Problem Sheet 6
Waves and Oscillations

1. Determine the period of oscillation of a mass $(\mathrm{m}=250 \mathrm{~g})$ linked to two elastic springs, $\mathrm{k}_{1}=30 \mathrm{~N} / \mathrm{m}$ and $\mathrm{k}_{2}=20 \mathrm{~N} / \mathrm{m}$. Neglect friction. Ans: $\pi / 5 \sqrt{2} \mathrm{~s}$

2. When we hang a mass $\mathrm{m}=1 \mathrm{~kg}$ from a vertical spring, this spring stretches $0,42 \mathrm{~m}$ until it reaches its equilibrium position. Now, we stretch the spring $0,10 \mathrm{~m}$ below this equilibrium point. Finally, we release the mass, leaving it oscillating freely. Neglect friction. a) Calculate the period of oscillation. b) Determine the equation of motion for the mass. Ans: a) $1,30 \mathrm{~s} ; \mathrm{b}) \mathrm{y}(\mathrm{t})=-0,1 \cos (4,8 \mathrm{t}) \mathrm{m}$
3. A mass 20 kg moves along X axis following a Simple Harmonic Motion. At $\mathrm{t}=0$, this mass is located at 4 m from origin ( $x=0$ ), and it has a velocity of $15 \mathrm{~m} / \mathrm{s}$, and an acceleration of $100 \mathrm{~m} / \mathrm{s}^{2}$, both vectors pointing to the origin. Determine: a) Period of oscillation. b) Equation for position at any time. c) Force acting on the mass at $t=\pi / 10 \mathrm{~s}$. Ans: a) $\mathrm{T}=1,26 \mathrm{~s} ; \mathrm{b}) \mathrm{x}(\mathrm{t})=5,0 \cdot \cos (5 \mathrm{t}+0,64) ; \mathrm{c}) \mathrm{F}=1500 \mathrm{~N}$.
4. A body linked to a spring follows a Simple Harmonic Motion (horizontally), with an amplitude of 6 cm . When the mass is at a distance 4 cm from equilibrium position, determine which fraction of the total energy is kinetic energy. Ans.: 5/9
5. A mass $m_{1}, 0,5 \mathrm{~kg}$, is on a horizontal plane, linked to a spring of elastic coefficient $\mathrm{k}=600 \mathrm{~N} / \mathrm{m}$. It is at rest, and no friction is present. Another mass $m_{2}, 0,5 \mathrm{~kg}$, collides elastically with $m_{1}$. As a consequence, the system spring-mass oscillates following a Simple Harmonic Motion of amplitude $0,25 \mathrm{~m}$. Which was the speed of the mass $\mathrm{m}_{2}$ right before the collision? Ans: $8,7 \mathrm{~m} / \mathrm{s}$.
6. The equation of a transverse harmonic wave propagating in a rope is $y(x, t)=0,45 \cdot \sin (3 \mathrm{t}-2 \mathrm{x})$, in S.I. units. Calculate: a) The propagation speed, wavelength and period of the wave. b) The speed of oscillation of a point $P$ located in $x=6 \mathrm{~m}$ at $\mathrm{t}=3 \mathrm{~s}$ and $\mathrm{t}=6 \mathrm{~s}$.
Ans: a) $1,5 \mathrm{~m} / \mathrm{s}, 3,1 \mathrm{~m}, 2,1 \mathrm{~s} ; \mathrm{b}) \mathrm{v}(\mathrm{t}=3)=0$, because at $\mathrm{t}=3 \mathrm{~s}$ the wave has not reached P yet; $\mathrm{v}(\mathrm{t}=6)=1,30 \mathrm{~m} / \mathrm{s}$
7. A harmonic wave propagates from left to right with a speed of $200 \mathrm{~m} / \mathrm{s}$. The amplitude is 4 m and the wavelength is 20 m . Which is the wave equation? Ans: $y(x, t)=4 \cdot \sin (20 \pi t-0,1 \pi x+\varphi) \mathrm{m}$
