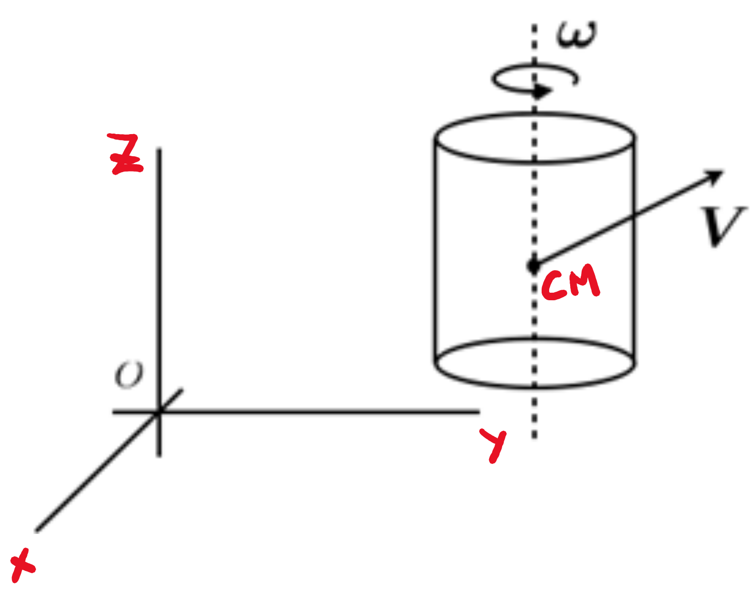
Problema 1 :

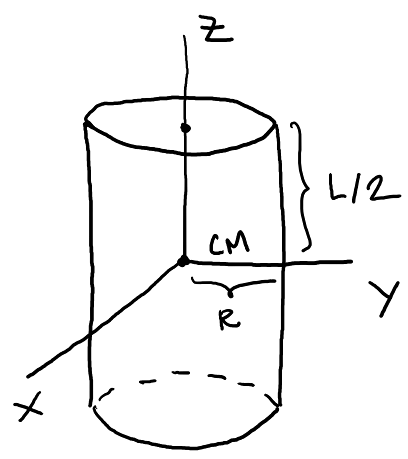


a) $\vec{L} = \vec{L}_{CM} + \vec{L}'$

$\vec{L}_{CM} = \vec{r}_{CM} \times \vec{p}_{CM}$

$$\left.\begin{array}{l} \vec{r}_{CM} = t\,\vec{V} \\ \vec{p}_{CM} = M\,\vec{V} \end{array}\right\} \Rightarrow \vec{L}_{CM} = \vec{0}$$

$\vec{L}' = I_{O'}\,\vec{\omega}_{O'|O}$, $O' = CM$, $\vec{\omega}_{O'|O} = \omega\,\vec{k}$



$$I_{CM} = \begin{pmatrix} \frac{1}{4}MR^2 + \frac{1}{12}ML^2 & 0 & 0 \\ 0 & \frac{1}{4}MR^2 + \frac{1}{12}ML^2 & 0 \\ 0 & 0 & \frac{1}{2}MR^2 \end{pmatrix}$$

$\Rightarrow \vec{L}' = \frac{1}{2}MR^2\omega\,\vec{k}$, $\boxed{\vec{L} = \frac{1}{2}MR^2\omega\,\vec{k}}$.

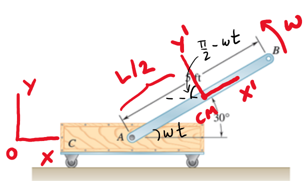
b) $K = K_{CM} + K'$

$K_{CM} = \frac{1}{2}M\vec{v}_{CM}^{\,2} = \frac{1}{2}MV^2$

$$K' = \frac{1}{2}\,\vec{\omega}_{O'|O}\,I_{O'}\,\vec{\omega}_{O'|O} = \frac{1}{2}\begin{pmatrix} 0 & 0 & \omega \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \frac{1}{2}MR^2\omega \end{pmatrix} = \frac{1}{4}MR^2\omega^2 .$$

$\Rightarrow \boxed{K = \frac{1}{2}MV^2 + \frac{1}{4}MR^2\omega^2}$.

