Universidad Carlos III de Madrid
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Escuela Politécnica Superior

Solución

Asignatura Prueba 1 22 Octubre 2015

Nombre del Alumno

Fecha Curso Grupo

(1) $e = 0,71 \quad s = 1,5 \quad r(\theta) = \cos^2\theta \quad \begin{cases} 0<\theta<\pi/2 \\ 0<\phi<\pi/2 \end{cases}$

$$s = \frac{1+|\rho|}{1-|\rho|} \Rightarrow |\rho| = \frac{s-1}{s+1} = \frac{0,5}{2,5} = \frac{1}{5}$$

$$1 - \left|\frac{1}{5}\right|^2 = \frac{24}{25} = 0,96.$$

$$\int \cos^2\theta \,\mathrm{sen}\,\theta \, d\theta \, d\phi = -\frac{\cos^3\theta}{3}\Big|_0^{\pi/2} \phi\Big|_0^{\pi/2} = \frac{1}{3}\cdot\frac{\pi}{2} = \frac{\pi}{6}$$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\frac{\pi}{6}} = 24$$

$$G = 0,71 \cdot \frac{24}{25} \cdot 24 = 16,36 \overset{10\log_{10}}{\Longrightarrow} \sout{11,26\ dB}\ \ 12,14\ dB$$

(2) $\bar{E} = \left[\frac{1-j}{\sqrt{2}}\,\mathrm{sen}\,\theta\,\hat{\theta} + \sqrt{3}\cos\phi\,\hat{\phi}\right]\frac{e^{-jkr}}{r}$ vectorial $\left[\frac{V}{m}\right]$

$$\langle S \rangle = \frac{1}{2\eta}\left[|E_\theta|^2 + |E_\phi|^2\right] = \frac{1}{2\eta}\left(\mathrm{sen}^2\theta + 3\cos^2\phi\right)\frac{1}{r^2}$$ escalar $[W/m^2]$

$$U = r^2\langle S \rangle = \frac{1}{2\eta}\left(\mathrm{sen}^2\theta + 3\cos^2\phi\right)$$ escalar $[W]$

$$r(\theta,\phi) = \frac{U}{U_{max}} = \frac{\mathrm{sen}^2\theta + 3\cos^2\phi}{4}$$

Papel ecológico.

(3) $D_1 = 23\,dB \quad D_2 = 11\,dB$

$$G_1 = 10^{\frac{23}{10}} = 199,53 \qquad G_2 = 10^{\frac{11}{10}} = 12,59$$

$$P_r = G_1 \cdot G_2 \left(\frac{\lambda}{4\pi r}\right)^2 P_t$$

$$P_t = P_r \left[\frac{4\pi r}{\lambda}\right]^2 \frac{1}{G_1 \cdot G_2} = 35 \cdot 10^{-3} \left(\frac{4\pi \cdot 100\lambda}{\lambda}\right)^2 \frac{1}{199,53 \cdot 12,59} =$$

$$= 22\,W$$

$$10 \log_{10} \frac{22\,W}{0,001\,W} = 43,4\,dBm.$$

(4) $RA(dB) = 20 \log_{10} \frac{Eje\ max}{Eje\ min} = 4\,dB \qquad 10^{\frac{4}{20}} \simeq 1,58$

$$\hat{e}_1 = \frac{\hat{\theta} + 1,58\, j\, \hat{\phi}}{\sqrt{1^2 + 1,58^2}} \qquad \hat{e}_2 = \begin{cases} \hat{\theta} & \text{vertical} \\ \hat{\phi} & \text{horizontal.} \end{cases}$$

caso peor $\left| \frac{1}{\sqrt{1^2 + 1,58^2}} \right|^2 = \frac{1}{3,496} \simeq 0,286 \cdot 22\,W \simeq 6,29\,W$

caso mejor $\left| \frac{1,58}{\sqrt{1^2 + 1,58^2}} \right|^2 = \frac{1,58^2}{3,496} \simeq 0,714 \cdot 22\,W \simeq 15,71\,W$

$$\Delta \text{Potencia} \simeq 15,71 - 6,29 \simeq 9,42\,W$$

(5) $A_e = \frac{\lambda^2}{4\pi} G(\theta, \phi) \qquad \lambda_0 = \frac{c}{f} = \frac{3 \cdot 10^8}{30 \cdot 10^6} = 10\,m.$

$$A_{e\,máx} = \frac{10^2}{4\pi}(1,64) = 13,05\,m^2$$

$$P_{rec} = P_{incidente} \cdot A_e = \frac{E_0^2}{2\eta} A_e = \frac{(2 \cdot 10^{-3})^2}{240\pi} 13,05 = 6,92 \cdot 10^{-8} W$$

$$= 69,2\,nW$$

(6) $\theta_{max} = 60°$ 11 lóbulos

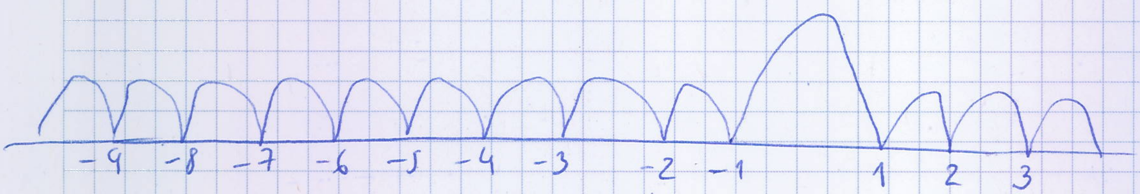
$$u = k\frac{L}{2}\left(\cos\theta - \frac{\beta}{k}\right)$$

