{ab} - M_{bc})i_1' + (L_b - M_{bc})i_2'$

$V_c = (M_{ca} - L_c)i_1' + (M_{bc} - L_c)i_2'$

$u_1 = u_a - u_c = (L_a + L_c - 2M_{ca}$

$+ (L_c + M_{ab} - M_{bc} - M_{ca})i_2'$

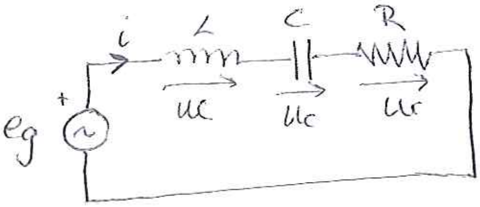
$u_2 = V_b - V_c = (L_c + M_{ab} - M_{bc} - M_{ca})$

M_{bc}

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \rightsquigarrow Z_{11} = (L_a + L_c - 2M_{ca})$$

y así...

RESPUESTA EN FRECUENCIA.



$u_r = R \cdot i \rightarrow \hat{U}_r = R\,\hat{I}$

$u_L = L\dfrac{di}{dt} \rightarrow \hat{U}_L = j\omega L\,\hat{I}$

$i = C\dfrac{du}{dt} \rightarrow \hat{I} = j\omega C\,\hat{U}$

$\omega = 2\pi f$ ⤳ varía de 0 a ∞

Para no arrastrar ω ⤳ llamamos $\omega_1 = 2\pi f_1 = 2\pi \cdot 50 = 100\pi$

$\Longrightarrow \omega = K \cdot \omega_1$

$X_L = \omega_1 L$
$B_C = \omega_1 C$
$\rightarrow \hat{U}_L = jKX_L\hat{I}$, $\hat{I} = jKB_C\hat{U}_C$ $\rightarrow \hat{E}_g = U_r + U_L + U_C$

$\rightarrow \hat{E}_g = R\hat{I} + jKX_L\hat{I} + \dfrac{1}{jKB_C}\hat{I} = \left[R + j\left(KX_L - \dfrac{1}{KB_C}\right)\right]\hat{I} =$

$\underline{Z} = \underset{\geq 0}{R} + j\underset{\gtrless 0}{X_K}$ $\quad X_K = KX_L - \dfrac{1}{KB_C}$ ⤳ Habrá un $K_r = K$ ... frecuencia de ...

$K = \dfrac{f}{f_1} = \dfrac{\omega}{\omega_1}$ $\qquad K_r = \dfrac{1}{\sqrt{X_L B_C}}$

Si $K > K_r \rightarrow X_K > 0$ induct
Si $K < K_r \rightarrow X_K < 0$ capacit
Si $K = K_r \rightarrow X_K = 0$ resistivo pu...

$Z = \sqrt{R^2 + X_K^2}$ ⤳ $K = K_r \rightarrow Z = R = Z_{min}$.

$f_r = f_1 K_r$
$\omega_r = K_r\omega_1$

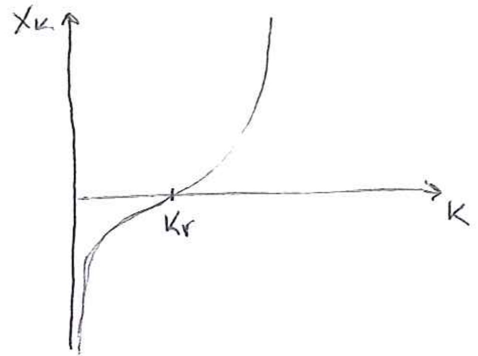
$\lambda_1 = \dfrac{Q_L}{P} = \dfrac{K_r X_L I^2}{RI^2} = \dfrac{K_r X_L}{R}$

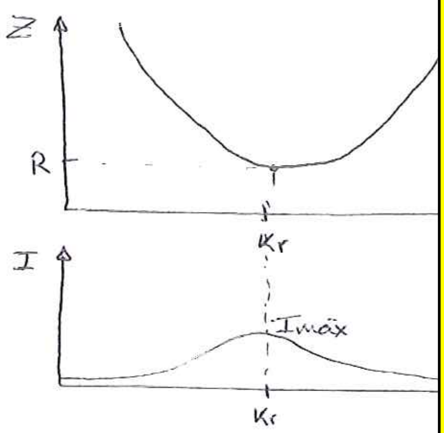
$X_K = K\dfrac{R\lambda_r}{K_r} - \dfrac{1}{K}K_r^2 X_L = K\dfrac{R\lambda_r}{K_r} - \dfrac{K_r^2}{K}\dfrac{R\lambda_r}{K_r} = R\lambda_r\left[\dfrac{K}{K_r}\right.$