Problema 2:



a) $\vec{L} = \vec{L}_{carrito} + \vec{L}_{barra}$

$\vec{L}_{carrito} = \vec{L}_{carrito,CM}$

$\vec{L}_{carrito,CM} = \vec{r}_{CM,carrito} \times \vec{P}_{CM,carrito}$

$$\left.\begin{array}{l} \vec{r}_{CM,carrito} = V \cdot t \, \vec{i} \\ \vec{P}_{CM,carrito} = M \cdot V \, \vec{i} \end{array}\right\} \Rightarrow \boxed{\vec{L}_{carrito,CM} = \vec{0}}$$

$\vec{L}_{barra} = \vec{L}_{CM,barra} + \vec{L}'_{barra}$

$\vec{L}_{CM,barra} = \vec{r}_{CM,barra} \times \vec{P}_{CM,barra}$

$\vec{r}_{CM,barra} = V t \vec{i} + \frac{L}{2} \left[\cos(\omega t) \vec{i} + \mathrm{sen}(\omega t) \vec{j} \right]$ (aquí asumimos que en $t=0$ la barra se encuentra en posición horizontal)

$\vec{P}_{CM,barra} = m \left[V \vec{i} + \omega \frac{L}{2} \left(-\cos(90° - \omega t) \vec{i} + \mathrm{sen}(90° - \omega t) \vec{j} \right) \right]$

$$\Rightarrow \boxed{\vec{L}_{CM,barra}} = V t \, m \, \omega \frac{L}{2} \mathrm{sen}(90° - \omega t) \vec{k} + \frac{L}{2} \cos(\omega t) \, m \, \omega \frac{L}{2} \mathrm{sen}(90° - \omega t) \vec{k}$$
$$- \frac{L}{2} \mathrm{sen}(\omega t) \, m V \vec{k} + \frac{L}{2} \mathrm{sen}(\omega t) \, m \frac{\omega L}{2} \cos(90° - \omega t) \vec{k}$$
$$= \boxed{\frac{VL}{2} m \left[\omega t \cos(\omega t) - \mathrm{sen}(\omega t) \right] + \frac{L^2 m \omega}{4}}$$

$\vec{L}'_{barra} = I_{CM} \vec{\omega}_{AB/O}$, $\vec{\omega}_{AB/O} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}$

$$I_{CM} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12} m L^2 & 0 \\ 0 & 0 & \frac{1}{12} m L^2 \end{pmatrix} \Rightarrow \boxed{\vec{L}'_{barra} = \frac{1}{12} m L^2 \omega \vec{k}}$$

sistema de referencia $x'y'z'$

b) $K = K_{carrito} + K_{barra}$

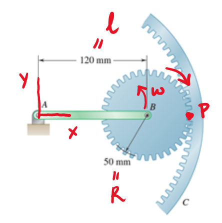
$\boxed{K_{carrito}} = K_{CM,carrito} = \boxed{\frac{1}{2} M V^2}$

$K_{barra} = K_{CM,barra} + K'_{barra}$

$$\boxed{K_{CM,barra}} = \frac{1}{2} m \vec{v}^{\,2}_{CM,barra} = \frac{1}{2} m \left[V \vec{i} + \omega \frac{L}{2} \left(-\cos(90° - \omega t) \vec{i} + \mathrm{sen}(90° - \omega t) \vec{j} \right) \right]^2$$
$$= \frac{1}{2} m \left[\left(V - \frac{\omega L}{2} \mathrm{sen}(\omega t) \right)^2 + \frac{\omega^2 L^2}{4} \cos^2(\omega t) \right]$$
$$= \boxed{\frac{1}{2} m \left[V^2 - V \omega L \, \mathrm{sen}(\omega t) + \frac{\omega^2 L^2}{4} \right]}$$

$$\boxed{K'_{barra}} = \frac{1}{2} \vec{\omega}_{AB/O} \cdot I_{AB} \cdot \vec{\omega}_{AB/O} = \frac{1}{2} (0 \;\; 0 \;\; \omega) \begin{pmatrix} 0 \\ 0 \\ \frac{1}{12} m L^2 \omega \end{pmatrix} = \boxed{\frac{1}{24} m L^2 \omega^2}$$