$\cos 60° = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ $u = k\frac{L}{2}\left(\cos\theta - \frac{1}{2}\right)$

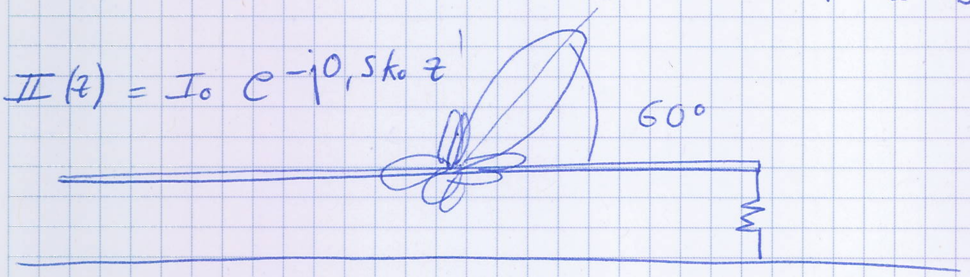
$L = 6\lambda$ $\theta \in [0, \pi]$ $u = \frac{2\pi}{\lambda}\frac{L}{2}\left(\cos\theta - \frac{1}{2}\right)$

$$\left[-\frac{3}{2}\frac{L}{\lambda}\pi, \ \frac{1}{2}\frac{L}{\lambda}\pi\right]$$

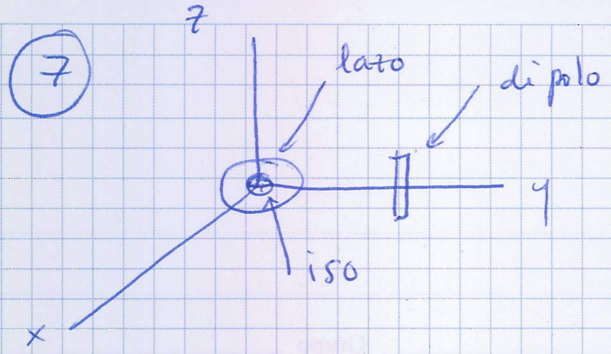
$L = 6\lambda \Rightarrow -9\pi, 3\pi$



$I(z) = I_0 e^{-j0{,}5k_0 z'}$



(7)



$$\bar{E}_{lazo} = E_0 \operatorname{sen}\theta\, \hat{\phi}\, \frac{e^{-jkr}}{r}$$

$$\bar{E}_{dip} = E_0 \operatorname{sen}\theta\, \hat{\theta}\, \frac{e^{-jkr}}{r}$$

$$\bar{E}_{iso} = E_0\, \hat{\phi}\, \frac{e^{-jkr}}{r}$$

$$\bar{E}_{rad} = \left[E_0 \operatorname{sen}\theta\, \hat{\phi} + E_0 \operatorname{sen}\theta\, \hat{\theta}\, e^{jk\frac{\lambda}{2}\hat{y}\cdot\hat{r}} - 2E_0\hat{\phi}\right]\frac{e^{-jkr}}{r}$$

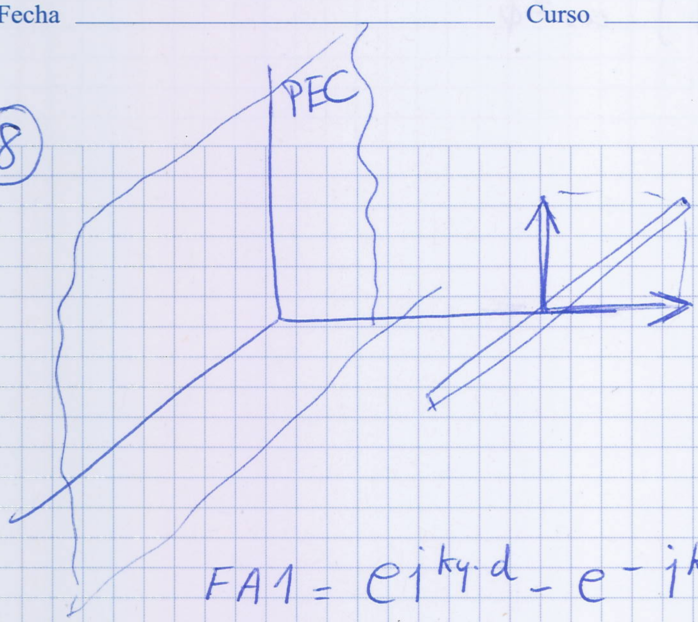
$$= E_0 \frac{e^{-jkr}}{r}\left[(\operatorname{sen}\theta - 2)\,\hat{\phi} + \operatorname{sen}\theta\, e^{j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\operatorname{sen}\theta \operatorname{sen}\phi}\,\hat{\theta}\right]$$