$\boxed{\nu = \dfrac{K}{K_r} - \dfrac{K_r}{K}} \Rightarrow \boxed{X_K = R\nu\lambda_r}$

$K = K_r \Rightarrow \nu = 0 \Rightarrow X_K$
$K < K_r \Rightarrow \nu < 0 \Rightarrow X_K$
$K > K_r \Rightarrow \nu > 0 \Rightarrow X_K$

$$\left.\begin{array}{l} K=0 \\ K=\infty \end{array}\right| \Rightarrow \nu = \infty \quad \begin{array}{l} \text{frecuencia } 0 \Rightarrow \text{Impedancia inf (corrie...} \\ \text{frecuencia } \infty \Rightarrow \text{Impedancia inf} \end{array}$$





$$\underline{E_g} = \underline{Z}\,\underline{I} \qquad E_g = Z I = R\sqrt{1+(\nu\lambda_r)^2}\, I \;;\; I = \frac{E_g}{Z}$$

$$Z = Z_{min} \Rightarrow I = I_{máx} = \frac{E_g}{R}$$

$$\underline{U_e} + \underline{U_c} = 0 \;;\; \underline{U_e} \neq 0 \quad \underline{U_c} \neq 0$$

$$K = K_r \Rightarrow \underline{U_e} = j K_r X_e \underline{I} = j K_r \left(\frac{\lambda_r R}{K_r}\right) \frac{E_g}{R} = j \lambda_r \underline{E_g}$$

$$U_c = \frac{1}{j K_r B_c} \underline{I} = \frac{K_r^2 X_e}{j K_r} \underline{I} = \frac{\cancel{K_r^2}\, \frac{\lambda_r R}{\cancel{K_r}}}{j \cancel{K_r}} \frac{E_g}{R} = -j \lambda_r \underline{E_g}$$

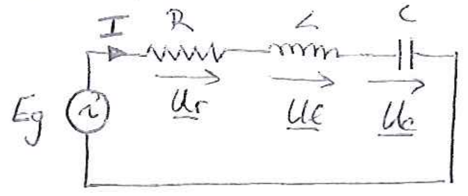
$$\frac{1}{B_c} = K_r^2 X_e$$

* Entorno de la frecuencia de resonancia.

$$\delta = \frac{K - K_r}{K_r} \;;\; \nu = \frac{K^2 - K_r^2}{K \cdot K_r} = \frac{(K+K_r)(K-K_r)}{K K_r} = \frac{K + K_r}{K}\delta$$

$$\left.\begin{array}{l} K \simeq K_r \Rightarrow \nu = 2\delta \\ X_K = R\lambda_r\nu \end{array}\right| \; X_K \simeq 2R\lambda_r\delta \Rightarrow \boxed{\underline{Z} \simeq R\left[1 + j\ldots\right.}$$

Resumen.



$k = \frac{\omega}{\omega_1} = \frac{f}{f_1}$

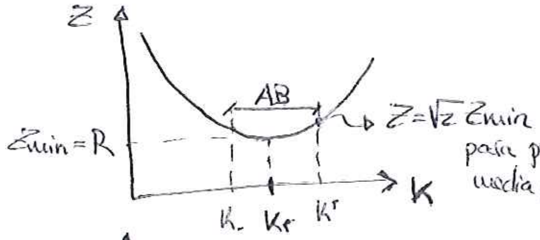
$B_c = \omega_1$

$X\ell = \omega_1$

$\underline{\underline{Z}} = R + j X_k = R\left[1 + j\nu\lambda_r\right]$ | $Z = R\sqrt{1+(\nu\lambda_r)^2}$