Problema 3 :



a) $\vec{r}_{CM,AB} = 60\,\vec{i}\ mm$

$\vec{r}_{CM,rueda} = 120\,\vec{i}\ mm$

$$\boxed{\vec{r}_{CM}} = \frac{1}{m_{AB}+m_{rueda}}\left(m_{AB}\,\vec{r}_{CM,AB} + m_{rueda}\,\vec{r}_{CM,rueda}\right)$$

$$= \frac{1}{1.5\ Kg}\left(0.5\ Kg \cdot 60\,\vec{i}\ mm + 1\ Kg\ 120\,\vec{i}\ mm\right)$$

$$\boxed{= 100\,\vec{i}\ mm}$$

b) $\vec{L} = \vec{L}_{barra} + \vec{L}_{rueda}$

$\vec{L}_{barra} = \vec{L}_{CM,barra} + \vec{L}'_{barra}$

$\vec{L}_{CM,barra} = \vec{r}_{CM,barra} \times \vec{P}_{CM,barra}$

$$\left.\begin{array}{l} \vec{r}_{CM,barra} = |\vec{r}_{CM,barra}|\,\vec{i} \\ \vec{P}_{CM,barra} = m\,\vec{v}_{CM,barra} = m\,\omega\,|\vec{r}_{CM,barra}|\,\vec{j} \end{array}\right\} \Rightarrow \boxed{\vec{L}_{CM,barra}} = m\,\omega\,|\vec{r}_{CM,barra}|^2\,\vec{k} = \boxed{1800\,\vec{k}\ Kg\,mm^2/s}.$$

$$\boxed{\vec{L}'_{barra}} = I_{CM}\,\vec{\omega}_{barra|A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}ml^2 & 0 \\ 0 & 0 & \frac{1}{12}ml^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} = \frac{1}{12}ml^2\omega\,\vec{k} = \boxed{600\,\vec{k}\ Kg\,mm^2/s}.$$

$\vec{L}_{rueda} = \vec{L}_{CM,rueda} + \vec{L}'_{rueda}$

$\vec{L}_{CM,rueda} = \vec{r}_{CM,rueda} \times \vec{P}_{CM,rueda}$

$$\left.\begin{array}{l} \vec{r}_{CM,rueda} = l\,\vec{i} \\ \vec{P}_{CM,rueda} = M\,\omega\,l\,\vec{j} \end{array}\right\} \Rightarrow \boxed{\vec{L}_{CM,rueda}} = M\omega l^2\,\vec{k} = \boxed{14400\,\vec{k}\ Kg\,mm^2/s}$$

$\vec{L}'_{rueda} = I_{CM}\,\vec{\omega}_{rotación|CM}$. La velocidad angular de la rueda viene dada por la condición de rodadura sin deslizamiento: $\vec{v}_{P|A} = \vec{v}_{CM,rueda|A} + \vec{\omega}_{rueda|A} \times \vec{r}_{P|CM,rueda} = \vec{0}$

Además, sabemos que $\vec{\omega}_{rueda|A} = \vec{\omega}_{barra|A} + \vec{\omega}_{rotación|CM}$, con $\vec{\omega}_{barra|A} = \omega\,\vec{k}$,

y $\vec{\omega}_{rotación|CM} = |\vec{\omega}_{rot|CM}|\,(-\vec{k})$. Así que

$$\vec{0} = \omega\,l\,\vec{j} + \left(\omega - |\vec{\omega}_{rot|CM}|\right)\vec{k} \times R\,\vec{i} = \left[\omega\,l + \left(\omega - |\vec{\omega}_{rot|CM}|\right)R\right]\vec{j}$$

$$\Rightarrow |\vec{\omega}_{rot|CM}| = \frac{l+R}{R}\,\omega \Rightarrow \vec{\omega}_{rueda|A} = \left(\omega - \frac{l+R}{R}\,\omega\right)\vec{k} = -\frac{l}{R}\,\omega\,\vec{k}$$

