Tema 9: Series de números

1. Estudiar la convergencia de las siguientes series y calcular su suma siempre que sea posible

   (a) $\sum_{n=2}^{\infty} \frac{3}{n(n+2)}$
   (Sol: $\frac{5}{4}$)

   (b) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n}\sqrt{n+1}}$
   (Sol: 1)

   (c) $\sum_{n=1}^{\infty} \frac{3n}{\sqrt{n^3+5}}$
   (Sol: divergente)

   (d) $\sum_{n=1}^{\infty} \frac{9^{n+1}}{11^n}$
   (Sol: $\frac{81}{2}$)

   (e) $\sum_{n=1}^{\infty} \frac{3n}{5n+1}$
   (Sol: divergente)

   (f) $\sum_{n=1}^{\infty} \frac{e^n}{2^{n-1}}$
   (Sol: divergente)

   (g) $\sum_{n=0}^{\infty} \frac{3^n}{\pi^{n+1}}$
   (Sol: $\frac{1}{\pi-3}$)

   (h) $\sum_{n=4}^{\infty} \frac{2n}{(n-3)^2}$
   (Sol: divergente)

   (i) $\sum_{n=1}^{\infty} \left(\frac{1}{e^{2n}} + \frac{1}{2n(n+1)} \right)$
   (Sol: $\frac{1+e^2}{2(e^2-1)}$)

   (j) $\sum_{n=1}^{\infty} \log\left(\frac{n}{n+1} \right)$
   (Sol: divergente)

   (k) $\sum_{n=1}^{\infty} \log\left(1 + \frac{1}{n} \right)$
   (Sol: divergente)

   (l) $\sum_{n=1}^{\infty} \frac{3n^2+3n+1}{(n^2+n)^3}$
   (Sol: 1)

2. Estudiar la convergencia de las siguientes series

   (a) $\sum_{n=1}^{\infty} \frac{7^n}{3+11^n}$
   (Sol: converge)

   (b) $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2}$
   (Sol: converge)

   (c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+2}}$
   (Sol: diverge)

   (d) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2n+1}$
   (Sol: diverge)

   (e) $\sum_{n=1}^{\infty} \frac{n\cos^4(n)}{1+n^5}$
   (Sol: converge)

3. Comprobar si las siguientes series alternantes son convergentes o divergentes

   (a) $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4+2}$
   (Sol: converge)

   (b) $\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{3^n}$
   (Sol: converge)

   (c) $\sum_{n=1}^{\infty} (-1)^n \frac{5n-2}{3n+1}$
   (Sol: diverge)

   (d) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$
   (Sol: converge)

4. Estudiar la convergencia de las series siguientes. En caso de serlo, determinar si la convergencia es condicional o absoluta

(a) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$
(Sol: diverge)

(b) $\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!}$
(Sol: convergencia absoluta)

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{\cos(5n)}{5^n}$
(Sol: convergencia absoluta)

(d) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{3n^2+1} \right)^n$
(Sol: convergencia absoluta)

(e) $\sum_{n=1}^{\infty} \frac{(3n)!}{(n!)^3}$
(Sol: diverge)

(f) $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$
(Sol: diverge)

5. Estudiar la convergencia de las siguientes series

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{\sqrt{n}}$
(Sol: converge)

(b) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}-1}{n(\sqrt{n}+1)}$
(Sol: converge)

(c) $\sum_{n=1}^{\infty} \frac{e^{\frac{2}{n}}}{n^3}$
(Sol: converge)

(d) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^3}$
(Sol: converge)

(e) $\sum_{n=1}^{\infty} (\sqrt[n]{3} - 1)^n$
(Sol: converge)

(f) $\sum_{n=1}^{\infty} \frac{1}{n + n \sin^2(n)}$
(Sol: diverge)

(g) $\sum_{n=1}^{\infty} \frac{7^n}{5^n + 6^n}$
(Sol: diverge)

(h) $\sum_{n=2}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}-1}$
(Sol: converge)