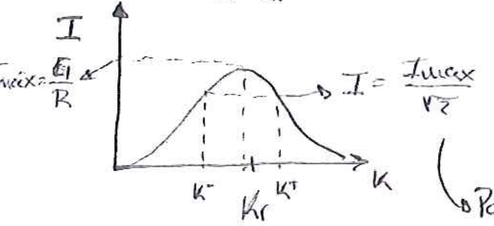
$X_k = k X\ell - \frac{1}{kB_c} = R\nu\lambda_r$ | $\underline{\underline{I}} = \frac{Eg}{\underline{\underline{Z}}}$; $I = \frac{Eg}{Z}$

$\boxed{\lambda_r = \frac{k_r X\ell}{R}}$ $\boxed{k_r^2 = \frac{1}{X\ell B_c}}$ $\boxed{\nu = \frac{k}{k_r} - \frac{k_r}{k}}$



$\delta = \frac{k-k_r}{k_r} = \frac{f-f_r}{f_r} = \frac{\omega-\omega_r}{\omega_r}$

$k \approx k_r \Rightarrow \nu \approx 2\delta$



$Z \approx R\left[1 + j \cdot 2 \cdot \lambda_r \cdot \delta\right]$

$Z = R\sqrt{1+4\lambda_r^2\delta^2} = Z_{min}\sqrt{1+4\lambda_r^2\delta^2}$

Para dar puntos de media potencia.

$I = \frac{Eg}{Z} = \frac{Eg}{Z_{min}\sqrt{1+4\lambda_r^2\delta^2}} = \frac{I_{max}}{\sqrt{1+4\lambda_r^2\delta^2}}$

- - - - - - - -

$\left.\begin{array}{l} P = RI^2 \\ P_{max} = R I_{max}^2 \end{array}\right\} \frac{P}{P_{max}} = \left(\frac{I}{I_{max}}\right)^2 = \frac{1}{1+4\lambda_r^2\delta^2}$ $\begin{cases} P_+ = \frac{1}{2} P_{max} \Rightarrow 2\lambda \\ P_- = \frac{1}{2} P_{max} \Rightarrow 2\lambda_r \end{cases}$

$\Rightarrow \delta_+ = + \frac{1}{2\lambda_r} = \frac{k_+ - k_r}{k_r}$

$\Rightarrow \delta_- = - \frac{1}{2\lambda_r} = \frac{k_- - k_r}{k_r}$

$\Rightarrow \delta_+ - \delta_- = \frac{1}{\lambda_r} = \frac{k_+ - k_-}{k_r} = \frac{f}{}$ ... ω / r

$\frac{I_+}{I_{max}} = \frac{1}{\sqrt{2}} = 2^{-1/2} \Rightarrow \log\frac{I_+}{I_{max}} = \frac{-1}{2}\log 2 = \frac{-1}{2}\cdot 0'3 \Rightarrow 20 \cdot \log\frac{I_+}{I_{max}}$

$AB = f_+ - f_-$; $\frac{AB}{f_1} = ab = k_+ - k_- = (\delta_+ - \delta_-)k_r = \frac{k_r}{\lambda_r}$

$$\boxed{\begin{aligned} \Delta\delta_{+-} &= \delta_+ - \delta_- = \frac{1}{\lambda_r} \\ \Delta K_{+-} &= K_+ - K_- = \frac{K_r}{\lambda_r} \\ \Delta f_{+-} &= f_+ - f_- = \frac{f_s}{\lambda_r} \end{aligned}}$$

$\downarrow AB \Rightarrow \uparrow$ selectividad.

$A \quad f$ = infinit…
contrario q…

$\nu_{K_r} = \pm 1$ → $\nu_+ \lambda_r = 1$; → $\nu_- K_r = -1$

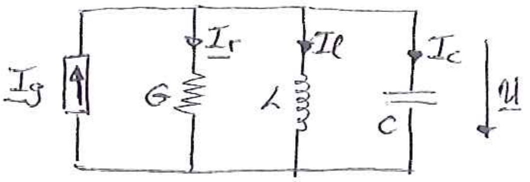
$$\nu = \frac{K}{K_r} - \frac{K_r}{K} = \frac{K^2 - K_r^2}{K_r K}.$$

De forma rigurosa, sin aproximación:

$$\nu_+ \lambda_r = 1 \longrightarrow \frac{K_+^2 - K_r^2}{K_+ K_r}\lambda_r = 1 \Rightarrow \left(K_+^2 - K_r^2\right)\lambda_r = K_r K_+ \quad y \ldots$$

$$\nu_- \lambda_r = -1$$

Circuito Paralelo.