Así que
$$\boxed{\vec{L}'_{rueda}} = \begin{pmatrix} \frac{1}{4}MR^2 & 0 & 0 \\ 0 & \frac{1}{4}MR^2 & 0 \\ 0 & 0 & \frac{1}{2}MR^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ -\frac{l+R}{R}\,\omega \end{pmatrix} = -\frac{1}{2}MR(l+R)\omega\,\vec{k} = \boxed{-4250\ Kg\,mm^2/s}.$$

c) $K = K_{barra} + K_{rueda}$

$K_{barra} = K_{CM,barra} + K'_{barra}$

$$\boxed{K_{CM,barra}} = \frac{1}{2}m\,v^2_{CM,barra} = \frac{1}{2}m\left(\omega\,\frac{l}{2}\right)^2 = \frac{1}{8}m\,\omega^2 l^2 = \boxed{400\ Kg\,mm^2/s}$$

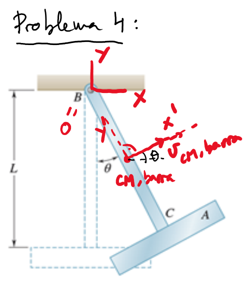
$$\boxed{K'_{barra}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & \omega \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}ml^2 & 0 \\ 0 & 0 & \frac{1}{12}ml^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} = \frac{1}{24}ml^2\omega^2 = \boxed{300\ Kg\,mm^2/s}$$

$K_{rueda} = K_{CM,rueda} + K'_{rueda}$

$$\boxed{K_{CM,rueda}} = \frac{1}{2}M\,v^2_{CM,rueda} = \frac{1}{2}M\,(\omega\,l)^2 = \frac{1}{2}M\,\omega^2 l^2 = \boxed{7200\ Kg\,mm^2/s}$$

$$\boxed{K'_{rueda}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & -\frac{l+R}{R}\,\omega \end{pmatrix}\begin{pmatrix} \frac{1}{4}MR^2 & 0 & 0 \\ 0 & \frac{1}{4}MR^2 & 0 \\ 0 & 0 & \frac{1}{2}MR^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ -\frac{l+R}{R}\,\omega \end{pmatrix} = \frac{1}{4}M(l+R)^2\omega^2 = \boxed{7225\ Kg\,mm^2/s}$$

Problema 4:



a) $\vec{r}_{CM} = \frac{1}{M+m}\left(m\,\vec{r}_{CM,BC} + M\,\vec{r}_{CM,A}\right)$

$\vec{r}_{CM,BC} = \frac{L}{2}\left(\operatorname{sen}\theta\,\vec{i} - \cos\theta\,\vec{j}\right)$

$\vec{r}_{CM,A} = L\left(\operatorname{sen}\theta\,\vec{i} - \cos\theta\,\vec{j}\right)$

$\Rightarrow \boxed{\vec{r}_{CM} = \frac{1}{M+m}\,L\left(\frac{m}{2}+M\right)\left(\operatorname{sen}\theta\,\vec{i} - \cos\theta\,\vec{j}\right)}$

b) $\vec{L} = \vec{L}_{barra} + \vec{L}_{disco}$

$\vec{L}_{barra} = \vec{L}_{CM,barra} + \vec{L}'_{barra}$

$\vec{L}_{CM,barra} = \vec{r}_{CM,barra} \times \vec{p}_{CM,barra}$

$\left.\begin{array}{l} \vec{r}_{CM,barra} = \frac{L}{2}\left(\operatorname{sen}\theta\,\vec{i} - \cos\theta\,\vec{j}\right) \\ \vec{p}_{CM,barra} = m\,\vec{v}_{CM,barra} = m\,\frac{L}{2}\,\dot\theta\left(\cos\theta\,\vec{i} + \operatorname{sen}\theta\,\vec{j}\right) \end{array}\right\} \Rightarrow \boxed{\vec{L}_{CM,barra} = m\,\frac{L^2}{4}\,\dot\theta\,\vec{k}}$

$\boxed{\vec{L}'_{barra}} = I_{CM}\,\vec{\omega}_{barra|B} = \begin{pmatrix} \frac{1}{12}mL^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{12}mL^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \dot\theta \end{pmatrix} = \boxed{\frac{1}{12}\,mL^2\dot\theta\,\vec{k}}$

