Análisis II - 2013 - 1ª Semana

1) ejer. 14 (secc. 8.3)

$$\left.\begin{array}{l} x = t - \cos t = \frac{\pi}{4} - \frac{1}{\sqrt{2}} \\ \frac{dx}{dt} = 1 + \sin t = 1 + \frac{1}{\sqrt{2}} \\ y = 1 - \sin t = 1 - \frac{1}{\sqrt{2}} \\ \frac{dy}{dt} = -\cos t = -\frac{1}{\sqrt{2}} \end{array}\right\} t = \pi/4$$

La recta tangente tiene de ecuaciones paramétricas

$$y = \left(\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) + \left(1 + \frac{1}{\sqrt{2}}\right)t$$

$$x = \left(1 - \frac{1}{\sqrt{2}}\right) - \frac{t}{\sqrt{2}}$$

2) ejemplo 5, pág 701

3) ejer 13 (secc. 12.8)

$u = u(x,y)$
$v = v(x,y)$

$$\begin{array}{l} x = u^3 + v^3 \\ y = uv - v^2 \end{array} \quad \begin{array}{l} 1 = 3u^2 \frac{\partial u}{\partial x} + 3v^2 \frac{\partial v}{\partial x} \\ 0 = v \frac{\partial u}{\partial x} + (u - 2v) \frac{\partial v}{\partial x} \end{array} \quad u = v = 1 \left\{ \begin{array}{l} 1 = 3\frac{\partial u}{\partial x} + 3\frac{\partial v}{\partial x} \\ 0 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} \end{array} \right\} \Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 1/6$$

Derivando respecto de y y haciendo $u = v = 1$, se tiene

$$\left\{ 0 = 3\frac{\partial u}{\partial y} + 3\frac{\partial v}{\partial y},\ 1 = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} \right\} \Rightarrow \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y} = 1/2$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1/6 & 1/6 \\ 1/2 & -1/2 \end{vmatrix} = \boxed{-\frac{1}{6}}$$

4) ... $\vec{i} + (2y \sin x - 4)\vec{j} + (3xz^2 + 2)\vec{k} =$

...

$= \boxed{15 + 4\pi}$

Análisis II – 2013 – 2ª semana

1) $\boxed{\text{ejer 15 (secc. 8.3)}}$ $\left.\begin{array}{l} x = t^3 - t \\ y = t^2 \end{array}\right\}$ $(0,1)$ $\left|\begin{array}{l} t=-1 \\ t=1 \end{array}\right.$

Puesto que $\dfrac{dy}{dx} = \dfrac{2t}{3t^2-1} = \dfrac{\pm 2}{2} = \pm 1$

Los tangentes en $(0,1)$ en $t = \pm 1$ tienen pendientes $\boxed{\pm 1}$

2) $\boxed{\text{ejemplo 3, pág. 715}}$

3) $\boxed{\text{ejerc. 14 (secc 14.4)}}$ $\displaystyle\iint_{x^2+y^2\leq 1} \ln(x^2+y^2)\,dA = \int_0^{2\pi} d\theta \int_0^1 (\ln r^2)\, r\, dr =$

$\displaystyle = 4\pi \int_0^1 r(\ln r)\,dr = 4\pi \left[\frac{r^2}{2}\ln r\right]_0^1 - \frac{1}{2}\int_0^1 r\,dr = 4\pi\left[0 - 0 - \frac{1}{4}\right] = \boxed{-\pi}$

$\left.\begin{array}{l} u = \ln r \\ du = \dfrac{dr}{r} \end{array}\right| \begin{array}{l} dv = r\,dr \\ v = \dfrac{r^2}{2} \end{array}$

4) $\boxed{\text{ejer. repaso 6 (Ejercicios 15.6)}}$ $\boxed{\text{– pág. 992}}$

$\hat{N}\,dS = \dfrac{-i - 2j - 3k}{3}\,dx\,dy$. Si S es la parte del plano en el primer octante, entonces, la proyección de S sobre xy es el triángulo $0 \leq y \leq 3$, $0 \leq x \leq 6 - 2y$. Luego: