Universidad Europea

# Complex Functions

- Write the following functions $f(z)$ like $f(x+iy) = u(x,y) + iv(x,y)$:
  - $f(z) = 2z^2 - 3z + 1$.
  - $f(z) = e^z$.
  - $f(z) = ze^{z^2}$.
- Write the following function $f(x+iy) = u(x,y) + iv(x,y)$ like $f(z)$.
  - $f(xi+y) = 4x^2 + i4y^2$.
  - $f(x+iy) = x + y + i(x^3y - y^2)$.
- Find the following limits:
  - $\text{lím}_{z\to i} \dfrac{z^4-1}{z-i}$.
  - $\text{lím}_{z\to 1+i} \dfrac{z-1-i}{z^2-2z+2}$.
- Show that the following complex functions are analytic.
  - $f(z) = z^3$.
  - $f(z) = e^z$.
  - $f(z) = e^{2xy}[\cos(y^2-x^2) + i\sin(y^2-x^2)]$.
  - $f(z) = \dfrac{y+ix}{x^2+y^2}$, $\forall (x,y) \neq (0,0)$.
- Show that the following functions $u(x,y)$ are harmonic and find $v$ such that $f = u + iv$ is analytic:
  - $u(x,y) = xy^3 - x^3y$.
  - $u(x,y) = y^3 - 3x^2y$.
  - $u(x,y) = \sin(y)\sinh(x)$.
- Calculate the following complex integrals:
  - $\int_C (2 - i + \overline{z})dz$ where $C$ is the line that connects $z_0 = 0$ and $z_1 = 1 - i$.
  - $\int_C z^2 dx$ if $C$ is the circumference of radius 1 and center $(0,0)$.