$$\underline{I}_g = \underline{I}_r + \underline{I}_l + \underline{I}_c = G\underline{U} + jB\ldots$$

$$\underline{Y} = G + jB_K \; ; \; B_K = KB_c - \frac{1}{KB_c}$$

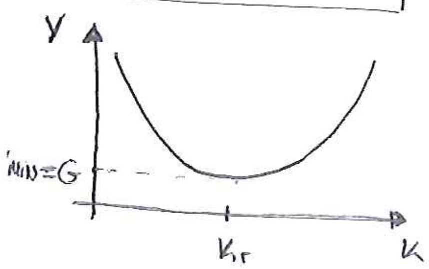
$$\boxed{K_r^2 = \frac{1}{X_e B_c}}$$

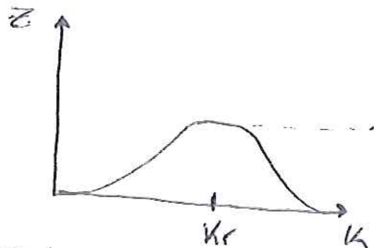
$$B_K = G\nu\lambda_r$$

En este circuito: $\lambda_r = \dfrac{U^2/(K_r\ldots)}{G\,U^2}$

factor de calidad

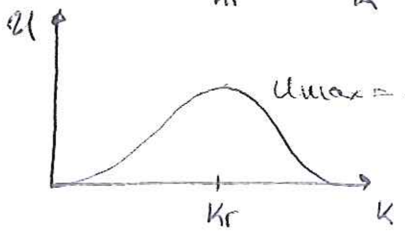
$$\boxed{Y = G\sqrt{1 + (\nu\lambda_r)^2}}$$





$$\frac{1}{Y_{min}} = \frac{1}{G} = Z_{max}.$$

$$U = \frac{I_g}{Y} \; ;$$

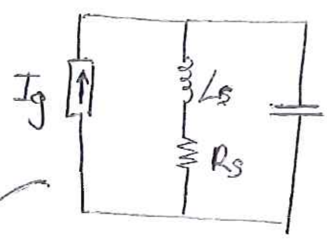


Aplicando dualidad:

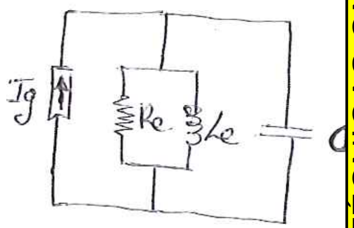
$$\left.\begin{array}{l}\underline{I}_l = -j\lambda_r \underline{I}_g \\ \underline{I}_c = +j\lambda_r \underline{I}_g\end{array}\right| \text{Si } k = k_r$$

Los puntos de potencia se definen igual, así

Problema. Caso práctico.



Buscamos uno equivalente en paralelo.



$$\begin{cases} X_{ls} = \omega_1 L_s \\ \lambda_s = \dfrac{kX_{ls}}{R_s}\end{cases}$$

$$\begin{cases} X_e = \omega L_e \\ \lambda_e = \dfrac{R_e}{kX_e}\end{cases} \Longrightarrow \begin{cases} R_e = R_s(1+\lambda_s^2) \\ X_{le} = X_{ls}\left(1+\dfrac{1}{\lambda_s^2}\right)\end{cases}$$

$\lambda_e = \lambda_s$

$$\text{Si } k = k_r \left|\ \lambda_{sr} = \frac{k_r X_{ls}}{R_s} \Rightarrow \begin{cases} R_e = R_s(1+\lambda_{sr}^2) \longrightarrow R_e \simeq \lambda_{sr}^2 R_s \text{ para} \\ X_{le} = X_{ls}\left(1+\dfrac{1}{\lambda_{sr}^2}\right) \longrightarrow X_{le} \simeq X_{ls} \text{ para}\end{cases}\right.$$

Se demuestra:

$$\boxed{K_r = K_{ri}\sqrt{1-\frac{1}{\lambda_{sri}^2}}} \quad (*)$$

$$\boxed{K_{ri}^2 = \frac{1}{X_{ls}B_c}} \qquad \boxed{\lambda_{sri} = \frac{k_{ri}X_{ls}}{R_s}}$$

