Universidad Europea

## Introduction Differential Equations

1. Classify the following differential equations:

   $(a)$ $(1-x)y'' - 4xy' + 5y = \cos x$

   $(b)$ $x\frac{d^3y}{dx^3} - 2(\frac{dy}{dx})^4 + y = 0$

   $(c)$ $yy' + 2y = 1 + x^2$

   $(d)$ $x^2dy + (y - xy - xe^x)dx = 0$

   $(e)$ $x^3y^{(4)} - x^2y'' + 4xy' - 3y = 0$

   $(f)$ $\frac{d^2y}{dx^2} + 9y = \sin y$

   $(g)$ $\frac{dy}{dx} = \sqrt{1 + (\frac{d^2y}{dx^2})^2}$

   $(h)$ $\frac{d^2r}{dt^2} = -\frac{k}{r^2}$

   $(i)$ $(\sin x)y''' - (\cos x)y' = 2$

2. Verify that the given function is a solution of each differential equation:

   $(a)$ $2y' + y = 0;\ y = e^{-x/2}$

   $(b)$ $y' + 4y = 32;\ y = 8$

   $(c)$ $\frac{dy}{dx} - 2y = e^{3x};\ y = e^{3x} + 10e^{2x}$

   $(d)$ $\frac{dy}{dt} + 20y = 24;\ y = \frac{6}{5} - \frac{6}{5}e^{-20t}$

   $(e)$ $y' = 25 + y^2;\ y = 5\tan 5x$

   $(f)$ $\frac{dy}{dx}\sqrt{\frac{y}{x}};\ y = (\sqrt{x} + c_1)^2;\ x > 0;\ c_1 > 0$

   $(g)$ $y' + y = \sin x;\ y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$

   $(h)$ $2xydx + (x^2 + 2y)dy = 0;\ x^2y + y^2 = c_1$

3. Determine a rectangle where the following initial value problems have a solution.

$$\begin{cases} \frac{dy}{dx} = y^{2/3} \\ y(1) = 2 \end{cases}$$

$$\begin{cases} \frac{dy}{dx} - y = x \\ y(0) = 1 \end{cases}$$