$\vec{L}_{disco} = \vec{L}_{CM,disco} + \vec{L}'_{disco}$

$\vec{L}_{CM,disco} = \vec{r}_{CM,disco} \times \vec{p}_{CM,disco}$

$\left.\begin{array}{l} \vec{r}_{CM,disco} = L\left(\operatorname{sen}\theta\,\vec{i} - \cos\theta\,\vec{j}\right) \\ \vec{p}_{CM,disco} = M\,\vec{v}_{CM,disco} = m\,L\,\dot\theta\left(\cos\theta\,\vec{i} + \operatorname{sen}\theta\,\vec{j}\right) \end{array}\right\} \boxed{\vec{L}_{CM,disco} = m\,L^2\,\dot\theta\,\vec{k}}$

$\boxed{\vec{L}'_{disco}} = I_{CM,disco}\,\vec{\omega}_{disco|B} = \begin{pmatrix} \frac{1}{4}MR^2 & 0 & 0 \\ 0 & \frac{1}{2}MR^2 & 0 \\ 0 & 0 & \frac{1}{4}MR^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \dot\theta \end{pmatrix} = \boxed{\frac{1}{4}MR^2\dot\theta\,\vec{k}}$

puesto que el disco está "soldado"

c) $K = K_{barra} + K_{disco}$

$K_{barra} = K_{CM,barra} + K'_{barra}$

$\boxed{K_{CM,barra}} = \frac{1}{2}\,m\,v^2_{CM,barra} = \frac{1}{2}\,m\left(\omega\,\frac{L}{2}\right)^2 = \boxed{\frac{1}{8}\,m\,\omega^2L^2}$

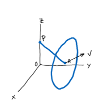
$\boxed{K'_{barra}} = \frac{1}{2}\,\vec{\omega}_{barra|B}\,I_{CM,barra}\,\vec{\omega}_{barra|B} = \frac{1}{2}\left(0\ \ 0\ \ \dot\theta\right)\begin{pmatrix} \frac{1}{12}mL^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{12}mL^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \dot\theta \end{pmatrix} = \frac{1}{2}\left(0\ \ 0\ \ \dot\theta\right)\begin{pmatrix} 0 \\ 0 \\ \frac{1}{12}mL^2\dot\theta \end{pmatrix} = \boxed{\frac{1}{24}\,mL^2\dot\theta^2}$

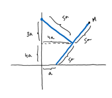
$K_{disco} = K_{CM,disco} + K'_{disco}$

$\boxed{K_{CM,disco}} = \frac{1}{2}\,M\,v^2_{CM,disco} = \frac{1}{2}\,M\left(\omega L\right)^2 = \boxed{\frac{1}{2}\,M\,\omega^2L^2}$

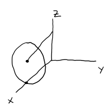
$\boxed{K'_{disco}} = \frac{1}{2}\,\vec{\omega}_{disco|B}\,I_{CM,disco}\,\vec{\omega}_{disco|B} = \frac{1}{2}\left(0\ 0\ \dot\theta\right)\begin{pmatrix} \frac{1}{4}MR^2 & 0 & 0 \\ 0 & \frac{1}{2}MR^2 & 0 \\ 0 & 0 & \frac{1}{4}MR^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \dot\theta \end{pmatrix} = \frac{1}{2}\left(0\ 0\ \dot\theta\right)\begin{pmatrix} 0 \\ 0 \\ \frac{1}{4}MR^2\dot\theta \end{pmatrix} = \boxed{\frac{1}{8}MR^2\dot\theta^2}$