Vamos a demostrar (*). Hay problemas similares:

$$\underline{Z}_s = R_s + jkX_{ls}$$

$$\underline{Y} = jkB_c + \frac{1}{\underline{Z}_s} = jkB_c + \frac{R_s - jkX_{ls}}{R_s^2 + k^2X_{ls}^2} = \frac{R_s}{R_s^2 + (kX_{ls})^2} + j\Big[kB_c -$$

$$= G_k + jB_k$$

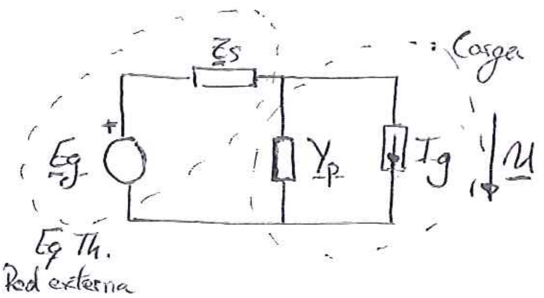
varía con f!

$$K = k_r \Rightarrow B_k = 0 \Rightarrow \boxed{B_c = \frac{X_{ls}}{R_s^2 + k_r^2 X_{ls}^2}} \Rightarrow$$

$$\Rightarrow R_s^2 + K_{ri}^2 X_{ls}^2 = \frac{X_{ls}}{B_c} \quad \Rightarrow K_{ri}^2 = \frac{1}{X_{ls}^2}\left[\frac{X_{ls}}{B_c} - R_s^2\right] = \frac{1}{X_{ls}B_c} -$$

$$= \underbrace{\frac{1}{X_{ls}B_c}}_{K_{ri}^2}\left[1 - \frac{R_s^2 \overbrace{X_{ls}B_c}^{1/K_{ri}^2}}{X_{ls}^2}\right] = K_{ri}^2\left[1 - \frac{R_s^2}{\underbrace{K_{ri}^2 X_{ls}^2}_{1/\lambda_{sri}^2}}\right] = K_{ri}^2\left[1 - \right.$$

Otro ejemplo:

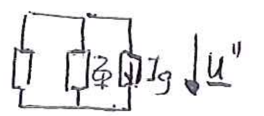


$Z_s = R_s + jkX_s$

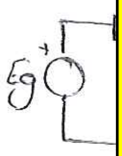
$Y_p = jkB_c$

$U = U' + U''$

$U' = \frac{Z_p}{Z_p + Z_s} E_g$



$Z_{eq} = Z_s + Z_p = R_s + jkX_s + \frac{1}{jkB_c}$



$Y_{ig} = Y_s + Y_p \Rightarrow Z_{ig} = \frac{1}{Y_{ig}} = \frac{1}{Y_s + Y}$; $U'' = Z_{ig} I_g$

$Y_{ig} = \frac{1}{R_s + jkX_s} + jkB_c$

Es el problema de un transformador de características:

20 kV / 400 V ; 1 MVA; $\varepsilon_{ccr} = 1\%$; $\varepsilon_{ccx} = 5\%$

407'83 kVA a 400 V

800 kVA a 400 V a 50 Hz

$I_g(k=7) = \frac{1}{7} I_g(k=1)$ | $E_g(k=1) =$

$E_g(k=7) = \frac{1}{100} E_g(k=1)$ | $\Rightarrow E_g($

$800\,kVA = \sqrt{3} \cdot 400 \cdot I_g(k=1) \Rightarrow I_g(k=7) = \frac{1}{7} \cdot \frac{800 \cdot 10^3}{\sqrt{3} \cdot 400} = \underline{165\,A}$

$$R_s = 0'01 \cdot \frac{(0'4\,kV)^2}{1\,MVA} = 1'6\,m\Omega \;; \quad X_s = 5 R_s = 8\,m\Omega$$

$$B_c = \frac{407'83}{(0'4\,kV)^2} = 2'54894\,S$$

• Caso 0 : $B_c = 0$

$\underline{U'} = \underline{E_g}$ $\quad U'(k=7) = \frac{4}{\sqrt{3}} V = 1\% U_{ref}$ (aguas arriba)

$$U'' = Z_{ig} I_g = Z_s I_g = \sqrt{R_s^2 + k X_s^2}\; I_g \longrightarrow k = 7 \longrightarrow U'' = 9'24378\,V$$