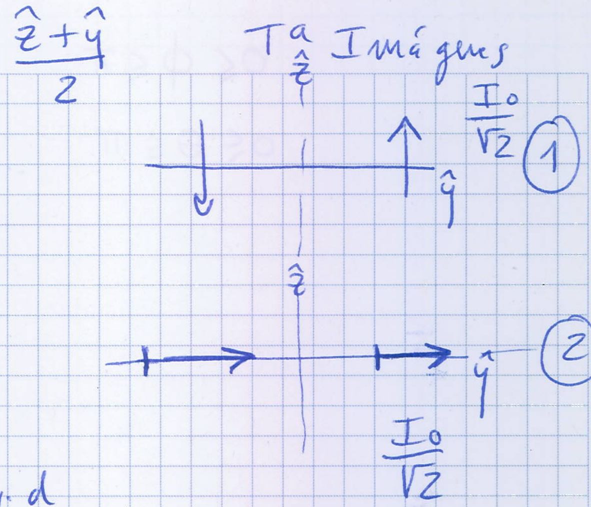
$$U = r^2\langle S\rangle = E_0^2\left[(\operatorname{sen}\theta - 2)^2 + \operatorname{sen}^2\theta\right]$$

$$= E_0^2\left(2\operatorname{sen}^2\theta - 2\operatorname{sen}\theta + 4\right)$$

$$t(\theta,\phi) = \frac{2\operatorname{sen}^2\theta - 2\operatorname{sen}\theta + 4}{4} = \frac{\operatorname{sen}\theta(\operatorname{sen}\theta - 1)}{2} + 1$$

(8)





$$FA1 = e^{jk_y \cdot d} - e^{-jk_y \cdot d} =$$

$$= e^{j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\,\text{sen}\,\theta\,\text{sen}\,\phi} - e^{-j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\,\text{sen}\,\theta\,\text{sen}\,\phi}$$

$$= 2j\,\text{sen}\left(\frac{\pi}{2}\,\text{sen}\,\theta\,\text{sen}\,\phi\right)$$

$$FA2 = e^{jk_y d} + e^{-jk_y d} =$$

$$= e^{j\frac{\pi}{2}\,\text{sen}\,\theta\,\text{sen}\,\phi} + e^{-j\frac{\pi}{2}\,\text{sen}\,\theta\,\text{sen}\,\phi} = 2\cos\left(\frac{\pi}{2}\,\text{sen}\,\theta\,\text{sen}\,\phi\right)$$

Dipolos 1

$$E_1 = \frac{j\omega\mu_o L}{4\pi}\,\frac{I_o}{\sqrt{2}}\,\frac{e^{-jkr}}{r}\,\text{sen}\,\theta\,\hat{\theta}$$

Dipolos 2

$$E_2 = \frac{-j\omega\mu_o L}{4\pi}\,\frac{I_o}{\sqrt{2}}\,\frac{e^{-jkr}}{r}\left[\cos\theta\,\text{sen}\,\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}\right]$$

$$\bar{E}_{total} = \frac{-j\omega\mu_o L}{4\pi}\,\frac{I_o}{\sqrt{2}}\,e^{-jkr}\left\{\left[2j\,\text{sen}\left(\frac{\pi}{2}\,\text{sen}\,\theta\,\text{sen}\,\phi\right)\cdot(-\text{sen}\,\theta)\right]\hat{\theta}\right.$$

$$\left.+\left[2\cos\left(\frac{\pi}{2}\,\text{sen}\,\theta\,\text{sen}\,\phi\right)\cdot\left(\cos\theta\,\text{sen}\,\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}\right)\right]\right\}$$

$$r(\theta,\phi) = \operatorname{sen}^2\left(\frac{\pi}{2}\operatorname{sen}\theta\operatorname{sen}\phi\right)\operatorname{sen}^2\theta + \cos^2\left(\frac{\pi}{2}\operatorname{sen}\theta\operatorname{sen}\phi\right)\cdot\cos^2\theta\operatorname{sen}^2\phi$$

$$+ 2\operatorname{sen}\left(\frac{\pi}{2}\operatorname{sen}\theta\operatorname{sen}\phi\right)\cdot\cos\left(\frac{\pi}{2}\operatorname{sen}\theta\operatorname{sen}\phi\right)\cdot\operatorname{sen}\theta\cos\theta\operatorname{sen}\phi$$

$$+ \cos^2\left(\frac{\pi}{2}\operatorname{sen}\theta\operatorname{sen}\phi\right)\cdot\cos^2\phi$$

$$0 \leq \phi \leq \pi$$

$$0 \leq \theta \leq \pi$$