Problema 5:





a) Escogemos un sistema de referencia conveniente:



$$\vec{r}_{CM} = \frac{1}{M+m}\left(m\,\vec{r}_{CM,barra} + M\,\vec{r}_{CM,disco}\right)$$

$$\vec{r}_{CM,barra} = 7a\vec{k} + \frac{1}{2}\left(-3a\vec{k} + 4a\vec{i}\right)$$

$$= a\left(\frac{11}{2}\vec{k} + 2\vec{i}\right)$$

$$\vec{r}_{CM,disco} = 7a\vec{k} + \left(-3a\vec{k} + 4a\vec{i}\right) = 4a\left(\vec{k}+\vec{i}\right)$$

$$\Rightarrow \boxed{\vec{r}_{CM} = \frac{1}{M+m}\,a\left[\left(\frac{11}{2}m + 4M\right)\vec{k} + \left(2m+4M\right)\vec{i}\right]}$$

b) $\vec{v}_{CM} = \frac{1}{M+m}\left(m\,\vec{v}_{CM,barra} + M\,\vec{v}_{CM,disco}\right)$

$$\vec{v}_{CM,barra|O} = \vec{\omega}_{barra|O} \times \vec{r}_{CM,barra|O}$$

$$\left.\begin{array}{l} \vec{\omega}_{barra|O} = \dfrac{V}{4a}\vec{k} \\ \vec{r}_{CM,barra|O} = a\left(\dfrac{11}{2}\vec{k} + 2\vec{i}\right) \end{array}\right\} \Rightarrow \vec{v}_{CM,barra|O} = \frac{V}{2}\vec{j}$$

$$\vec{v}_{CM,disco} = \vec{v}_{A|O} = V\vec{j}$$

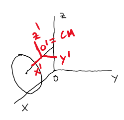
$$\Rightarrow \boxed{\vec{v}_{CM|O}} = \frac{1}{M+m}\left(m\frac{V}{2}\vec{j} + MV\vec{j}\right) = \boxed{\frac{1}{M+m}V\left(\frac{m}{2}+M\right)\vec{j}}$$

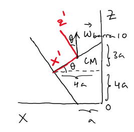
c) $\vec{L}_{eje+rueda} = \vec{L}_{eje} + \vec{L}_{rueda}$

$$\vec{L}_{eje} = \vec{L}_{CM,eje} + \vec{L}'_{eje}$$

$$\vec{L}_{CM,eje} = \vec{r}_{CM,eje} \times \vec{P}_{CM,eje}$$

$$\left.\begin{array}{l} \vec{r}_{CM,eje} = a\left(\dfrac{11}{2}\vec{k} + 2\vec{i}\right) \\ \vec{P}_{CM,eje} = m\,\vec{v}_{CM,eje|O} = m\dfrac{V}{2}\vec{j} \end{array}\right\} \Rightarrow \boxed{\vec{L}_{CM,eje} = am\frac{V}{2}\left(-\frac{11}{2}\vec{i} + 2\vec{k}\right).}$$





$$\vec{L}'_{eje} = I_{CM,eje}\,\vec{\omega}_{barra|O}$$

$$\vec{\omega}_{barra|O} = \frac{V}{4a}\vec{k} = \frac{V}{4a}\left(-\operatorname{sen}\theta\,\vec{i}' + \cos\theta\,\vec{k}'\right)$$

$$= \frac{V}{4a}\left(-\frac{3a}{5a}\vec{i}' + \frac{4a}{5a}\vec{k}'\right) = \frac{V}{20a}\left(-3\vec{i}' + 4\vec{k}'\right)$$

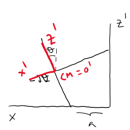
$$\Rightarrow \boxed{\vec{L}'_{eje}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}m(5a)^2 & 0 \\ 0 & 0 & \frac{1}{12}m(5a)^2 \end{pmatrix}\begin{pmatrix} -\frac{V}{20a}3 \\ 0 \\ \frac{V}{20a}4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{5}{12}mVa \end{pmatrix} = \frac{5}{12}mVa\,\vec{k}' = \frac{5}{12}mVa\left(\cos\theta\,\vec{k} - \operatorname{sen}\theta\,\vec{i}\right)$$

$$= \frac{5}{12}mVa\left(\frac{4}{5}\vec{k} - \frac{3}{5}\vec{i}\right) = \boxed{\frac{1}{12}mVa\left(4\vec{k} - 3\vec{i}\right)}$$