• Caso 1: $B_c \neq 0$

E_g:

$$K_{rs} = \frac{1}{\sqrt{X_s B_c}} = \frac{1}{\sqrt{8 \cdot 10^{-5} \cdot 2'54}} = 7'00286 \approx 7 \longrightarrow \lambda_{rs} = \frac{7'00286 \cdot 8 \cdot 10^{-3}}{1'6 \cdot 10^{-3}} = 35'0$$

$$U'(k=7) = \lambda_{rs}\, E_g(k=7) = 35'0143 \cdot \frac{4}{\sqrt{3}} V = 80'8621 = \underline{35'01\% U_{ref}}.$$

I_g:

$$K_{rp} = 7'00286 \sqrt{1 - \frac{1}{35'0143^2}} = 7 \;;$$

$$\lambda_{rp} = K_{rp} X_s / R_s = 35 \implies R_e \simeq R_s \lambda_{rp}^2 = 1'6 \cdot 10^{-3} \cdot 35^2$$

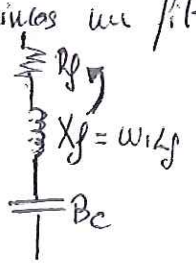
$$U''(k=7) = R_e\, I_g(k=7) = 323'664\,V = \underline{140'15\% U_{ref}}$$

$$\left. \begin{array}{l} I_c'(k=7) = k B_c U' \\ I_{ref}(k=1) = B_c U_{ref} \end{array} \right| \quad \frac{I_c'(k=7)}{I_{cref}} = k \frac{U'}{U_{ref}} = 7 \cdot 35'01 = 245\%$$

$$\frac{I_c''(k=7)}{I_{cref}} = 7 \cdot \frac{U''}{U_{ref}} = 7 \cdot 140'15 = 980\%.$$

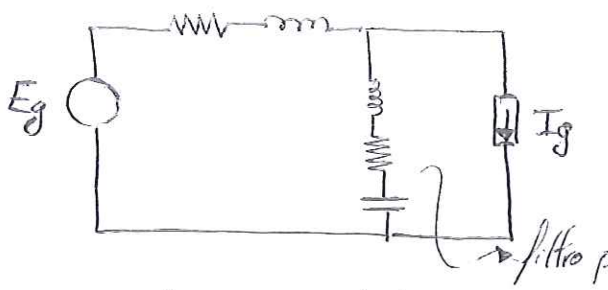
Con efecto Skin
$\Rightarrow \downarrow \lambda_{rs}$ ate

→ Añadimos un filtro

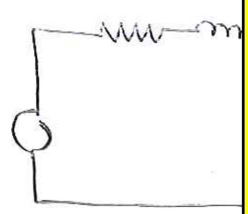


$$\underline{Y_p} = \frac{1}{R_f + jkX_f + \frac{1}{jkB_c}}$$

$$\underline{Z_p} = R_f + jkX_f - \frac{j}{kB_c} \implies K_0 X_f - \frac{1}{K_0 B_c} = 0$$



Si $K = K_o$ ⇒



Si fuese un filtro ideal sería un cortocircuito para el sé[...]co.

$$\underline{Z}_p = R_f + jK\frac{1}{K_o^2 B_c} - j\frac{1}{KB_c} = R_f + j\frac{1}{B_c}\left(\frac{K}{K_o^2} - \frac{1}{K}\right) = \frac{1}{\lambda_o K_o B[\dots]} \quad [\dots] \frac{1}{K}\Big)$$

$$\lambda_o = \frac{K_o X_f}{R_f} \Rightarrow R_f = \frac{K_o X_f}{\lambda_o} = \frac{K_o}{\lambda_o}\left(\frac{1}{K_o^2 B_c}\right) = \frac{1}{\lambda_o K_o B_c}$$

* Si $K = K_o \Rightarrow \underline{Z}_p = \frac{1}{\lambda_o K_o B_o}$

* Si $\begin{cases} K = 1 \\ R_f = 0 \end{cases} \Rightarrow \underline{Z}_p = j\frac{1}{B_c}\left(\frac{1}{K_o^2} - 1\right) \Rightarrow \underline{Y}_p = j\frac{K_o^2}{K_o^2 - 1}B_c \Rightarrow B_p = \frac{K_o^2}{K_o^2 - 1}B[\dots]$

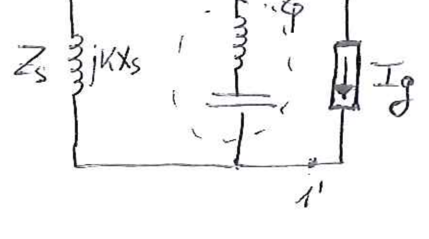
$\Rightarrow Q_p = B_p U_{ref}^2 = \frac{K_o^2}{K_o^2 - 1}B_c \cdot U_{ref}^2$

reactiva

$0 \le K \le K_o \Rightarrow B_p \ge 0 \rightarrow$ Ca[...]tivo

$K_o \le K < \infty \Rightarrow B_p \le 0 \rightarrow$ [...]tivo

Si quitamos las resistencias:

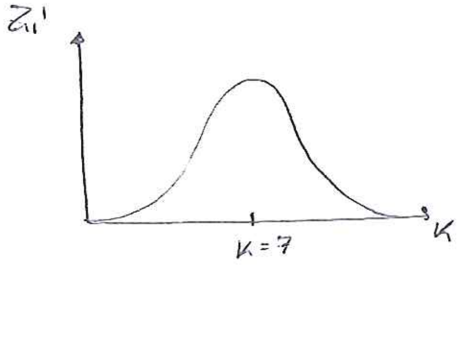


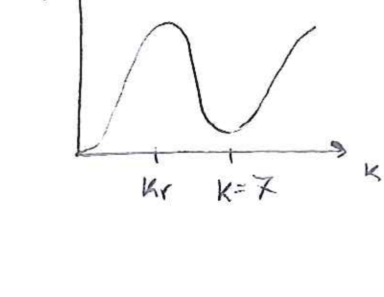
$$\underline{Z}_p = j\frac{1}{B_c}\left(\frac{K}{K_o^2} - \frac{1}{K}\right) = j\cdot\frac{1}{B_c}\cdot\frac{K^2 - K_o^2}{K_o^2 K} \Rightarrow [\dots]$$

$$\underline{Y}_{11'} = \underline{Y}_p + \underline{Y}_s = jB_c\frac{K_o^2 K}{K_o^2 - K} - \frac{j}{KX_s} \Rightarrow B_c[\dots] \quad [\dots] \Rightarrow$$

$$\Rightarrow B_c K_o^2 K_r^2 X_s = K_o^2 - K_r^2 \Rightarrow K_r^2\left[X[\dots]\right.$$

$K_r < K_o$





Ejemplo con la batería de condensadores:

$K_0 = 7$
$\lambda_0 = 50$
$B_c = 2'54894$

• Calcular frecuencia de antirresonancia y $U''(k=5)$

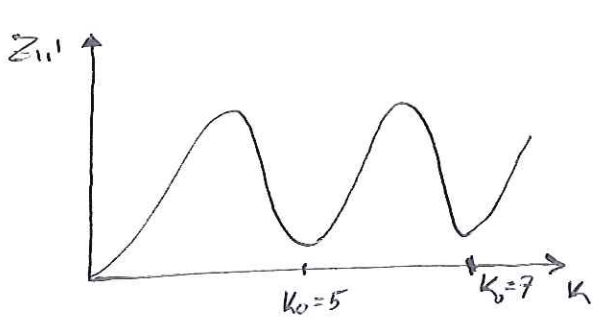
$I_g(k=5) = \frac{1}{5} I_g(k=1) \approx 165\,A$.

$$K_r = \frac{7}{\sqrt{1+8\cdot 10^{-3}\cdot 2'54\cdot 7}} = 4'95 \approx 5 \qquad \lambda_0 = \frac{K_0 X_f}{R_f}$$

