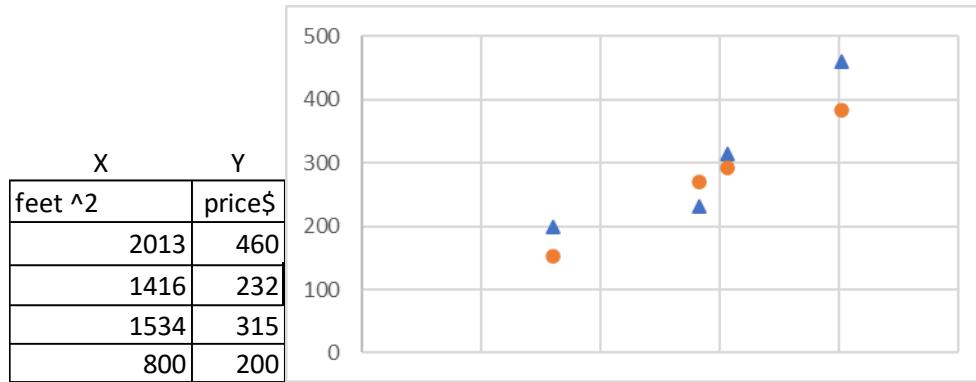


### Exercise 1\_ Prediction and inference\_ big data\_machine learning

1.- 1.- We want to predict the price of a new house in the USA. To run that prediction, we know some prices (Y dependent variable) and their corresponding surface (X feet ^2) (independent variable or predictor). We suppose a linear relationship between this  $h_{\theta} = \Theta_0 + \Theta_1 X$ , being  $X$  the feet ^2 and  $h_{\theta}$  the prediction. To be able to have a prediction, we need to calculate  $\Theta_0$  and  $\Theta_1$  fitting the linear function to the data. In the graph below circles are the real prices and triangles the prediction



The criterion to find what is the best possible solution is to minimise the summation of squared errors using the formula (m is the size of the sample, in our case 4)

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

#### ITERATION 0

To find what is the minimum error we begin guessing some solution  $\Theta_0=0$  and  $\Theta_1=0,21$ , and we calculate the summation of the square error (please fill in the blanks)

| $\Theta_0$        | 0                                | feet ^2 | price\$ | $h_{\theta}$            | $h_{\theta}-y_i$ | $(h_{\theta}-y_i)^2$ |
|-------------------|----------------------------------|---------|---------|-------------------------|------------------|----------------------|
| $\Theta_1$        | 0,19                             | 2013    | 460     | 382,47                  | -77,53           | 6010,90              |
|                   |                                  | 1416    | 232     |                         |                  |                      |
|                   |                                  | 1534    | 315     | 291,46                  | -23,54           | 554,13               |
|                   |                                  | 800     | 200     | 152,00                  | -48,00           | 2304,00              |
|                   |                                  |         |         | suma                    | -112,03          | 10240,99             |
| number of train m | 4                                |         |         | $J(\Theta_0, \Theta_1)$ |                  |                      |
| linear regression | $h_{\theta}=\Theta_0+\Theta_1 X$ |         |         |                         |                  |                      |

Then we calculate the new  $\Theta_0$  and  $\Theta_1$ . To choose this new parameters we consider the previous solution, and we apply the algorithm called gradient descent. Lets be alfa the learning parameter

|            | j=0                  | j=1                          |
|------------|----------------------|------------------------------|
| feet ^2    | $h_{\theta_1} - y_i$ | $(h_{\theta_1} - y_i) * x_i$ |
| 2013       | -77,53               | -156067,89                   |
| 1416       |                      |                              |
| 1534       | -23,54               | -36110,36                    |
| 800        | -48                  | -38400                       |
| sum/m      |                      |                              |
| alfa       | 0,00000001           |                              |
| iter1      |                      |                              |
| $\Theta_0$ |                      |                              |
| $\Theta_1$ |                      |                              |