$$\vec{L}_{disco} = \vec{L}_{CM,disco} + \vec{L}'_{disco}$$

$$\vec{L}_{CM,disco} = \vec{r}_{CM,disco} \times \vec{P}_{CM,disco}$$

$$\left.\begin{array}{l} \vec{r}_{CM,disco} = 4a\left(\vec{k}+\vec{i}\right) \\ \vec{P}_{CM,disco} = M\,\vec{v}_{CM,disco} = MV\vec{j} \end{array}\right\} \Rightarrow \boxed{\vec{L}_{CM,disco} = 4aMV\left(-\vec{i}+\vec{k}\right)}$$



$$\boxed{\vec{L}'_{disco}} = I_{CM,disco}\cdot\vec{\omega}_{disco|O'} = \begin{pmatrix} \frac{1}{2}MR^2 & 0 & 0 \\ 0 & \frac{1}{4}MR^2 & 0 \\ 0 & 0 & \frac{1}{4}MR^2 \end{pmatrix}\begin{pmatrix} -\frac{V}{20a} \\ 0 \\ 0 \end{pmatrix} = -\frac{1}{40}\frac{MR^2V}{a}\vec{i}' = \frac{1}{40}\frac{MR^2V}{a}\left(-\cos\theta\,\vec{i} + \operatorname{sen}\theta\,\vec{k}\right)$$

$$= \boxed{\frac{1}{40}\frac{MR^2V}{a}\left(-\frac{4}{5}\vec{i} + \frac{3}{5}\vec{k}\right)}$$

por la condición de rodadura sin deslizamiento, vemos que $\vec{\omega}_{disco|O'} = \frac{V}{20a}\left(-\vec{i}'\right)$

d) $K_{eje+rueda} = K_{eje} + K_{rueda}$

$$K_{eje} = K_{CM,eje} + K'_{eje}$$

$$\boxed{K_{CM,eje}} = \frac{1}{2}m\,v^2_{CM,eje} = \frac{1}{2}m\frac{V^2}{4} = \boxed{\frac{1}{8}mV^2}$$

$$\boxed{K'_{eje}} = \frac{1}{2}\vec{\omega}_{eje|O}\cdot I_{CM,eje}\cdot\vec{\omega}_{eje|O} = \frac{1}{2}\left(-\frac{3V}{20a}\;\; 0\;\; \frac{4V}{20a}\right)\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}m(5a)^2 & 0 \\ 0 & 0 & \frac{1}{12}m(5a)^2 \end{pmatrix}\begin{pmatrix} -\frac{3V}{20a} \\ 0 \\ \frac{4V}{20a} \end{pmatrix} = \frac{1}{2}\left(-\frac{3V}{20a}\;\; 0\;\; \frac{4V}{20a}\right)\begin{pmatrix} 0 \\ 0 \\ \frac{5}{12}mVa \end{pmatrix} = \boxed{\frac{1}{24}mV^2}$$

$$K_{disco} = K_{CM,disco} + K'_{disco}$$

$$\boxed{K_{CM,disco}} = \frac{1}{2}M\,v^2_{CM,disco} = \boxed{\frac{1}{2}MV^2}$$

$$\boxed{K'_{disco}} = \frac{1}{2}\vec{\omega}_{disco|O'}\,I_{CM,disco}\,\vec{\omega}_{disco|O'} = \frac{1}{2}\left(-\frac{V}{20a}\;\; 0\;\; 0\right)\begin{pmatrix} \frac{1}{2}MR^2 & 0 & 0 \\ 0 & \frac{1}{4}MR^2 & 0 \\ 0 & 0 & \frac{1}{4}MR^2 \end{pmatrix}\begin{pmatrix} -\frac{V}{20a} \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\left(-\frac{V}{20a}\;\; 0\;\; 0\right)\begin{pmatrix} -\frac{MR^2V}{40a} \\ 0 \\ 0 \end{pmatrix} = \boxed{\frac{1}{1600}\frac{MR^2V^2}{a^2}}$$

e) $\vec{L}_M = \vec{r}_M \times \vec{P}_M$

Pero un punto tiene una masa infinitesimalmente pequeña, así que $\boxed{\vec{L}_M = \vec{0}.}$

f) Por la misma razón, $\boxed{K_M = 0}$