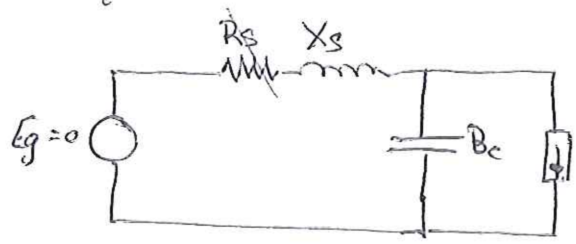
$$K_0^2 = \frac{1}{X_f B_c} \implies X_f = \frac{1}{K_0^2 B_c} = \frac{1}{7^2\, 2'54} \Rightarrow R_f = \frac{K_0 X}{\lambda} \quad '54$$

$$\underline{Y}_{11}'(k=5) = \frac{1}{R_s + j5X_s} + \frac{1}{R_f + j5X_f - j\frac{1}{5B_c}} \Rightarrow \underset{(k=5)}{\underline{Z}_{11}'} = \frac{1}{\underline{Y}_{11}'} \Rightarrow$$

$$\Rightarrow u''(k=5) = Z_{11}'(k=5)\cdot I_g(k=5) = \underline{49'02\%\, U_{ref}}$$

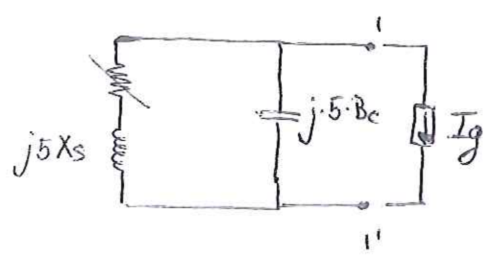


Junio 2012.



$B_c = 4'13\,S$

$X_s = 8\,m\Omega$

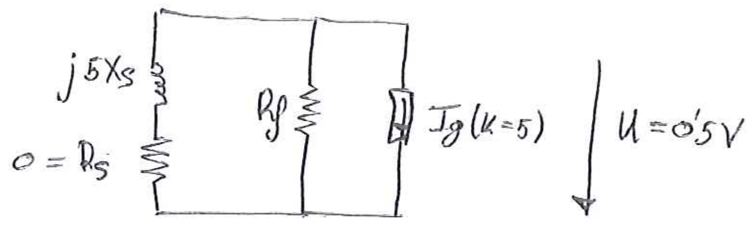


$$\underline{Y}_{ig} = j5B_c + \frac{1}{j5X_s} \Rightarrow \underline{Z}_{ig} =$$

$$\Rightarrow u = \frac{5\cdot 8\cdot 10^{-3}}{1-5^2\cdot 8\cdot 10^{-3}\cdot 4'13}\cdot 200 = 45$$

2)
$$K_0^2 = \frac{1}{X_f B_c} \Rightarrow 5^2 = \frac{1}{X_f \cdot 4'13} \Rightarrow X_f = \frac{1}{25 \cdot 4'13} = 9'68 \ m\Omega$$

3) $$\lambda_{f_1} = \frac{1 \times X_f}{R_f}$$



K=5

$$Z_{ig} = \frac{R_f(R_s + j5X_s)}{R_f + R_s + j5X_s} \Rightarrow Z_{ig} = \frac{R_f\sqrt{R_s^2 + 25X_s^2}}{\sqrt{(R_f+R_s)^2 + 25X_s^2}} = \frac{U}{I_g} = \frac{0}{20}$$

$$\Rightarrow \underline{R_f = 2'5049 \ m\Omega} \qquad \text{ya } \underline{R_s = 0}$$

$$\lambda_{f_1} = \frac{9'68}{2'50} = 3'86 \rightarrow \lambda_{f_0} = 5\lambda_{f_1} = 19'33$$

$$U_c(K=5) = \lambda_{f_0} \cdot U = 19'33 \cdot 0'5 = \underline{9'66 \ V}$$

$$\Delta P_f = R_f \cdot I_f^2 = R_f \cdot \frac{U^2}{R_f^2} = \frac{U^2}{R_f} = \frac{0'5^2}{2'5049 \cdot 10^{-3}} = \underline{99'804 \ W}$$

Soluciones:

Ejercicio 1. Julio 2012

$R_s = 1'6 \ m\Omega$ y $R_s = \cancel{1\Omega}$ → muy grande.

$B_c = 4'992 \ S$

Enero 2013. Ejercicio 1.

$R_s = 1'6 \ m\Omega$

$B_c = 2'44507 \ S$

$X_f = 8'3467 \ mH$.