$$\begin{aligned} \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) &= \frac{2}{2m} \sum_{i=1}^m (h_{\theta_0}(x^{(i)}) - y^{(i)})^2 \\ &= \frac{2}{2m} \sum_{i=1}^m (\theta_0 + \theta_1 x^{(i)} - y^{(i)})^2 \end{aligned}$$

$$\begin{aligned} j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) &= \frac{1}{m} \sum_{i=1}^m (h_{\theta_0}(x^{(i)}) - y^{(i)}) \\ j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) &= \frac{1}{m} \sum_{i=1}^m (h_{\theta_0}(x^{(i)}) - y^{(i)}) \cdot x^{(i)} \end{aligned}$$

- To calculate this fill in the blanks in the table and obtain the new  $\Theta_0$  and  $\Theta_1$

## ITERATION 1

Using the last  $\Theta_0$  and  $\Theta_1$  calculated in the table above do the process again and calculate the iteration 2 parameters

|             |  | X       | Y       |              |                           |                        |
|-------------|--|---------|---------|--------------|---------------------------|------------------------|
|             |  | feet ^2 | price\$ | $h_{\theta}$ | $h_{\theta} - y_i$        | $(h_{\theta} - y_i)^2$ |
| 2,80075E-07 |  | 2013    | 460     | 383,37       | -76,63                    | 5872,70                |
|             |  | 1416    | 232     | 269,67       | 37,67                     | 1419,07                |
|             |  | 1534    | 315     | 292,14       | -22,86                    | 522,44                 |
|             |  | 800     | 200     | 152,36       | -47,64                    | 2269,93                |
|             |  |         |         | suma         | -109,46                   | 10084,14               |
|             |  |         |         |              | J( $\Theta_0, \Theta_1$ ) | 1.261                  |

number of training m 4  
 linear regression  $h_{\theta} = \Theta_0 + \Theta_1 X$

|            | j=0                  | j=1                          |
|------------|----------------------|------------------------------|
|            | $h_{\theta_1} - y_i$ | $(h_{\theta_1} - y_i) * x_i$ |
| 2013       | -76,63               | -154263,36                   |
| 1416       | 37,67                | 53341,54                     |
| 1534       | -22,86               | -35062,44                    |
| 800        | -47,64               | -38114,99                    |
| sum/m      |                      |                              |
| alfa       | 0,00000001           |                              |
| iter 2     |                      |                              |
| $\Theta_0$ | 5,53734E-07          |                              |
| $\Theta_1$ |                      |                              |

## ITERATION 2

| X       |         | Y            |                    |                        |
|---------|---------|--------------|--------------------|------------------------|
| feet ^2 | price\$ | $h_{\theta}$ | $h_{\theta} - y_i$ | $(h_{\theta} - y_i)^2$ |
| 2013    | 460     | 384,24       | -75,76             | 5739,18                |
| 1416    | 232     |              |                    |                        |
| 1534    | 315     | 292,81       | -22,19             | 492,36                 |
| 800     | 200     | 152,70       | -47,30             | 2236,87                |
|         |         | suma         | -106,96            | 9934,30                |

number of train m 4  
 linear regression  $h_{\theta} = \Theta_0 + \Theta_1 X$

|            | j=0                  | j=1                          |
|------------|----------------------|------------------------------|
| feet ^2    | $h_{\theta_1} - y_i$ | $(h_{\theta_1} - y_i) * x_i$ |
| 2013       | -75,75740768         | -152499,6617                 |
| 1416       |                      |                              |
| 1534       | -22,18920174         | -34038,23548                 |
| 800        | -47,29554171         | -37836,43337                 |
| sum/m      | -26,7388151          | -42540,0233                  |
| alfa       |                      | 0,00000001                   |
|            | iter3                |                              |
| $\Theta_0$ | <b>8,21122E-07</b>   |                              |
| $\Theta_1$ |                      |                              |