Separable and Exact differential equations

1. Solve the following separable differential equations:

1. $\frac{dy}{dx} = \sin 5x$

2. $\frac{dy}{dx} = (x+1)^2$

3. $dx + e^{3x}dy = 0$

4. $(x+1)\frac{dy}{dx} = x+6$

5. $e^x \frac{dy}{dx} = 2x$

6. $xy' = 4y$

7. $\frac{dy}{dx} + 2xy = 0$

8. $\frac{dy}{dx} = \frac{y^3}{x^2}$

9. $\frac{dy}{dx} = \frac{y+1}{x}$

10. $\frac{dx}{dy} = \frac{x^2 y^2}{1+x}$

11. $\frac{dx}{dy} = \frac{1+2y^2}{y \sin x}$

12. $\frac{dy}{dx} = e^{3x+2y}$

13. $e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

14. $(4y + yx^2)dy - (2x + xy^2)dx = 0$

15. $x^2 y^2 dy = (y+1)dx$

16. $y \ln x \frac{dx}{dy} = (\frac{y+1}{x})^2$

17. $\frac{dP}{dt} = P - P^2$

18. $\frac{dN}{dt} + N = Nte^{t+2}$

19. $e^y \sin 2x dx + \cos x(e^{2y} - y)dy = 0$

20. $(e^y + 1)^2 e^{-y}dx + (e^x + 1)^3 e^{-x}dy = 0$

21. $2\frac{dy}{dx} - \frac{1}{y} = \frac{2x}{y}$

22. $\frac{y}{x}\frac{dy}{dx} = (1+x^2)^{-1/2}(1+y^2)^{1/2}$

23. $\frac{dy}{dx} = \frac{xy+3x-y-3}{xy-2x+4y-8}$

2. Solve the following initial value problems:

1. $(e^{-y} + 1)\sin x dx = (1 + \cos x)dy$ $y(0) = 0$
2. $ydy = 4x(y^2 + 1)^{1/2}dx$ $y(0) = 1$
3. $\frac{dy}{dt} + ty = y$ $y(1) = 3$

3. Determine if the following differential equations are exact or not. If the equation is exact, solve it.

1. $(2x - 1)dx + (3y + 7)dy = 0$

2. $(2x + y)dx - (x + 6y)dy = 0$

3. $(5x + 4y)dx + (4x - 8y^3)dy = 0$

4. $(\sin y - y \sin x)dx + (\cos x + x \cos y - y)dy = 0$

5. $(2y^2 x - 3)dx + (2yx^2 + 4)dy = 0$

6. $(2y - \frac{1}{x} + \cos 3x)\frac{dy}{dx} + \frac{y}{x^2} - 4x^3 + 3y \sin 3x = 0$

7. $(x + y)(x - y)dx + x(x - 2y)dy = 0$

4. Solve the following initial value problems:

1. $(x + y)^2 dx + (2xy + x^2 - 1)dy = 0$ $y(1) = 1$

2. $(e^x + y)dx + (2 + x + ye^y)dy = 0$ $y(0) = 1$

3. $(4y + 2x - 5)dx + (6y + 4x - 1)dy = 0$ $y(-1) = 2$

5. Calculate the value of k so that the following differential equations are exact:

1. $(y^3 + kxy^4 - 2x)dx + (3xy^2 + 20x^2y^3)dy = 0$

2. $(2x - y\sin xy + ky^4)dx - (20xy^3 + x\sin xy)dy = 0$

3. $(2xy^2 + ye^x)dx + (2x^2y + ke^x - 1)dy = 0$

6. Find a function $M(x, y)$ so that the following differential equation is exact:
$M(x, y)dx + (xe^{xy} + 2xy + \frac{1}{x})dy = 0$

7. Solve the following differential equations using the integration factor $\mu(x, y)$.

1. $6xy dx + (4y + 9x^2)dy = 0$ $\mu(x, y) = y^2$

2. $(-xy\sin x + 2y\cos x)dx + 2x\cos x dy$ $\mu(x, y) = xy$

8. Solve the following non-exact differential equations by making them exact using an integration factor:

1. $(x^2 + 3y^2)dx - 6xy dy = 0$ $\mu(x, y)$ depende solo de x

2. $dx - (e^y + x)dy = 0$ $\mu(x, y)$ depende solo de y

3. $(1 - x^2y)dx + x^2(y - x)dy = 0$ $\mu(x, y)$ depende solo de x